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A novel damping model for earthquake induced structural response simulation

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ABSTRACT

In modelling damping phenomenon in buildings subjected to seismic loading, several proportional viscous damping models are usually considered: Rayleigh, Caughey, and Wilson-Penzien modal damping models. However, the Rayleigh model can only be calibrated to match damping ratios for two modes. The Caughey damping model, which extends the Rayleigh model, allows matching damping ratios for more than two modes, but its damping ratio curve in the frequency domain often oscillates within a short frequency interval, and some part of the curve could fall into a negative region. The Wilson-Penzien modal damping model allows matching damping ratio directly at each mode without forming a damping ratio curve. However, it requires a very costly computation of solving a generalized eigenvalue problem to determine natural frequencies. In this study, a new proportional damping model is proposed. This model gives a smooth curve with negligible oscillation and provides greater flexibility in matching damping ratios for a broad range of frequencies. It is particularly accurate in forming a constant modal damping ratio curve across a practical range of frequencies, a practice commonly used in earthquake engineering communities. The computationally intensive calculation of natural frequencies could be avoided if their range can be reliably estimated, which is often the case. A case study of response history analysis is reported here to showcase the accuracy of the proposed damping model compared to the existing models.

1 INTRODUCTION

In response history analysis of large-scale structures, proportional viscous damping models are commonly used to simulate energy dissipation due to structural damping mechanisms not already accounted for by material hysteretic models, e.g. interactions between structural members and non-structural components (Chopra 2017). Popular proportional models typically considered in the earthquake engineering community include Rayleigh model (Rayleigh 1896), Caughey model (Caughey 1960, Caughey and O'Kelly 1965), and

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Wilson-Penzien model (Wilson and Penzien 1972). Generalized proportional models have recently been proposed by Adhikari (2000, 2006) but are not as popular. In the following, the pros and cons of these models are briefly discussed, particularly for matching a constant modal damping ratio, a practice commonly used in earthquake engineering community for simulating structural dynamic response to seismic loading.

The Rayleigh model matches damping ratios by using two curves, one inversely proportional to the structural circular natural frequency ω in the unit of radian per second (mass proportional term), and the other linearly proportional to ω (stiffness proportional term). These curves can be understood as the basis functions of frequency for the Rayleigh model to form an arbitrary damping ratio curve in the frequency domain as required by structural engineers. With only two basis functions, each with a user-calibrated coefficient, it, however, allows the damping ratios of the structure to be matched only at two modes. The damping ratios at other modes are forced to follow a curve dictated by these two modes. This model is, therefore, very inflexible in matching a damping ratio curve for a structure with more than two modes, let alone a constant modal damping ratio for all the modes. In addition, the resultant damping ratio is infinite at zero frequency and infinite frequency (or zero period). This, however, has non-physical consequences on the resultant damping forces. In particular, as discussed by Chopra and McKenna (2016), it would result in spurious damping forces at degrees of freedom (DOFs) without mass inertia during inelastic response, even if there is no dynamic force applied on those DOFs.

The Caughey model, aka the Caughey series, aims to extend the Rayleigh model with more basis functions. The basis functions are odd power polynomial functions of frequencies with increasing order. The damping ratios of the structure can be matched at as many modes as desired. Unfortunately, this damping model results in a damping ratio curve often oscillating drastically within a short frequency interval, due to the characteristics of high order polynomials. The damping ratios within some frequency intervals could also potentially fall into a negative region. The infinite damping ratio at zero frequency and infinite frequency also remains. In addition, when more and more terms are considered, the matrix used to calculate the coefficients of basis functions becomes more and more ill-conditioned and the calculated coefficients become sensitive to round-off errors.

The Wilson-Penzien model allows a damping ratio to be matched directly at each mode without forming a damping ratio curve, avoiding oscillation and negative values found in the Caughey model. Unfortunately, this model requires a costly computation of solving a generalized eigenvalue problem to determine the natural frequencies, which is particularly undesirable for a large-scale structure with lots of modes. To reduce this computational cost, it is often recommended to determine a smaller set of modes and assign damping ratios to those modes, but it results in undamped response in higher modes. To avoid this undamped response, a stiffness proportional term is usually added as recommended by Clough and Penzien (2003). It would, unfortunately, result in having non-physical spurious damping forces.

