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Development of practical method for incorporation of elemental damping in inelastic dynamic time history analysis

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ABSTRACT

A practical method for considering elemental viscous damping, rather than the traditional method of damping associated with lumped nodal mass, is developed for use by practitioners wishing to conduct inelastic dynamic time history analysis. It was firstly shown that there are conceptual, as well as implementation, advantages of such a method. The methodology consists of calibrating the elemental viscous damping model response against that of the initial stiffness proportional constant modal damping model with lumped nodal mass. This was successfully undertaken using the first mode elastic free vibration response and a logarithmic decrement approximation. Sometimes elemental damping ratios in excess of 1000% were obtained. Damping ratios in the higher modes were also obtained and were found to be sometimes higher and sometimes lower in the elemental damping model. For elastic time history analysis of structures with different configurations, heights and distributions of element mass subject to a suite of 19 ground motions, the average peak drift calibrated by the method above varied by less than 1.7%. Also, in the case of inelastic analyses with a lateral force reduction factor of about 4 and elastoplastic hysteresis curves, the difference in average peak drift response was up to 8.8%. Since the elemental damping obtained from the calibration depended on the structure and model considered, it is recommended that a first mode

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free vibration calibration of this type be conducted for the elemental damping structural model of interest before undertaking inelastic dynamic time history analysis.

1 INTRODUCTION

Viscous damping is often included in modelling of structures to consider the effect of energy being dissipated which leads to reduction of response amplitude with time during free vibration. Carr (1997) stated that the choice of damping model would have a significant effect on the structure's inelastic response even if the forces and moments in structures are small. Therefore, a guidance for the selection of an appropriate damping model, together with appropriate damping values, is essential for engineers for performing time-history analysis.

However, unlike inertia forces and stiffness forces, for which the mechanism is well understood, the mechanism of damping still remains cryptic and elusive; yet to quantify the damping force, many different approaches to model damping have been proposed. An acceptable damping model should have:

- Damping forces of appropriate magnitude (i.e. not excessively large)
- A small sensitivity to different parameters
- A rational damping magnitude and distribution during inelastic response
- A damping force sign consistent with the direction of velocity
- Easy and fast progressing solution

Of the many proposed damping models, the Rayleigh damping model is popular because of its computational efficiency, however, it may have excessively large damping response in the higher modes if special care is not taken (Carr, 2000, Puthanpurayil et al. 2016). To deal with the problem, Chrisp (1980) suggests that the constant modal-damping method provides constant levels of damping in each mode, but the damping matrix is fully populated making solution times slower. It should be noted that damping models have little physical justification, and this may give rise to many issues in time- history analysis (Rayleigh and Lindsay 1945).

Therefore, from a physical point of view, it has been argued that it is more reasonable to base the model on the element level. Based on this concept, Chrisp (1980) proposed elemental damping. Such an approach only provides damping in the elements of the structure that are deforming. The advantages of the elemental damping are:

- Damping is an the appropriate location.
- Unrealistic damping forces are avoided.
- Damping is easily separated from hysteretic dissipation.
- The approach is consistent with the tangent stiffness effect.
- There is no need to form the damping matrix, and the analysis is simple.

However, the following issues need to be considered when using elemental damping:

- Mass needs to be assigned to all damping elements.
- Appropriate damping ratios for these elements need to be determined.

This project aims to provide guidance to engineers who wish to use elemental damping in time history analysis. In particular, answers are sought to the following questions:

1. How can elemental damping be assigned to a structure to represent behaviour given by the constant modal damping method with nodal mass/damping?

2. Does different element mass distribution affect the required elemental damping?
3. Is the relationship between nodal damping ratio and required elemental damping ratio the same for all structures?
4. How do the elastic and inelastic time history responses of a structure with calibrated elemental damping models compare with those of the same structure with a nodal damping model?
5. Based on the findings above, what are the steps required to select an appropriate value of damping for the element damping model for use in time history analysis?

That the term 'nodal damping' is used is because for the constant modal damping method considered in this paper, the damping actions act on the nodes.

2 METHODOLGY

2.1 Models

The analysis was conducted by using Ruaumoko (Carr 2018) structural dynamic analysis. The software allows easy application of the constant modal damping model (which uses nodal damping), and the Wilson-Penzien elemental damping model which were used in the analyses. Matlab was used to generate structure model input script for Ruaumoko.

