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# An Improvement of the Extended N2 Method for Plan-Asymmetric and Torsionally Stiff Structures

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## ABSTRACT

The present paper reports on the performance of the nonlinear static procedures (NSPs), applied to a group of 3D steel structures designed with MRF and CBF as lateral resisting systems, with plan asymmetric configurations. The studied NSPs were the conventional and Extended N2 methods, the Capacity Spectrum method (CSM) and ACSM. The main objective was to assess their effectiveness through a comparison of the results obtained with nonlinear time-history analyses. In terms of translational response, the results indicated that all the NSPs overestimated the response in both directions, being the N2 and CSM methods more accurate than the ACSM. The analysis of the results also indicated that, in general, the Extended N2 method is more appropriate to estimate the torsional response of the studied structures, with the exception of the torsionally stiff structures composed by CBF. To overcome this limitation, an improvement of the Extended N2 method was proposed for this type of structures. It consists on the amplification of the displacements obtained at the flexible edge through the application of a corrective factor that is function of the linear torsional response of the equivalent torsional flexible structure.

## 1 INTRODUCTION

The Nonlinear Static Procedures (NSPs) are a very useful tool recommend by several seismic codes , such as FEMA 440 (ATC, 2005) and Eurocode 8 (CEN, 2004)). These methods are frequently used in regular engineering applications to avoid the inherent complexity and additional computational effort required by the

incremental nonlinear time-history analyses. Nevertheless, most of the part of the existing methods are developed for regular structures and its extension to irregular structures is not straightforward. The aim of this work was to investigate the performance of a designated group of NSPs procedures when applied to different types of plan asymmetric steel buildings composed by moment-resisting frames (MRF) and concentrically-braced frames (CBF). The Nonlinear Static Procedures examined in this work are the original N2 method (CEN, 2004), the Extended N2 method (Fajfar *et al.*, 2005), the Capacity Spectrum Method (ATC, 2005) and the Adaptive Capacity Spectrum Method (Pinho *et al.*, 2007).

## 2 STRUCTURES STUDIED

The study consisted in the assessment of the response of two types plan asymmetric steel structures: torsionally restrained and torsionally unrestrained structures composed by moment-resisting frames (MRF) and concentrically-braced frames (CBF). The plan configuration of the torsionally restrained and unrestrained structures depends on the location of the lateral resisting systems on the outside or inside perimeter of the structures, respectively.

The plan dimensions of the structures are 30m x 18m with a 6m x 4m core in which are located the elevator and stairs. Regarding the elevation profile, the storey height is equal to 4.5m in the first storey and 3.5m in the second and third storeys. These structural configurations can be associated with real industrial buildings. The plan layouts of the structures are illustrated in Figures 2.1 and 2.2 and the 4 cases considered in the study are identified in Table 2.1.

*Table 2.1: Structures designation*

<b>Lateral Resisting System</b>	<b>Torsional Characteristics</b>	<b>Case</b>
Moment-resisting Frames ( <b>MRF</b> )	Torsionally unrestrained and plan-irregular structures	MRF-TU-PI
	Torsionally restrained and plan-irregular structures	MRF-TR-PI
Concentrically- braced Frames ( <b>CBF</b> )	Torsionally unrestrained and plan-irregular structures	CBF-TU-PI
	Torsionally restrained and plan-irregular structures	CBF-TR-PI

The static design followed the Eurocode 3 (CEN, 2005) provisions. The gravity loads comprised the self-weight of the structure (3.93 kN/m<sup>2</sup>), cladding (13.3kN/m) and live load of (2 kN/m<sup>2</sup>). The beam and columns sections used in the design were European IPE and HEB sections, respectively, and the braces were hot finished structural hollow sections. The beams and columns steel grades were S275, while the braces and gusset plates were S355.

The seismic design followed the Extended Improved Forced Based Design (IFBD) method proposed by Peres *et al.* (2016). This procedure consists of a more rational sequence of the design steps prescribed in Eurocode 8 (CEN, 2004) involving the evaluation of the behaviour factor in the two plan directions, instead of the selection of an empirical value based on the ductility class and the lateral resisting system adopted. The seismic action was evaluated as recommended by EC8 (CEN, 2004), assuming a Type 1 response spectrum and soil type B for a peak ground acceleration of 0.3g.

The static and seismic design of the gusset plates was made according to the International Normative CIDECT DG1 (Wardenier *et al.*, 2010), as described by Araújo (2012).

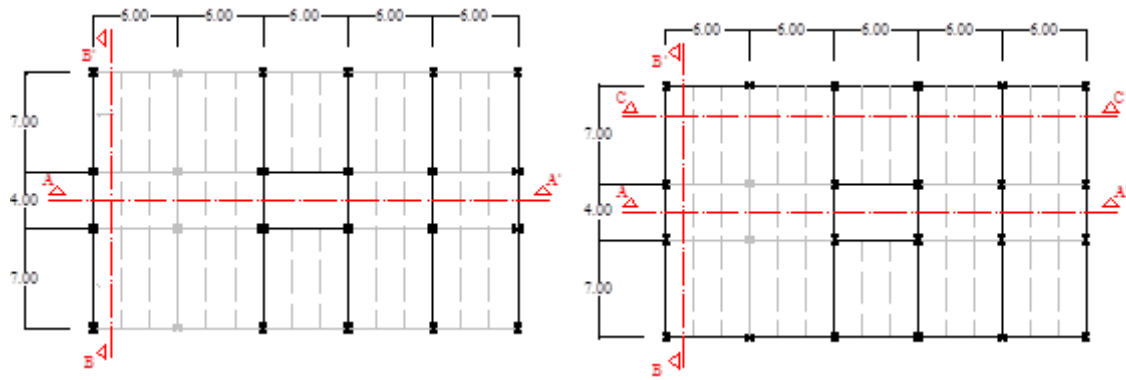


Figure 2.1: Plan view of the MRF-TU-PI and MRF-TR-PI (dimensions in meters)