The generalized proportional damping models, which aims to extend the Rayleigh model by having generalized basis functions, include square root, trigonometric, hyperbolic, exponential, or logarithmic functions. With these basis functions, these models are much more versatile than the above models in matching arbitrary damping ratios for a wide range of frequencies. Unfortunately, it would require computing matrix functions of matrices $\boldsymbol{M}^{-1}\boldsymbol{K}$, $\boldsymbol{K}^{-1}\boldsymbol{M}$, $\boldsymbol{K}\boldsymbol{M}^{-1}$, or $\boldsymbol{M}\boldsymbol{K}^{-1}$, where $\boldsymbol{M}$ and $\boldsymbol{K}$ are mass and stiffness matrices, respectively, when the damping matrix is formed. It is not always computationally efficient to compute functions of matrices. The computational cost is likely in the same order of solving a generalized eigenvalue problem.

In summary, the existing proportional viscous damping models are either very inflexible in matching modal damping ratios or very costly to form a damping matrix, or both. They also result in spurious damping forces if a stiffness proportional term is included. A recent attempt to have a computationally efficient $\boldsymbol{C}$ matrix is to form this matrix by assembling elemental damping matrices (Puthanpurayil et al. 2016).

Unfortunately, the resultant ***C*** matrix is no longer proportional, making the calibration against global modal damping ratios not necessarily unique and well-defined.

This study sets out to propose a new proportional damping model that overcomes the limitations of the current proportional models, particularly in terms of friendliness in matching desired modal damping ratios at selected or a range of frequencies. The proposed model aims to have the following properties:

- Damping ratios are allowed to be matched at as many frequencies as desired,
- The damping ratios at the two extreme limits of frequencies (zero or infinite) are set to be zero, avoiding spurious damping forces on DOFs without mass inertia and unrealistic damping forces, and
- The resultant damping ratio curve in the frequency domain is to be smooth without large oscillation and is particularly suitable for a constant modal damping ratio across a broad range of frequencies.

In the following, before the proposed damping model is introduced, the performance of the Rayleigh and the Caughey models are discussed in detail about their accuracy in matching damping ratios. It is followed by introducing the proposed damping model and its accuracy in matching damping ratios. This proposed damping model is compared against the Rayleigh and the Caughey models in matching a constant modal damping ratio for a broad range of frequencies. An inelastic response history analysis on a steel frame is also carried out to showcase the accuracy of the model.

In this paper, only key figures will be presented in this paper to showcase the excellent performance of the proposed damping model. Detailed discussions on the formulas and computer implementations of the model have been submitted to a journal and will soon be made available to keen readers.

2 BASIS FUNCTIONS OF RAYLEIGH AND CAUGHEY MODELS

In the frequency domain, the curves of the damping ratio ζ in terms of the structural frequency ω generated by the Rayleigh and the Caughey models are given in Equations 1 and 2, respectively, as

Rayleigh: $\zeta(\omega) = 0.5a_0/\omega + 0.5a_1\omega$ (1)

Caughey: $\zeta(\omega) = 0.5a_0/\omega + 0.5a_1\omega + 0.5a_2\omega^3 + 0.5a_3\omega^5 + 0.5a_4\omega^7 + \cdots$ (2)

where a_i's , i=0,1,2,…, are coefficients calculated by matching damping ratios at user-specified frequencies.

In other words, the functions of $0.5/\omega$ (mass proportional term) and 0.5ω (stiffness proportional term) can be understood as the basis functions of the Rayleigh model, and the functions of $0.5\omega^{2l-1}$, where l is an integer, are the basis functions of the Caughey model, to form an arbitrary damping ratio curve. These basis functions are shown in Figure 1.