Four structures shown in Figure 1 were investigated. The first structure (Model 1) was designed and modelled by Hall (2006), where rotational degrees-of-freedom were restrained on all floors. To investigate the potential effect of building height, a two-storey one-bay moment frame with rotational degrees-of-freedom restraints was also considered (Model 2). Models 3 and 4 are identical to that of Models 1 and 2, with nodal rotation unrestrained. All those models have bilinear material behaviour with the ratio of post-yield stiffness to initial stiffness being 0.03. For ease of reference, models with rotational degrees-of-freedom being restrained at each floor (Model 1 and Model 2) were refereed as shear type model as they would most likely deform in shear and, models with no nodal rotation restraints (Model 3 and Model 4) were called wall type model for they would most likely deform in bending behaving like a wall.

Assigning mass to elements instead of nodes is one of the most important requirements/limitations for an elemental damping model. One of the differences between the models considered in this study versus that from Hall (2006) is that the mass was distributed along the elements rather than lumped at nodes. The stiffness of Model 1 was adjusted to ensure that the fundamental period is identical to that from Hall's (2006) model. The distributed masses and fundamental periods of Models 1 and 2 are shown in Table 1. The stiffness of Models 3 and 4 were also modified such that their fundamental periods also matches that from Models 1 and 2, respectively.

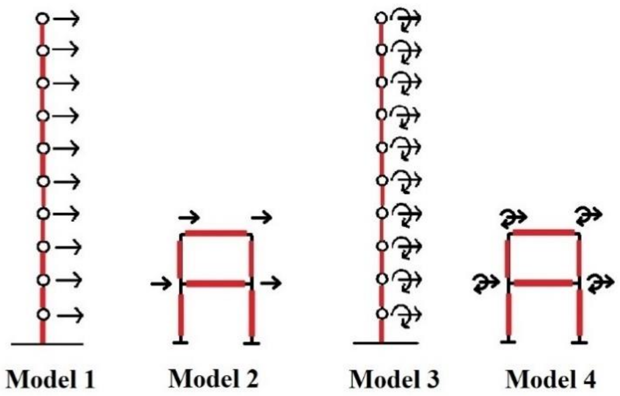

Figure 1: Structure models with mass indicated in red

Table 1: Fundamental periods and distributed masses of Model 1 and Model 2

Model Number	Fundamental Period (s)	Description of Evenly Distributed Member Mass	Description of Lumped Mass at Nodes
Model 1	1.4	100 kN on each member	50 kN at top node, 100 kN at other nodes
Model 2	0.273	50 kN on members in first storey	100 kN at each node
		100 kN on members in second storey	

2.2 Calibration of damping ratio at fundamental mode

In this project, it is assumed that the constant modal-damping model (CDM) is an adequate benchmark model for elastically responding structures. Thus, structural analysis results using CDM can be used to obtain the relationship between damping ratios used in CDM and elemental damping models. This was done by using structural analyses considering (i) ground shaking excitations, and (ii) free vibration response.

In the first approach (Method One), structural analyses using CDM with nodal damping ratios ranging from 0% to 5% were performed for the 19 records (Somerville 1997), and the elastic peak roof displacement relative to the ground was recorded for each case. The analyses were repeated using the elemental damping model to find the elemental damping ratio which results in the same peak peak displacement. Linear interpolation was used on the plot of peak roof displacement versus damping ratio to find the elemental damping ratio that results in the same displacement of using certain nodal damping ratio.

In the second approach (Method Two), the structures were slowly pushed with an inverse triangular force distribution as shown in Figure 2, and then released so that it underwent free vibration. The reason of using an inverse triangular force instead of first-mode-shape force is that inverse triangular force is easier for engineers to conduct analysis and, the response is close enough to the one using first-mode-shape force as shown later in Section 3.3. The equivalent system damping ratio was calculated by using the logarithmic decrement method on top-level displacement response, shown in Equation 1,

$$\delta = \frac{1}{n} \ln \frac{x(t)}{x(t+nT)} \qquad (1)$$

where $x(t)$ is the amplitude at time t, $x\ (t+nT)$ is the amplitude of the peak after n cycles, and T is the period of the free vibration . To keep consistent, the damping ratio was calculated based on number of cycles, n, of ten, although this parameter was also investigated. It is noted that because the elemental damping distribution is not Rayleigh and the inverse triangular force distribution is not exactly the first mode, the logarithmic decrement method does not perfectly describe the free vibration displacement response. However, the logarithmic decrement method was still considered sufficient enough to find the equivalent system damping ratio.