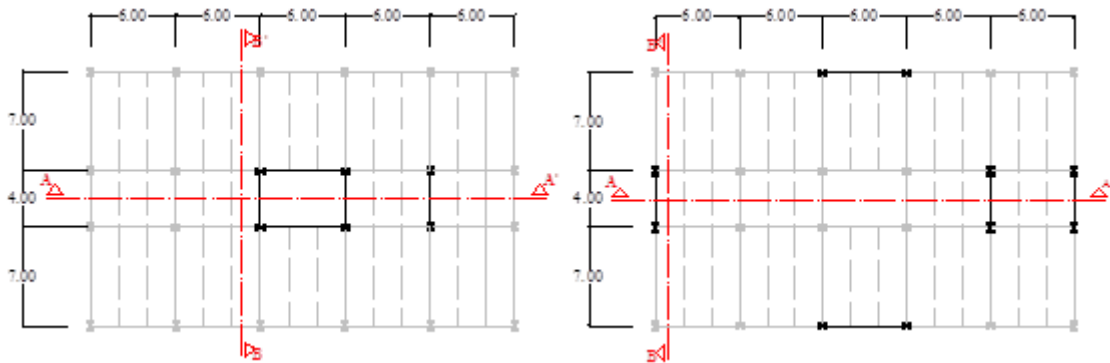


Figure 2.2: Plan view of the CBF-TU-PI and CBF-TR-PI (dimensions in meters)

### 3 NUMERICAL MODELLING AND ANALYSIS PROCEDURES

The seismic assessment of the structures was carried out by means of the selected NSPs and the results obtained compared with results provide by nonlinear time-history analyses. Both the nonlinear static analyses (pushover analysis) and the nonlinear time-history analyses were conducted with the finite element analysis package OpenSEES (PEER, 2006).

The material nonlinear behaviour was considered through a fibre modelling approach for both types of structures (MRF and CBF). Force-based elements were applied using a single element for beam and column members. The braces were modelled with two force-based elements, allowing the consideration of an initial geometrical imperfection with values within the range of 0.1 % to 1.5 % of the brace length. The number of integration points varied from four to ten, depending of the element and model that was being analysed. This modelling approach allows for the yielding of cross sections as well as the flexural buckling of the braces, which are the main factors contributing to stiffness and strength degradation effects in CBFs.

The connections in the MRF were not explicitly modelled since they were considered to be non-dissipative components in the design process. A centreline modelling approach was applied and therefore the panel zones were not discretely modelled.

Regarding the material model, a simplified bilinear stress-strain constitutive relationship was assumed for all elements of the MRF structures and beams and columns of the CBF structures. The Giuffre-Menegotto-Pinto steel material with isotropic strain hardening, designated as Steel 02 in OpenSEES, was adopted for the braces and gusset plates of the CBF structures. The strain hardening ratio considered was 1% for both material models.

The geometrical nonlinearities were included in the analyses through the application in the columns of the P-Delta transformation available in OpenSEES.

The conventional pushover analyses were performed assuming a lateral force load vector following the first mode of vibration of the structures, whereas the adaptive pushover analyses were carried out through the application of a lateral displacement load vector that is dependent on the actual deformed pattern calculated at each step of the analysis (Pinho et al., 2007). The loads were applied independently in the two horizontal directions with positive/negative signs, resulting in four different nonlinear static analyses. The initial lateral displacement load vector is a three-component displacement vector, which explicitly incorporates the torsional characteristics of the structures through the addition of the orthogonal and rotational load components in the direction of the analysis (Adhikari, 2010). The consideration of all component sign combinations requires the conduction of sixteen adaptive pushover analyses.

The seismic input for the nonlinear time-history analysis consisted of fifteen records (Figure 3.1) obtained from real earthquake events. The selection of the ground motion records was made with the SeIEQ tool (Macedo and Castro, 2017) following the recommendations of Araújo et al. (2016). The records were firstly pre-selected from the PEER's database based on geophysical parameters, considering both horizontal components, and after scaled (both components of the record with the same scale factor) in order that the average of the group has a good match with the Eurocode 8 spectrum (Type 1; Soil type B, PGA=0.30g) in the period range of interest

The nonlinear time-history analyses were conducted with the group of records previously mentioned. The two horizontal record components are considered acting simultaneously, first on the x and z directions (AA' and BB' of Figure 2.1, respectively). After that, the two components are rotated by 90 degrees and applied on the z and x directions, resulting in a final set of 30 nonlinear dynamic analyses.

The viscous damping was modelled using the Rayleigh damping formulation, considering a damping matrix proportional to the tangent stiffness. A damping coefficient equal to 2.5% was assumed for the first mode (Peres, R., 2010).

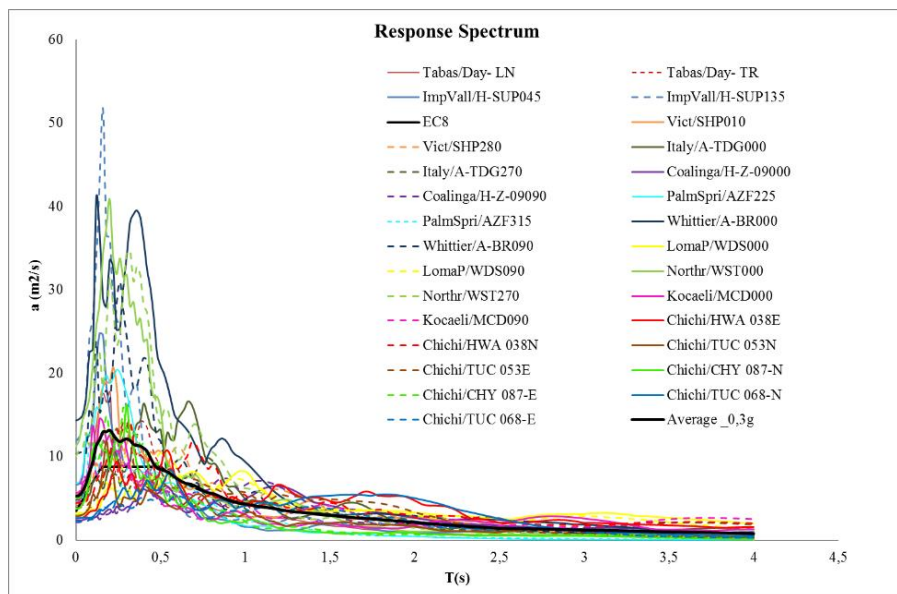


Figure 3.1: Average and targeted elastic response spectrum of the ground motion records