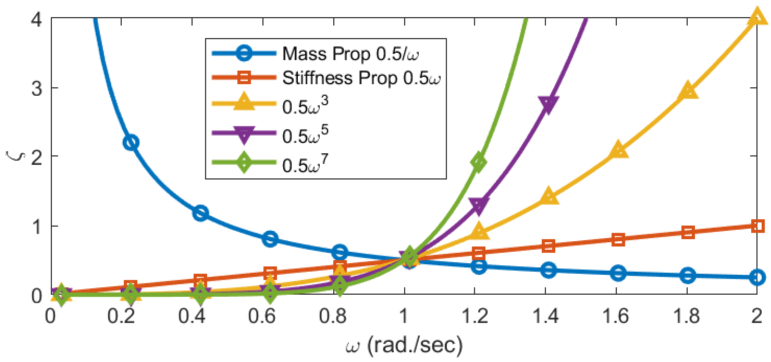
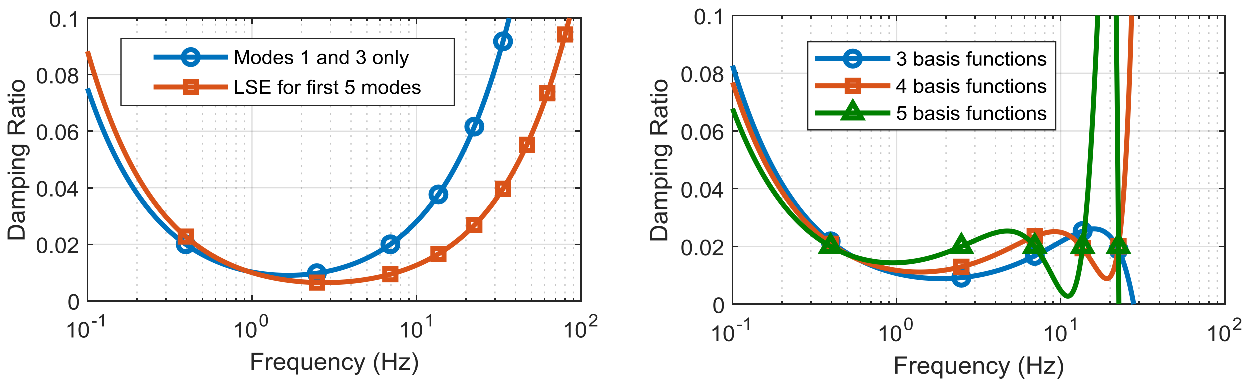


Figure 1: Basis functions for Rayleigh and Caughey models

As shown in Figure 1, as well as already discussed in the introduction section, the basis functions of these models are not ideal in matching damping ratios accurately, because the number of basis functions of the Rayleigh model is not enough, and the basis functions of the Caughey model are problematic. In particular, the basis functions for higher order terms are getting more and more parallel with each other. Their coefficients would be determined by an ill-conditioned matrix and are, therefore, sensitive to round-off errors. In addition, both models result in an infinite damping ratio at ω=0 and $\omega=\infty$, which will lead to spurious and unrealistic damping forces. Both models also could not be used to match a constant modal damping ratio across a broad range of structural frequencies, a common practice typically assumed in the earthquake engineering community.

For example, given a two-dimensional chimney of 100 m height. Its lowest five natural frequencies are given as 0.40 Hz, 2.48 Hz, 6.94 Hz, 13.6 Hz, and 22.5 Hz. These five modes likely govern the dynamic response of the structure subjected to seismic loading. Suppose a constant modal damping ratio of 2% is required to be matched for these five modes using the Rayleigh model. Two options are considered: the damping ratios of modes 1 and 3 are matched to 2%, and the damping ratios of these five modes are matched by using the least squares errors. Figure 2a shows the resultant damping ratio curves matched by using the Rayleigh model.



(a) Rayleigh model

(b) Caughey model

Figure 2: Damping ratio curves matched by using the Rayleigh and Caughey models

As shown in the figure, the constant modal damping ratio is matched very poorly. The damping ratio could be as low as about 0.6% or as high as 6% for the first five modes.