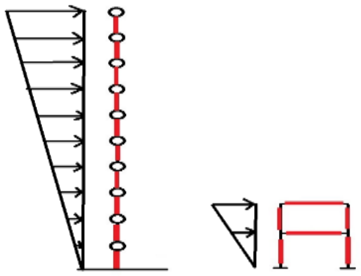

Figure 2: Force distribution

2.3 Determining damping ratios of higher-order modes

After calibration of the damping ratio at fundamental mode, a modified version of the Free Vibration Approach was used to calculate the equivalent system damping ratios corresponding to higher-order modes. This was done by pushing the structure to the mode shape of interest instead of an approximated one, and the mode shape was obtained from Ruaumoko (Carr 2018) using elastic modal analysis.

2.4 Inelastic response evaluation with different damping models

To evaluate the difference in the building's inelastic response due to the use of different damping models, the nodal damping ratio was firstly set to be a specific value and the equivalent elemental damping ratio obtained using Method Two was applied for the elemental damping model. Then elastic and inelastic time history analysis was conducted on all structure models with different damping models using 19 different earthquake excitations used in Method One.

For the inelastic analysis, the strength was defined so that for Model 1 and Model 3, there was no displacement concentration at certain level and for Model 2 and Model 4, the beams yielded on both ends. The earthquake scale factor was defined in Equation 2 to give an equivalent force reduction factor of 4 where F_y is the yield strength of the critical member and F_e is the elastic earthquake demand of the critical member

$$SR = (4 \times F_y)/(F_e) \tag{2}$$

3 RESULTS AND DISCUSSION

3.1 Calibration using earthquake record approach

The elemental damping ratio, which returned the same maximum relative displacement at top node as that from CDM in time history analysis, is illustrated in the graph on the left of Figure 3. It can be seen from the graph that the relationship between the nodal damping ratio and the elemental damping ratio is linear until a nodal damping ratio of 4.4% for Model 3. Also, for Model 1, 1% nodal damping is equivalent to 319% elemental damping and for Model 3, 1% nodal damping is equivalent to 91% elemental damping. This indicates that there are different relationships between nodal damping ratio and elemental damping ratio for different structures.

While a relationship was obtained, Method One was time-consuming and the relationship was sensitive to the number of interpolation points used. This is shown in the graph on the right of Figure 3 where the result would be very different if 30 interpolation points were used instead of 50 points. Because of those limitations, Method One was not used further in this study.

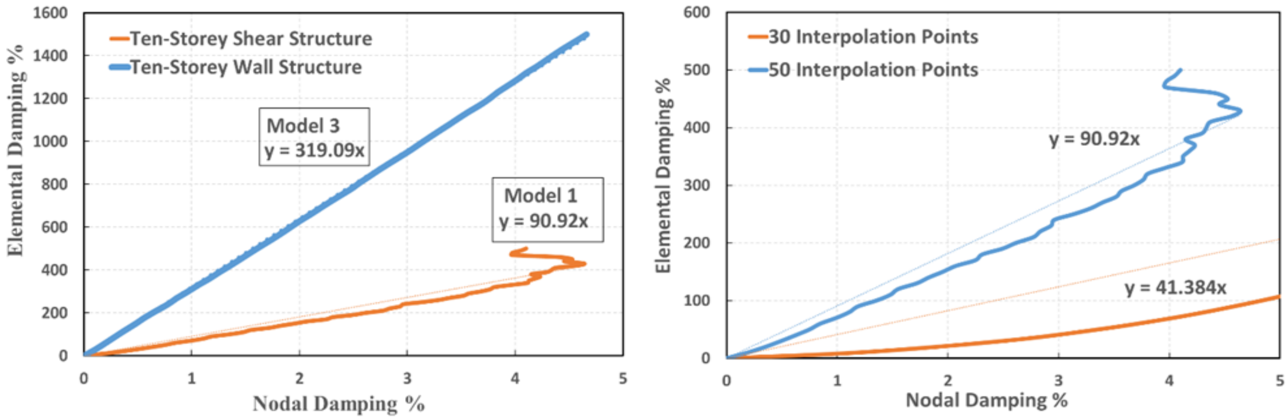

Figure 3: Relationships between nodal damping and elemental damping based on Method One (left) and Model One interpolation points sensitivity

3.2 Calibration using free vibration approach

The relationships for four structures between nodal damping and elemental damping, which were determined by Method Two, are shown in Figure 4.