Regarding the application of the NSPs, an elastic-plastic perfectly plastic (EPP) bilinear representation of the capacity curve was assumed. The CSM iterative process applied corresponds to procedure B (ATC, 2005) and

the choice of the expressions to compute the ACSM equivalent damping and the spectral reduction factor is based on the work of Pinho et al. (2007, 2013) and Monteiro et al. (2014). The equivalent damping is computed using the approach proposed by Gulkan and Sozen (1974), which is based on the Takeda model without hardening, while the spectral reduction factor is obtained using the Lin and Chan (2003) relationships.

The gusset plates were modelled as a force-based element where the element cross-section is defined by the thickness ( $t_w$ ) and effective width ( $b_w$ ) of the plate and the length of the element is taken equal to  $2 * t_w$ .

The rigidity of the gusset plate, the gusset-to-beam and gusset-to-column connections are modelled using rigid elastic elements as proposed by Hsiao et al. (2012, 2013).

## 4 RESULTS OF THE ANALYSIS

The global behaviour of the structures was qualitatively assessed through the comparison of the pushover curves obtained using the conventional and adaptive pushover analyses with the results from incremental nonlinear time-history analysis. The positive and negative signs considered in the conventional analyses result in coincident curves, therefore no reference is made regarding the sign considered in the analysis. However, the adaptive pushover curves, which are obtained with a three components vector, depend on the sign combinations considered in each displacement vector, resulting in a variety of curves. The results presented for x and z directions correspond, respectively, to directions AA' and BB' indicated in Figures 2.1 and 2.3.

The nonlinear time-history results are presented in terms of the mean of the maximum displacements obtained at the centre of mass at the roof level and the corresponding values of base shear:

i) obtained at time step  $t$  corresponding to the occurrence of the maximum top displacement,  $V_{dmax}$ ;

ii) the maximum value of base shear,  $V_{max\_dmax}$ , found on the interval  $[t-0.5s, t+0.5s]$ , where  $t$  is the time step in which the maximum top displacement is attained (Antoniou and Pinho, 2004).

The seismic responses of the structures are estimated for four levels of peak ground acceleration, namely 0.15g, 0.3g, 0.45g and 0.60g and not for the range of top displacement values that were considered in pushover analyses. The intensity level of 0.30g corresponds to the design intensity and the 0.15g level corresponds to the serviceability limit state (where the structural response is largely linear elastic). The values of 0.45g and 0.60g were selected in order to assess the structural response in the nonlinear range. Nevertheless, due to numerical convergence problems related with the values of the geometric imperfections adopted, it was not possible to get results for some structures, specifically for the CBFs and the intensity levels of 0.45g and 0.60g.

To evaluate the nonlinear static procedures (the conventional and Extended N2 method, CSM and ACSM), inter-storey drifts and torsional responses were evaluated for the same levels of seismic intensity considered in the nonlinear time-history analyses.

### 4.1 Pushover Curves

The capacity (pushover) curves and the results from the nonlinear time-history analyses obtained for the examined structures are presented in Figures 4.1 to 4.4.

The pushover curves obtained with the conventional and adaptive pushover methods for the MRF structures (Figures 4.1 and 4.2) suggests that the conventional pushover curves provide an upper bound estimate and the adaptive pushover curves provide lower bound estimates of lateral strength. Moreover, the adaptive pushover analysis always leads to the lowest pushover curves, being the difference between the pushover curves resistance more significant for the torsionally unrestrained structures (more flexible structures).

The results obtained for the CBF structures (Figures 4.3 and 4.4) indicate that the adaptive pushover analysis is able to capture the dynamic behaviour of the structures, which mainly depends on the response of the braces. It is interesting to note that the expected softening response, due to brace instability, is captured

much better by the adaptive pushover analysis. Regarding the different adaptive pushover curves obtained, the results are identical to those obtained for the MRF structures.

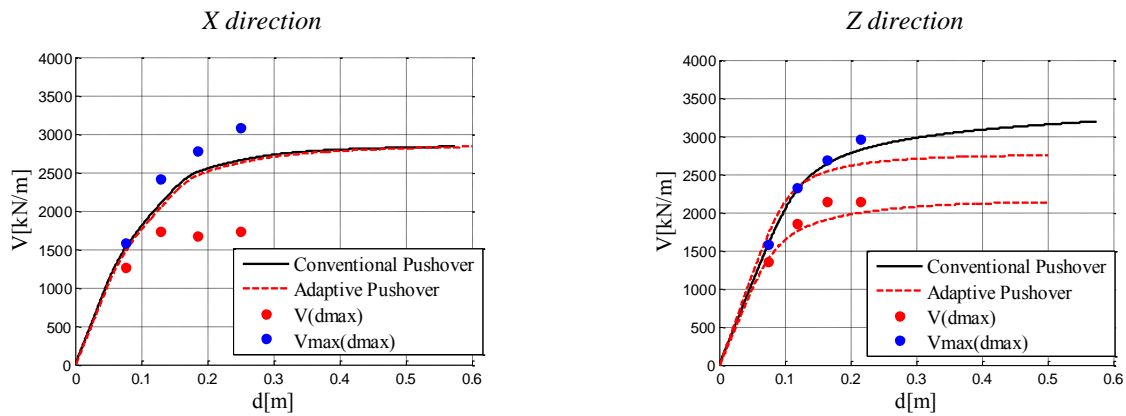


Figure 4.1: Pushover Curves and time-history results obtained for the MRF-TU-PI structure