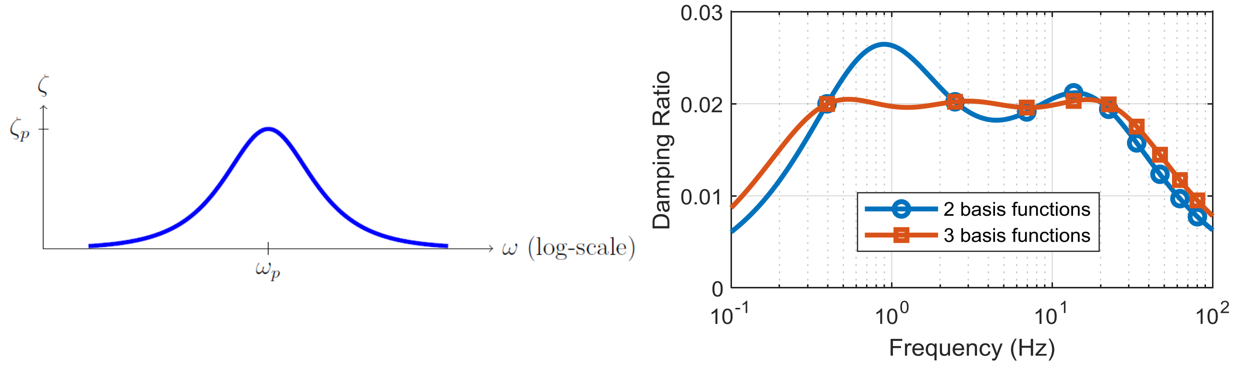
Suppose the damping ratio curve is matched using the Caughey model. The numbers of basis functions considered are 3, 4 and 5. The resultant damping ratio curves are shown in Figure 2b.

The matching quality is even worse, as the damping ratios of higher modes could either fall into a negative region for the case with an odd number of basis functions or grow rapidly towards infinity. To avoid negative damping ratios for higher modes, it is, therefore, a common practice that the number of basis functions must always be even (Clough and Penzien 2003).

3 BASIS FUNCTIONS OF PROPOSED DAMPING MODEL

To solve the above-mentioned problems, a new damping model is proposed here. It has a better basis function shown in Equation 1 and plotted in Figure 3a.

$$\zeta(\omega) = 2\zeta_p \omega_p \omega / \left(\omega_p^2 + \omega^2\right) \tag{3}$$



(a) Basis function *(b) Damping ratio curves matched*

Figure 3: Basis function of the proposed model and damping ratio curves matched

where ζ_p and ω_p are the damping ratio and frequency at the peak of the function, serving as two parameters controlling the value and location of the peak. This basis function is non-negative, is symmetric with respect to the line $\omega = \omega_p$, decays asymptotically to zero at $\omega = 0$ and $\omega = \infty$, and has a constant bandwidth on the logarithmic scale, regardless of the value of ω_p. Note, since it is always non-zero, except at $\omega = 0$ and $\omega = \infty$, the damping ratio curve formed using this basis function will always remain non-zero, except at $\omega = 0$ and $\omega = \infty$. In other words, there will never be undamped response in higher modes, a concern when using the Wilson-Penzien damping model (Clough and Penzien 2003).

The basis function of the proposed model with an ω_p is quite different from another basis function with a different ω_p. This is very different from the case of the Caughey model that has basis functions getting closer and closer with increasing order. In other words, when calculating the coefficients of the basis functions of the proposed model, the matrix used would be well-conditioned, so long as the ω_p's are well separated, in contrast to the ill-conditioned matrix used to compute the coefficients of the Caughey model.

The basis function of the proposed damping model is much more versatile than the basis functions of the Rayleigh and the Caughey models in matching an arbitrary curve of damping ratios. For example, given the same two-dimensional chimney introduced in the previous section, the damping ratio curves using the proposed model with 2 and 3 basis functions fitted with the least squares error method are shown in Figure 3b.