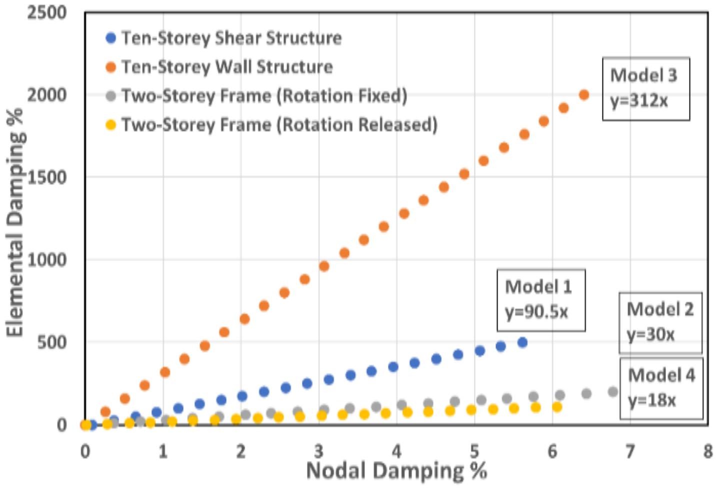

Figure 4: Relationship between nodal and elemental damping based on Method Two (n = 10)

Figure 4 shows that all relationships are linear, and that 1% nodal damping resulted in similar equivalent system damping with 90.5% elemental damping for Model 1, 312% elemental damping for Model 3, 30% elemental damping for Model 2 and 18% for Model 4.

The amount of elemental damping required seems big, but it is not necessarily wrong. The elemental damping model no longer has proportional damping and large damping in the element does not necessarily have the same meaning as it does for a single-degree-of-freedom (SDOF) lumped mass model, especially for a complex structure. The relationship obtained from the analytical results is only a nominal damping ratio ascribed to the structural elements to get a similar nodal damping for the structure system.

By comparing Figure 4 with the graph on the left of Figure 3, it can be seen that the calibration relationships obtained from Method One and Method Two respectively are consistent. This is expected and the reason is that if both damping models have the same equivalent damping ratio in a first mode response (Method Two) then they will have the same energy damping ability (i.e. they damp energy at the same rate). That means when subjected to earthquake excitations, the structure with different damping models will have similar response, especially in the elastic range where the response is first-mode dominant. Method One is to calibrate by equalling the elastic response so the calibration results should be similar.

Figure 4 also indicates that the relationship is significantly different for the shear structures (Model 1 and Model 2) and wall structures (Model 3 and Model 4). This is because the structures have significantly different elastic response. For the shear structure, the elemental damping ratio was 312 times larger than that in constant modal-damping model (CDM). For the wall structure, the elemental damping ratio was 90.5 times larger than that in constant modal-damping model (CDM). Therefore, there is no fixed general relationship between elemental damping and nodal damping ratios.

3.3 Higher mode considerations

After calibrating based on a nodal damping of 5%, Figure 5 shows the damping in higher modes of Models 1, 2, 3 and 4. Two, four and six cycles of free vibration were used in the logarithmic decrement method. The nodal damping base value of 5% on which the calibration was conducted is also shown. Figure 5 indicates that the equivalent system damping of first mode for elemental damping is close to the nodal damping value of 5%. This shows that the response of structures under inverse triangular force distribution used to calibrate the elemental damping is close to the one under the first-mode-shape force.