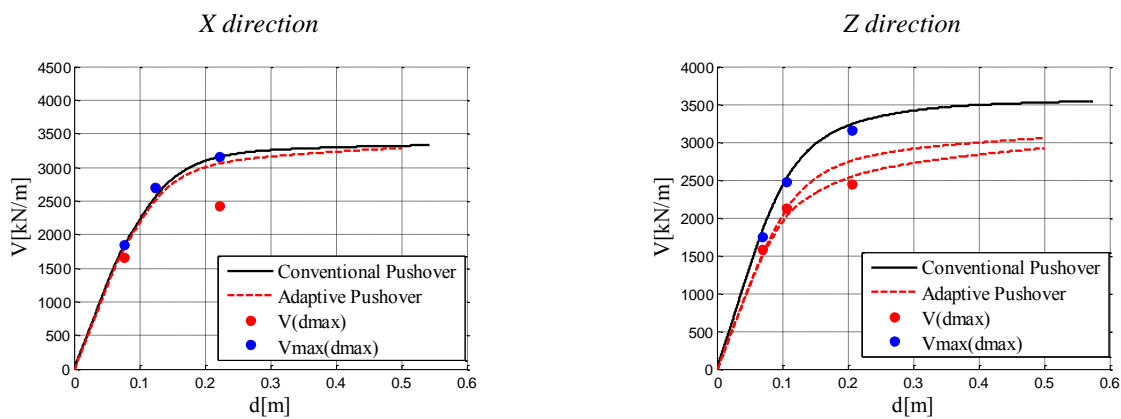


Figure 4.2: Pushover Curves and time-history results obtained for the MRF-TR-PI structure

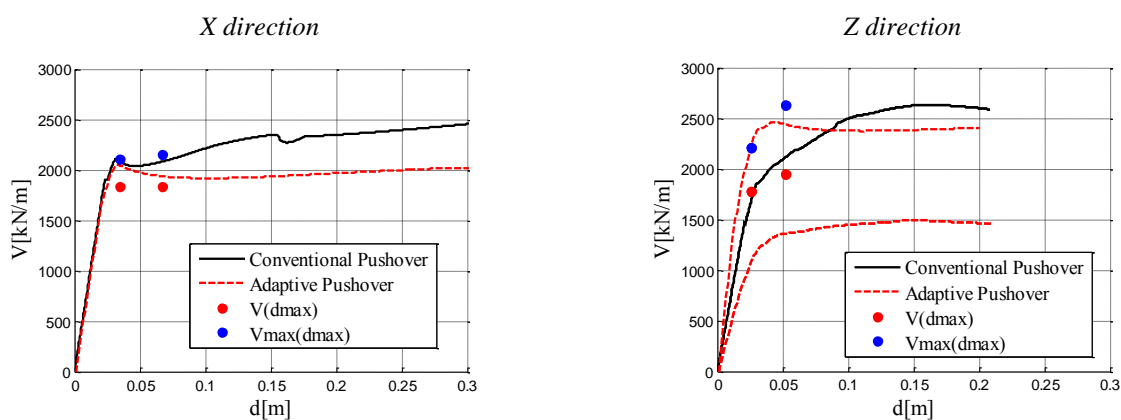


Figure 4.3: Pushover Curves and time-history results obtained for the CBF-TU-PI structure

X direction

Z direction

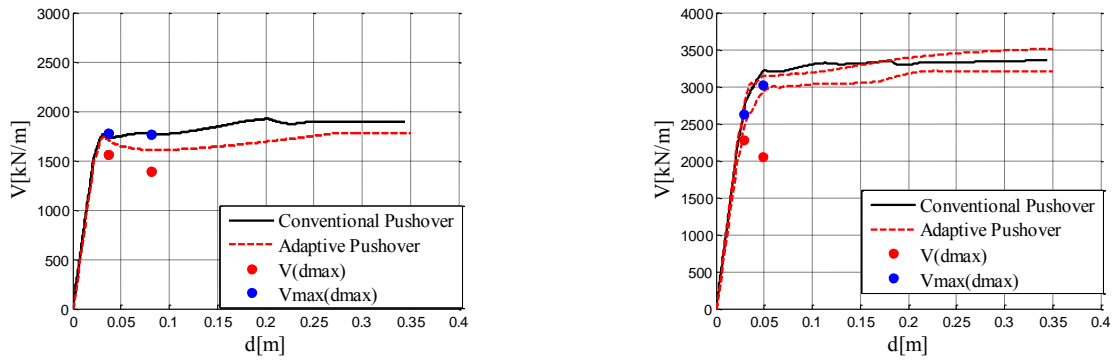


Figure 4.4: Pushover Curves and time-history results obtained for the CBF-TR-PI structure

## 4.2 Interstorey drifts

To assess the performance of the NSPs, the target displacements and the corresponding inter-storey drifts, measured at the centre of mass, were evaluated for the same levels of seismic intensity considered in the nonlinear time-history analyses, and compared with the inter-storey drifts obtained with the latter. The target of the nonlinear time-history results is given by the mean of the maximum top displacements and corresponding inter-storey drifts (TH), while the values of the mean plus and minus the standard deviation (TH  $\pm$  SD) are given to provide the reader additional information about the dispersion of the results.

Figures 4.5 to 4.8 illustrate the inter-storey drifts obtained for the MRF structures for the intensity levels of 0.30g and 0.60g. The analysis of the figures indicates that, for both intensities, all the NSPs overestimate the seismic response in both directions, both the N2 methods and the CSM being more accurate than the ACSM. An exception to this trend is the set of results obtained for the design intensity level in the z direction, where the three procedures provide very close estimates of the inter-storey drifts.