It shows that the relative errors of the damping ratio of the first five modes are as little as about 5%, which is almost negligible. The damping ratios for higher modes also remain positive and are smaller than 2%, an improvement over the damping ratios given by the Rayleigh and the Caughey models.

4 MATCHING CONSTANT MODAL DAMPING RATIO

A constant modal damping ratio across a range of frequencies covering dominant modes, if not all, is usually adopted in a response history analysis of large-scale structures subjected to seismic response. This is considered good practice when there is little to no information about the energy dissipation mechanisms of the structure.

In practice, the range of the frequencies of those dominant modes is not exactly known until their natural frequencies are determined through solving a generalized eigenvalue problem. Unfortunately, solving a generalized eigenvalue problem for a large-scale structure is very costly in computation, even if only the lowest few frequencies are sought for. If the natural frequencies of those dominant modes could be assumed to fall within a possible frequency interval, e.g. the interval between 0.1 Hz and 10 Hz, a practical interval commonly considered in simulating seismic response of a large-scale structure, it is possible to avoid the cost of calculating natural frequencies by matching a constant modal damping ratio curve within that interval. It is, therefore, of interest to know how the Rayleigh, the Caughey, and the proposed damping models would perform in matching a constant modal damping ratio for this frequency interval.

The 2% constant modal damping ratio curves matched by the Rayleigh, the Caughey with 4 basis functions, and the proposed model with 3 basis functions using the least squares error method for this frequency interval between 0.1 Hz and 10 Hz are shown in Figure 4.

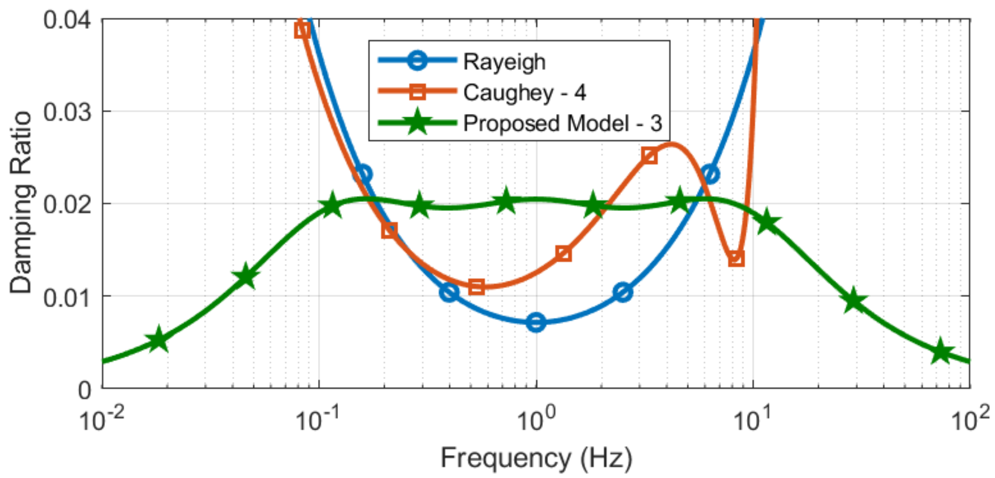


Figure 4: Constant modal damping ratio curves matched by Rayleigh, Caughey and proposed models

As shown in Figure 4, the curves obtained using the Rayleigh and the Caughey models are very poor in matching this constant modal damping ratio. The relative error could be larger than 50%. To reduce this error, it is, therefore, necessary to determine the natural frequencies a priori, and strategically minimise the errors only for those frequency points. In other words, determining the natural frequencies is unavoidable for these two models.

On the contrary, the curve obtained using the proposed damping model matches this constant modal damping ratio with excellent accuracy. The relative error stays 2 to 3% within the interval and reaches 5% only at both ends of the interval. The little oscillation within the interval can always be minimised within an acceptable tolerance by adding a few more basis functions. In fact, the more the basis functions, the better the accuracy. Therefore, the proposed damping model avoids the costly computation of natural frequencies if the range covering them can be reliably predicted.