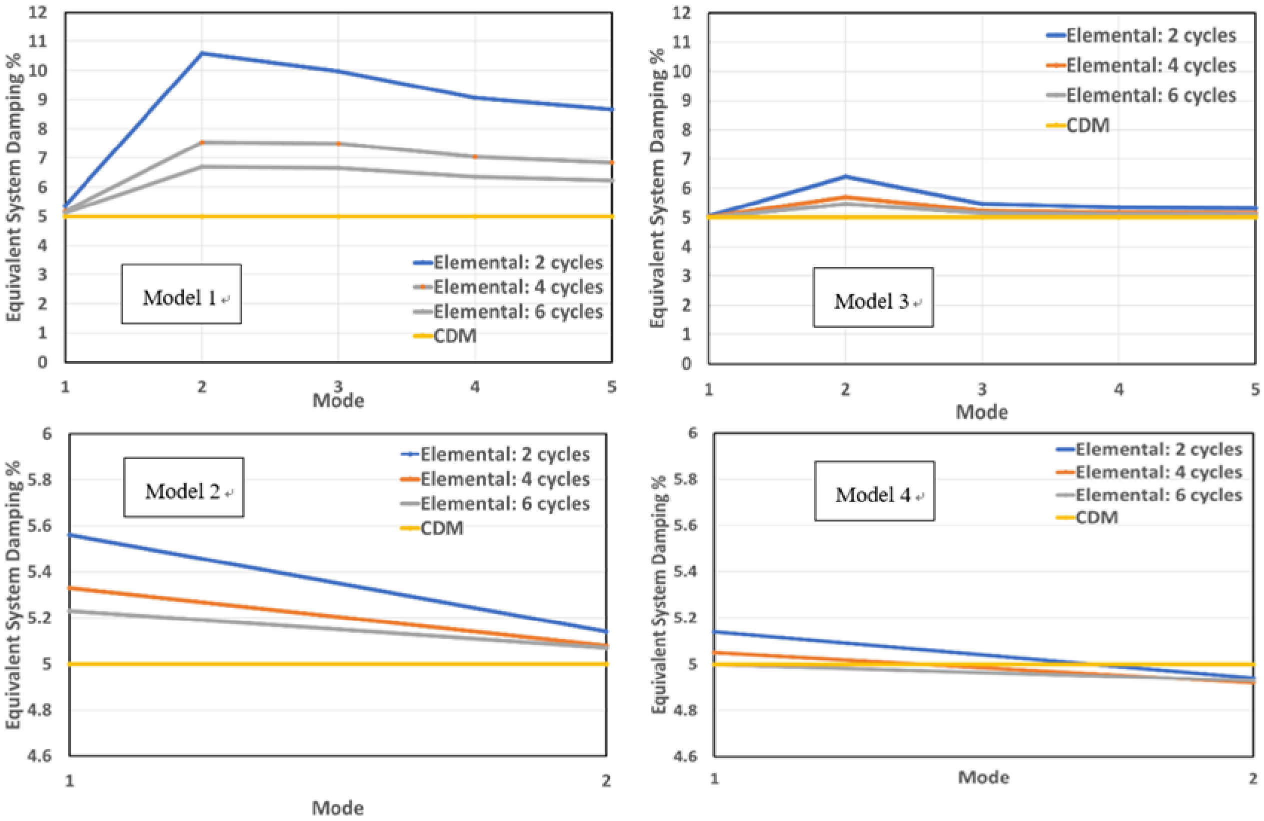

Figure 5: Elemental damping ratio in higher modes for all structure models (CDM damping = 5%)

Figure 5 shows that the damping ratio depends on number of cycles of free vibration used in the logarithmic decrement method. This is expected because the elemental damping is different from the Rayleigh damping, which invalidates the logarithmic decrement rule. Also, other modes are likely to participate during the free vibration, which would affect the results. The figure also shows that the structure has larger damping in first several cycles.

Compared to the CDM for Model 1 and Model 3, the EDM has a larger equivalent damping ratio at the second mode than it does at other modes. For Model 2 and Model 4, the first mode has a higher damping ratio, but the second mode has a slightly smaller value. Therefore, from these results, the peak elastic response for Model 1 and Model 3 is expected to be lower using EDM than that using CMD.

Also, the shear structures (Model 1 and 2) have larger damping at other modes than the wall structures (Model 3 and 4). Therefore, the difference of peak elastic response between the CMD and the EDM is expected to be bigger for the shear structure than the wall structure.

3.4 Time history analysis

With the calibrated elemental damping model (EDM), comparisons of elastic and inelastic time history analyses with the nineteen earthquake records are shown in Figure 6. The CDM nodal damping ratio of 5% and the elemental damping ratio from the calibrated EDM was equally assigned to tall elements.