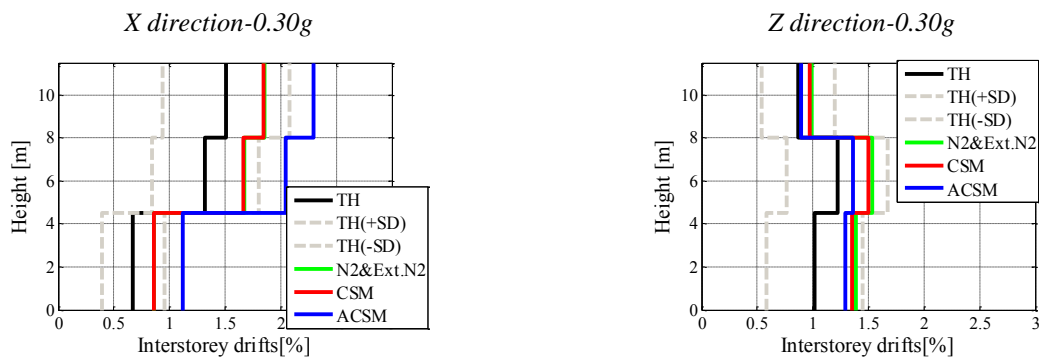


Figure 4.5 – Inter-storey drifts obtained for the MRF-TU-PI structure for the intensity level of 0.3g

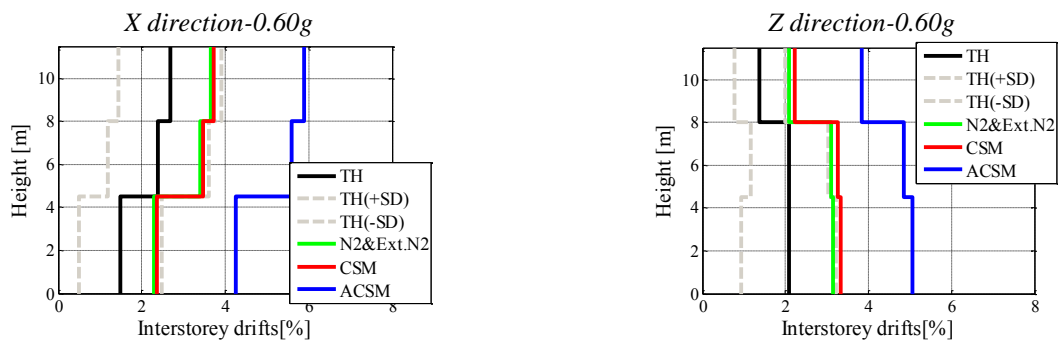


Figure 4.6 – Inter-storey drifts obtained for the MRF-TU-PI structure for the intensity level of 0.6g

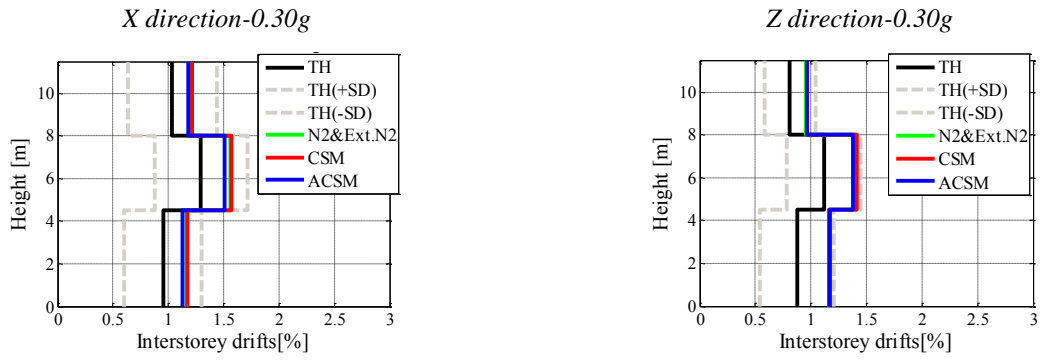


Figure 4.7 – Inter-storey drifts obtained for the MRF-TR-PI structure for the intensity level of 0.3g

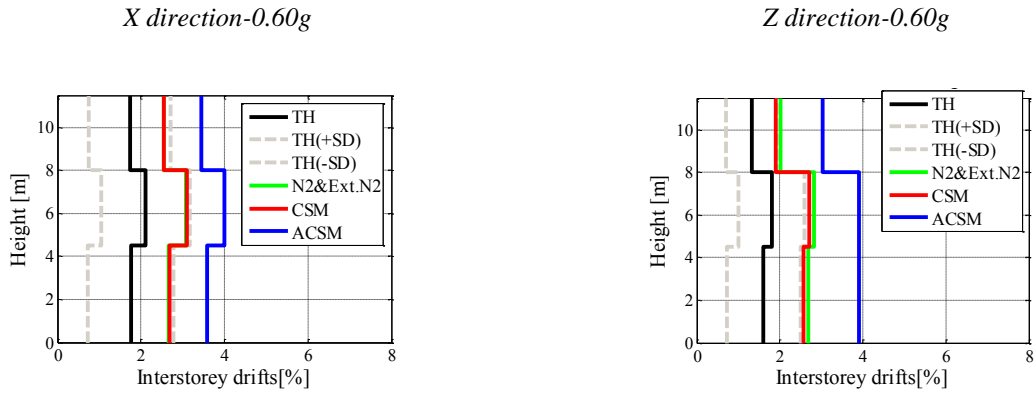


Figure 4.8 – Inter-storey drifts obtained for the MRF-TR-PI structure for the intensity level of 0.6g