5 EXAMPLE OF RESPONSE HISTORY ANALYSIS

This example evaluates the accuracy of the proposed damping model in a response history analysis compared to the Wilson-Penzien model. The structural model is a 3-storey 1-bay steel frame shown in Figure 5. All the beams and columns are 610UB125kg/m and 310UC158kg/m, respectively.

The lumped mass at each node is 15.3 tonne. The first four natural frequencies are given in Figure 5. The modal damping ratio is assumed to be 2% for the first four modes. For the proposed damping model, two basis functions are used with the relative error of less than 2% within the range of frequencies covering the first four modes. The damping ratio curves of these two models are shown in Figure 5. Markers are put at the points corresponding to the structural frequencies. The damping matrix is assumed unchanged even when inelastic response occur, according to the suggestion by Chopra (2017).

The material is assumed to be Grade 300 steel with the yield strength of 300 MPa and no hardening. The elements used to model beams and columns are concentrated plastic hinge element with N-M interactions incorporated. The ground motion is the first component of NGA0779 during the 1989 Loma Prieta earthquake with a scaled PGA of 2g.

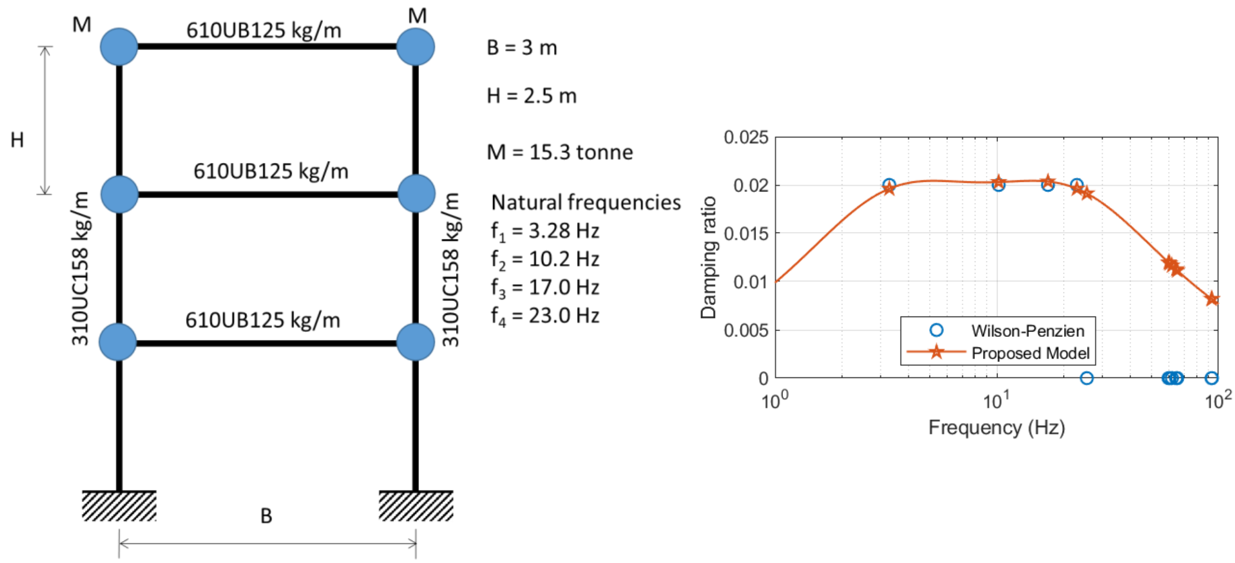


Figure 5: Three-storey 1-bay steel frame and damping ratio curves

The roof displacement time histories using the two damping models are shown in Figure 6. The comparison shows that, for this simple structure where lower modes dominate the response, both models result in nearly identical response for the roof displacement time history, which indicates the proposed damping model performs equally well with the Wilson-Penzien model in terms of accuracy.