For Model 1, the EDM average storey drifts are smaller than those from CDM for both elastic and inelastic time history analysis. Differences between the inelastic response with different damping models are larger than that for the elastic response. Those are consistent with the finding in Section 3.3 that the damping ratio for the higher-order modes is larger than 5%. For Model 3, the average storey drift resulting from EDM and CDM in time history analysis are of similar magnitude (i.e. 4% difference in inelastic time history analysis).

It can be also seen from Figure 6 that the elastic response resulted from EDM matches that from CDM for both Model 2 and Model 4 well. For Model 2, the average inelastic storey drift, 1.3%, resulting from CDM is 18% larger than that from EDM, whereas for Model 4 the average inelastic storey drift, 0.85%, from CDM is 8.8% larger than that resulted from EDM. The larger difference between EDM and CDM inelastic response of Model 2 and Model 4 is expected because the higher mode damping effect as discussed in Section 3.3.

In general, the elastic response indicates a good agreement between CDM and calibrated EDM, which validates the calibration method based on first-mode response. The inelastic response follows the findings in Section 3.3 where higher mode damping is larger for the shear type rather than for the wall type structures.

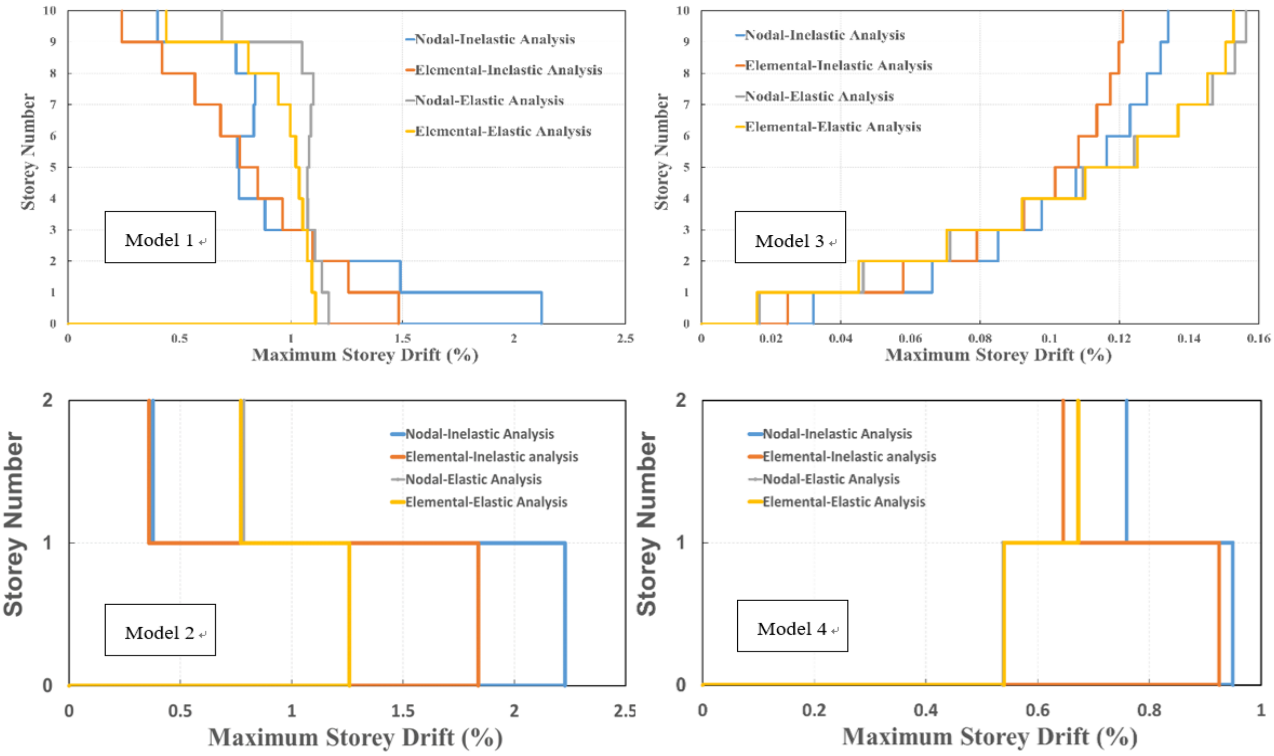

Figure 6: Comparison of elastic and inelastic time history analysis responses using different damping models for all structure models

3.5 Parameter investigation

The effects from magnitude of the pushing force, and from mass distribution on the calibration result were also investigated. The figure in the left of Figure 7 shows free vibration displacement with time of Hall’s structures being pushed to different displacements using the same force increase rate. In both cases, the equivalent system damping is found to be the same, which is 5%. Therefore, the magnitude of pushing force before free vibration is shown to have no effect on the calibration.