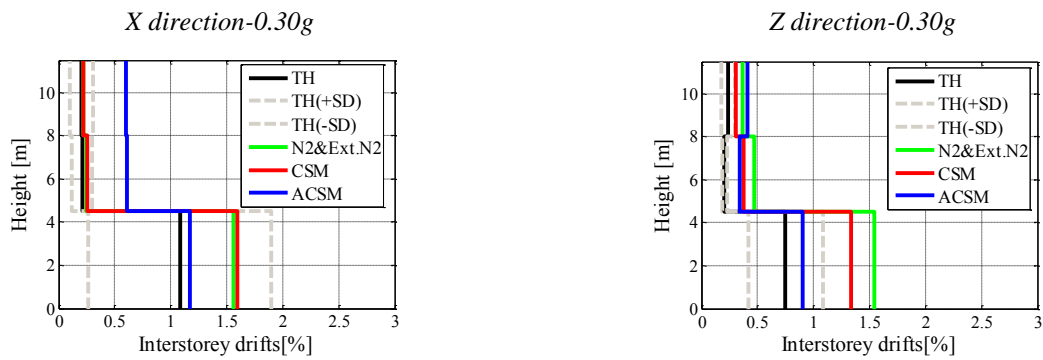


Figure 4.9 – Inter-storey drifts obtained for the CBF-TU-PI structure for the intensity level of 0.3g

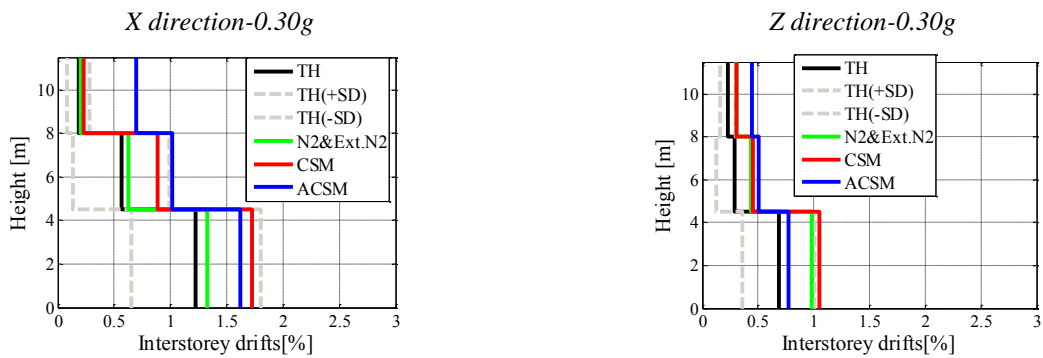


Figure 4.10 – Inter-storey drifts obtained for the CBF-TR-PI structure for the intensity level of 0.3g

Figures 4.9 and 4.10 show the inter-storey drifts obtained for the CBF structures for the intensity level of 0.30g. The results obtained indicate that, in general, N2 methods and the CSM provide very accurate estimates of the seismic response of the studied frames in both directions, at second and third storey levels, while the ACSM provides accurate approximations at the first storey level. The ACSM is the only that provides a good estimate of the distribution of inter-storey drifts for the CBF-TU-PI structure, in the direction of the eccentricity.

### 4.3 Torsional responses

The torsional responses were estimated through the normalization of the edge displacements at the roof level with respect to the corresponding centre of mass displacements. The results obtained in the direction of the irregularity (direction z) are presented in Figures 4.11 to 4.14.

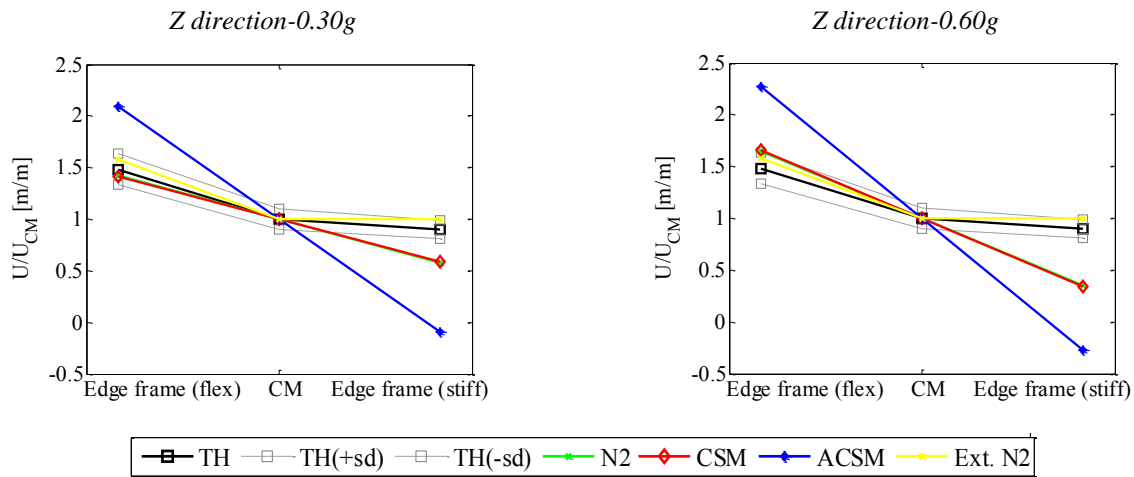


Figure 4.11 -Torsional displacements obtained for MRF-TU-PI structure

The examination of the results obtained for the MRF-TU-PI structure (Figure 4.11) indicate that, for the design intensity level, the N2 and the CSM methods predict lower deformations than the nonlinear time-history results at both edges of the structure, although these deformations are similar at the flexible edge. On the other hand, for the intensity level of 0.60g the N2 and the CSM methods underestimate the edge displacements at the stiff edge of the structure (right edge frame) and slightly overestimate the displacements at the flexible edge (left edge frame). Concerning the performance of the ACSM method, it significantly underestimates the edge displacements at the stiff edge and overestimates the edge displacements at the flexible edge, for both intensities levels considered.