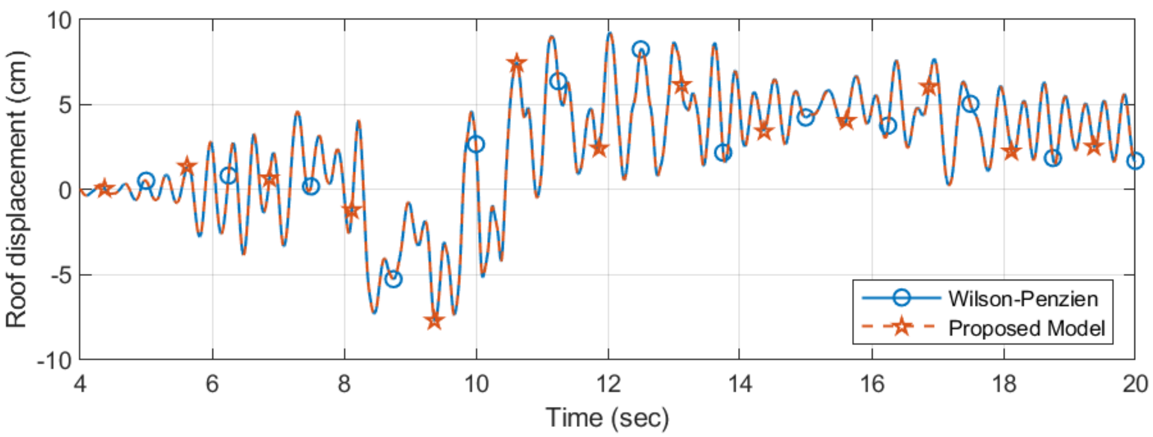


Figure 6: Roof displacement time histories.

6 CONCLUSIONS

A new proportional viscous damping model has been proposed. It aims to overcome the limitations of the current proportional viscous damping models commonly used in the seismic analysis of large-scale structures.

The proposed damping model provides great flexibility in matching damping ratios for a broad range of frequencies with great accuracy. It is particularly accurate for a constant modal damping ratio within a practical range of structural frequencies typically considered in the seismic analysis of large-scale structures. With this remarkable performance, it is possible to avoid the cost of computing natural frequencies, so long as their range can be reliably estimated. Since the basis functions are always independent of each other, so long as the peak frequencies are well separated, the coefficients of the proposed damping model can be calculated with a well-conditioned matrix. An example has been conducted to demonstrate the accuracy of the proposed damping model in a response history analysis of a 3-storey steel frame.

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REFERENCES

Adhikari, S. 2000. *Damping models for structural vibration*. Ph.D. thesis, Department of Engineering, University of Cambridge.

Adhikari, S. 2006. Damping modelling using generalized proportional damping, *Journal of Sound and Vibration*, Vol 293(1-2) 156-170.

Caughey, T. 1960. Classical normal modes in damped linear dynamic systems, *Journal of Applied Mechanics*, Vol 27(2) 269-271.

Caughey, T. & O'Kelly, M.E. 1965. Classical normal modes in damped linear dynamic systems, *Journal of Applied Mechanics*, Vol 32(3) 583-588.

Chopra, A. 2017. *Dynamics of structures: theory and applications to earthquake engineering*, Pearson, Hoboken, NJ, 5th edition.

Clough, R. & Penzien, J. 2003. *Dynamics of structures*, Computers and Structures, Inc, Berkeley, Calif.

MATLAB Optimization Toolbox 2018. *Matlab optimization toolbox*, The MathWorks, Natick, MA, USA.

Puthanpurayil, A.M., Lavan, O., Carr, A.J. & Dhakal, R.P. 2016. Elemental damping formulation: an alternative modelling of inherent damping in nonlinear dynamic analysis, *Bulletin of Earthquake Engineering*, Vol 14(8) 2405-2434.

Rayleigh, J.W.S.B. 1896. *The theory of sound*, Vol. 2. Macmillan.

Wilson, E. & Penzien, J. 1972. Evaluation of orthogonal damping matrices, *International Journal for Numerical Methods in Engineering*, Vol 4(1) 5-10.