To investigate the effect of mass distribution on the calibration result, Model 4 was adapted. The figure in the right of Figure 7 shows that the structure with all mass on the beam requires less elemental damping than the structure, which has mass on both beams and columns, to have the same response as the CDM structure with nodal damping. This shows that mass distribution affects the structure elemental damping. This is expected since the damping is related to the mass matrix. Since beams and columns contribute differently to the decay in structural response due to damping, calibration of the EDM value should be conducted with the damping value that is to be used in the analysis.

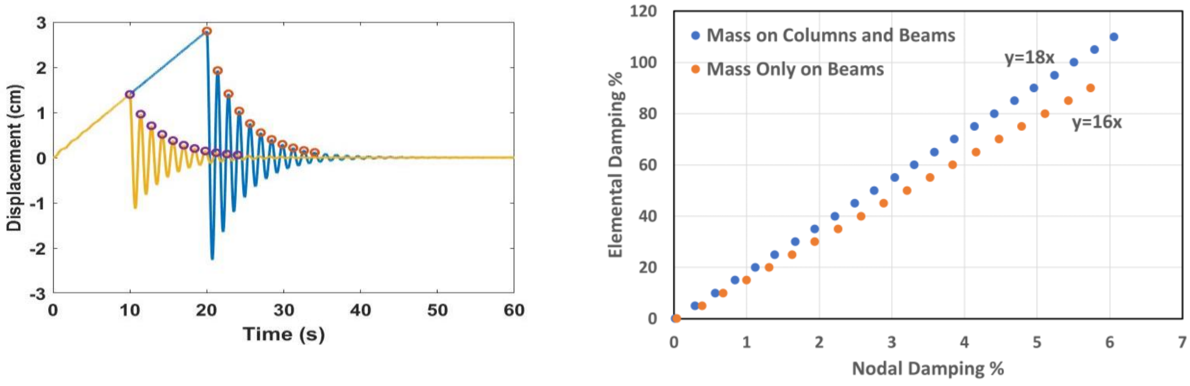

Figure 7: Displacement versus time with various force amplitude (left) and Calibration relationship for Model 4 with different mass distributions (right)

4 ANALYSIS RECOMMENDATIONS

It is recommended that the following method may be used by practitioners and others who wish to use an elemental damping model (EDM) in their time history analyses. This should be done in the following way:

1. Masses should be assigned to the different elements in a realistic way for the EDM.
2. By changing the element damping ratio for the EDM and constant damping model (CDM), the relationship between EDM and CDM damping ratio should be obtained from the free vibration response after release of the structure from an approximately first mode deformed shape using the logarithmic decrement method. Since this relationship is dependent on the structure/model used, it should be found for each structure/model.
3. The appropriate element damping value should be obtained from the relationship between EDM and CDM damping for the CDM damping ratio of interest.
4. Time history analysis may then be conducted using the CDM damping ratio.

5 CONCLUSIONS AND RECOMMENDATIONS

Methods of selecting elemental damping ratios for use in time history analysis are described, and the response between structures modelled with the calibrated elemental damping method (EDM), and with the constant damping method (CDM) are described. It was found that:

1. The EDM value could be obtained using the logarithmic decrement method from the response of structures pushed using the inverted triangular force distribution, and then released to vibrate in free vibration method. The value causing a similar response to that of the CDM with a specific damping ratio can then be obtained.
2. The elements upon which mass is distributed, and the amount of element mass, influences the response.
3. The relationship between CDM damping ratio and EDM damping ration varies significantly for different structures. Therefore, the damping ratio needs to be calibrated for each structure individually.
4. Elastic results indicate a good agreement between CDM and calibrated EDM. In general, for the models studied, the calibrated EDM had lower inelastic response. This was consistent with the fact that the higher mode damping was more generally greater in the EDM than in the CDM.
5. Recommendations for analysis with the EDM are provided.

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