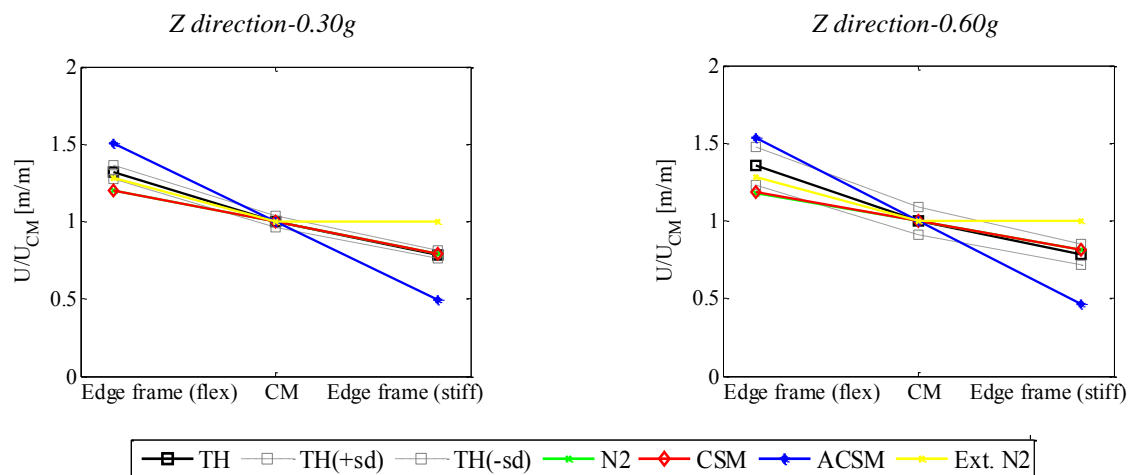
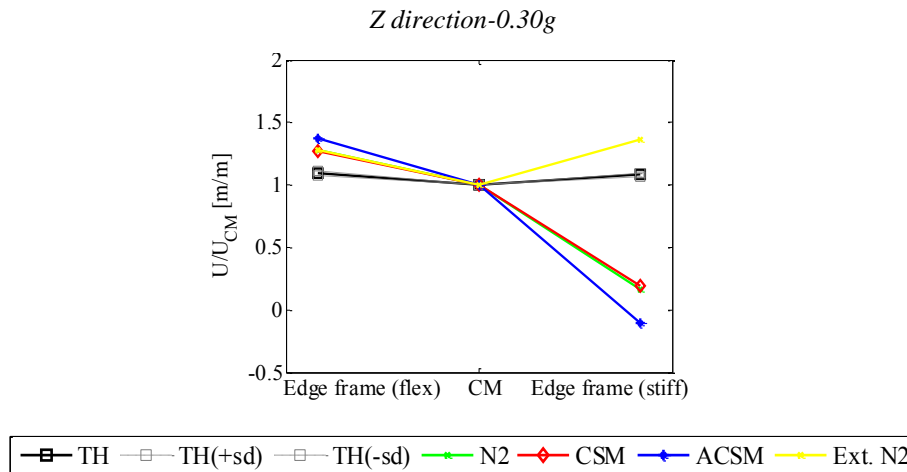


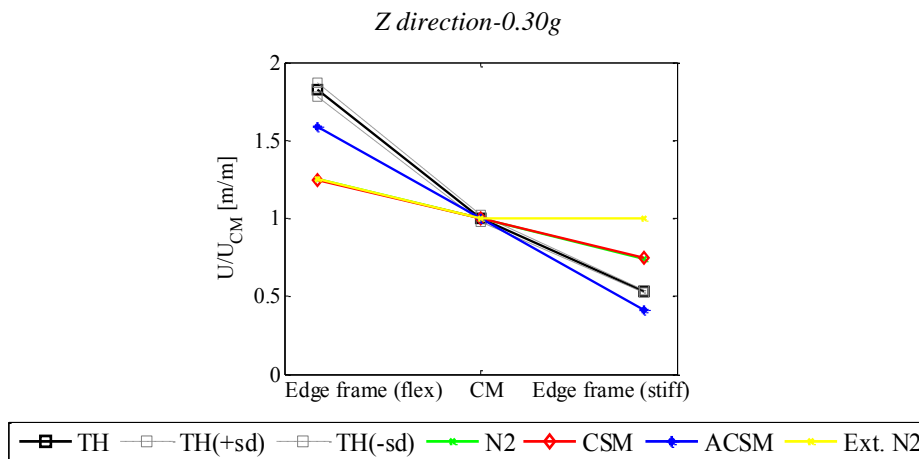
Figure 4.12 -Torsional displacements obtained for MRF-TR-PI structure

Regarding the MRF-TR-PI structure (Figure 4.12), it can be concluded that the N2 and the CSM methods underestimate the displacements at the flexible edge of the structure (left edge), for the two intensities levels, whereas the ACSM overestimates the edge displacements. Regarding the stiff edge (right edge), both the N2 and the CSM provide an accurate estimation of the displacements, while the ACSM clearly underestimates the same deformations. The Extended N2 method provides a better estimation than the conventional N2 method and the CSM of the displacements at the flexible edge. However, it overestimates the deformations at the stiff edge.



*Figure 4.13 -Torsional displacements obtained for CBF-TU-PI structure*

Concerning the CBF-TU-PI structure (Figure 4.13), all the nonlinear static procedures overestimate the edge displacement at the flexible edge (left edge) and highly underestimate the edge displacements at the stiff edge (right edge), except for Extended N2 method, which provides a closer and safe estimation at the stiff edge.



*Figure 4.14 -Torsional displacements obtained for CBF-TR-PI structure*

As for the CBF-TR-PI structure (Figure 4.14), both conventional and extended N2 methods, as well as the CSM, underestimate the displacements at the flexible edge (left edge) and overestimate the displacements at the stiff edge (right edge). For this structure, the ACSM underestimates the displacements at both stiff and flexible edges.

In conclusion, the extended N2 method appears to estimate more accurately the torsional response of the asymmetric steel structures considered in this study, except for the CBF-TR-PI structure at the flexible edge, which exhibits higher torsional displacements in the nonlinear range than in the linear range. This is due to the amplification of displacements that results from the concentration of plasticity on the flexible side of the

structure, combined with the degradative hysteretic behaviour associated to brace buckling. Erduran et al. (2011) have reported similar conclusions in their study on the effect of torsion on the behaviour of peripheral steel-braced frames.

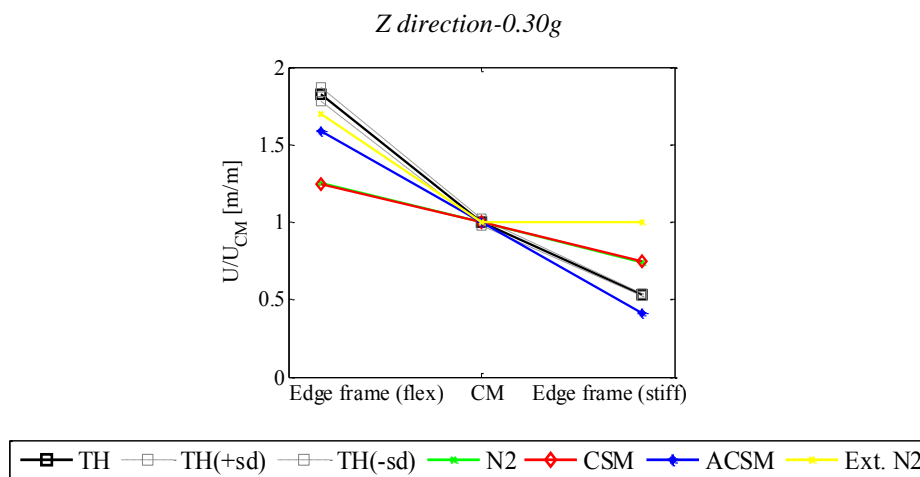
## **5 AN IMPROVEMENT OF THE EXTENDED N2 METHOD FOR PLAN-ASYMMETRIC AND TORSIONALLY STIFF STRUCTURES**

To overcome the limitation identified above regarding the application of the Extended N2 method to torsionally stiff structures with concentrically-braced frames (CBF-TR-PI), an improvement of the Extended N2 method is proposed herein. It consists on the amplification of the demands obtained at the flexible edge through the application of a corrective factor that is a function of the linear torsional response of the equivalent torsional flexible structure. The equivalent torsionally flexible structure has to be a structure that has the same number of lateral resisting systems in both directions, but positioned in a way that the structure becomes torsionally flexible instead of torsionally stiff. This can be achieved by placing the lateral resisting systems inside the structure's perimeter. Additionally, a torsionally flexible structure is characterized by having a torsional fundamental period of vibration. In this study, the equivalent torsional flexible structure corresponds to the CBF-TU-PI structure.

The improvement to the Extended N2 method is to be applied to torsionally stiff asymmetric structures. It consists in the following steps:

1. All the steps until the target displacement is obtained are the same as for the original N2 method.
2. The following step is the performance of a linear analysis of the 3D model in the two horizontal directions.
3. Evaluation of the Extended N2 corrective factors. The correction factors are the ratio of the elastic by the inelastic normalized roof displacements, being the latter obtained by a pushover analysis. The normalized roof displacement is the roof displacement at an arbitrary location divided by the roof displacement at the CM. The correction factors should be defined for each horizontal direction separately. The amplification due to torsion is obtained by multiplying the relevant results of pushover analysis, determined in the beginning of the procedure, by these correction factors, while the reduction of the demands due to torsion is not considered.
4. Definition of the equivalent torsionally flexible structure and linear analysis of the corresponding 3D model in the two horizontal directions. Evaluation of the linear torsional displacements (or linear corrective factors) for both horizontal directions, through the normalization of the roof edge displacements with respect to the CM displacements.
5. The amplification of the demands at the flexible edge of the plan-asymmetric torsionally stiff structure is determined through the application of the linear corrective factor, obtained in step 4, to the demands obtained at the end of step 3. At the stiff edge, the Extended N2 method already provides a good estimation of the edge displacements since the method does not consider the reduction of seismic demand due to torsional effects.

Figure 5.1 illustrates the application of the linear corrective factor to the torsionally stiff structure (CBF-TR-PI). The Extended N2 method provides a conservative estimate of the torsional response of the CBF-TR-PI structure, although it is preferable to the underestimated response obtained with the ACSM method, even if the latter is closer to the nonlinear time-history response.



*Figure 5.1- Torsional displacements obtained for CBF-TR-PI structure after applying the corrective factor*

## 6 CONCLUSIONS

This paper reports on the application of a group of NSPs to a group of plan asymmetric steel structures with three storeys height, designed using MRF and CBF as lateral seismic resisting. The main objective was to assess the effectiveness of these methods through a comparison with the results obtained by nonlinear time-history analysis.

The global behaviour of the structures was evaluated through the comparison of the pushover curves obtained using the conventional and adaptive pushover methods with the results from TH analyses.

The NSPs were assessed in terms of translational and torsional responses through the evaluation of the inter-storey drifts and normalization of the edge displacements with respect to the centre of mass displacements, respectively.

In terms of translational response, the results indicate that, in general, all the NSP provide very accurate estimates of the structural response, with exception of the ACSM that overestimates the response at the upper levels.

With reference to the torsional behaviour, the results indicates that, depending on the type and location of the lateral resisting systems, different conclusions can be drawn. However, in general, the Extended N2 method provided very accurate estimates of the torsional response of the studied structures, with exception of the CBF-TR-PI structure at the flexible edge. To overcome this limitation, an improvement of the Extended N2 method was proposed for torsionally stiff structures and plan asymmetric structures composed by CBF. It consists on the amplification of the displacements obtained at the flexible edge through the application of a corrective factor that is a function of the linear torsional response of the equivalent torsional flexible structure.

Given the limited number of cases studied, further analyses on torsionally stiff plan asymmetric structures should be carried out in order to extract definite conclusions regarding the validity of this corrective factor.

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