

# Effect of choice of inherent damping models on reliability of Incremental Dynamic Analysis

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**ABSTRACT:** Incremental Dynamic Analysis (IDA) is the present state of the art the method for assessing the performance of a structure. For majority of IDA based studies damping matrix is computed using the classical Rayleigh damping model. Although a lot of issues have been identified with the classical Rayleigh damping model, to date this model remains the most popular choice for nonlinear time-history analysis. The use of Rayleigh damping can have very adverse effects on the IDA estimates of engineering demand parameters like inter-story drift. To bring this drawback of Rayleigh damping to light, this paper consolidates the recent research effort in this direction by compiling the performances of different damping models (both global and elemental models) and their effects on the IDA responses. It has been shown that different damping models produces very different estimates of engineering demand parameters and hence different IDA curves. Critically evaluating all the responses a probable direction for damping modelling is also identified for IDA analysis.

## 1 INTRODUCTION

Incremental Dynamic Analysis (IDA) can be conceptually viewed as the dynamic counterpart of Nonlinear Static Pushover (NSP) analysis and is the present state of the art method in analytically predicting the seismic performance of a structure in a performance based earthquake engineering framework. IDA conceptually involves mainly subjecting an analytical model of a structure into a series of ground motions of increasing the intensities and computing probabilistic responses through nonlinear time history analyses. So, in short IDA framework requires a series of nonlinear time history analyses and the accuracy of the estimate is highly dependent on the way the nonlinear time history analyses are performed.

Now a pressing question that appears in this context is as follows: Considering the uncertainties inherent in the nonlinear time history analysis, can we really rely on the IDA for our estimate of the seismic performance? Or in other words are we getting a better benefit by adopting IDA over NSP as all the uncertainties inherent in NSP are also present in IDA? From a true philosophical perspective, yes, a real benefit is obtained by adopting IDA mainly because no approximation to the physics of the seismic phenomenon occurs in this method. The entire dynamic seismic process is represented as it is and the system equilibrium is checked in real time whereas the NSP converts the whole dynamic problem into an equivalent static one in an incremental Hookean format. Then the next immediate question would be:

does that mean we are getting more accurate or reliable solutions via IDA? This is a more subjective one as it depends on a large number of factors starting from the choice of the analysis program to the selection of the ground motions.

Broadly the uncertainties inherent in the IDA process are two folded: one is the phenomenon uncertainty called the Aleatory uncertainty (for e.g. ground motions) where the analyst has no control (e.g. as with the present state of art no can predict the characteristics of a future ground motion); the other one is the modelling/model uncertainty called the epistemic uncertainty, mainly caused by the choice of analytical model. The present focus of this paper is mainly into the modelling uncertainty inherent in IDA. The argument which the authors wants to emphasise is that for a specific selection of ground motions, if the mathematical modelling of the structural system is relatively precise (in terms of geometry, material and choice of dissipation model), the stochastic inference deduced from IDA would be enlightening. A corollary statement of the above argument could be that, if the mathematical model representing the system is relatively imprecise (in terms of geometry, material or dissipation model choice) then it does not matter how many ground motions are used or how many data points are generated, the stochastic inferences would be of little value.

Mass, stiffness and damping matrices characterize the physical dynamic system. In these, an estimation of mass and stiffness maybe achieved with a relatively reasonable precision, whereas the estimation of damping model is completely an *ad hoc* procedure. As the damping model estimation affects the whole system response, it is imperative that in an IDA analysis sufficient care is taken to represent the damping phenomenon.

### 1.1 Scope of the present paper

Focus of this paper is to consolidate some of the recent research findings in the inherent damping modelling and to propose better ways to reduce the epistemic uncertainty posed by the choice of damping models in IDA analysis. To this extent, the scope of the present paper includes a detail critical overview of present state of the art in inherent damping modelling and a consolidated summary of some of the very recent research results published by the authors in Puthanpurayil et al. (2016) and Carr et al. (2017) which emphasises the effect of choice of damping models on nonlinear time history response computation. The results compiled clearly shows that based on the choice of damping models there is a high level of variability observed in IDA stochastic response estimation. Based on the evaluation of the effect of different models on the IDA curves, a possible direction for damping modelling is also recommended.

## 2 REVIEW OF THE PRESENT STATE OF THE ART IN INHERENT DAMPING MODELLING

Nonlinear time history analysis necessitates the explicit definition of a damping matrix. The major dissipation phenomenon in nonlinear analysis happens mainly through the member hysteresis. The main purpose of damping matrix is to reflect the *unmodeled dissipation* exhibited by non-structural components and the elastic components of an inelastic element inherent in a structural system. To explicitly model the phenomenon responsible for the *unmodeled dissipation* is close to impossible (Scanlan 1976). So, the main purpose of a mathematical model depicting the damping matrix is to reflect this *un-modelled dissipation* in such a way that these dissipation effects are reflected indirectly in the analysis with no untoward effects.

### 2.1 Popular damping models in practice and their issues

Majority of the models popular in practice is viscous based; i.e. damping force is a function of velocity. In this section a brief overview of these models prevalent in practice is described; for more details interested readers should refer to Puthanpurayil et al. (2016).

#### 2.1.1 Rayleigh damping model

Up to the present time the most popular model used to represent the damping phenomenon in nonlinear time history analysis is the constant stiffness based Rayleigh damping. Rayleigh damping is a proportional damping obtained in an *ad hoc* manner by the weighted combination of stiffness and mass

matrices. The weights are determined as a function of the system frequency using a predetermined damping ratio. The use of Rayleigh damping dates to 1887 when Lord Rayleigh first used the viscous dissipation function based on instantaneous velocities as the only *state variables* to represent the dissipation phenomenon exhibited in acoustics (Lord Rayleigh, 1887). The adoption of this model in linear/nonlinear structural dynamics and the present form in which this model is implemented in the current existing software, has no theoretical or physical justification other than the mathematical convenience. The majority of the commercially available software packages (e.g. SAP, ETABS, STAAD Pro. etc.) only have the initial stiffness based Rayleigh damping for nonlinear structural dynamics.

The extension of Rayleigh model into nonlinear dynamics was a result of the popularity and familiarity of this model in linear dynamics and is accompanied with some major shortcomings. Crisp (1980) for the first time observed that on the onset of nonlinearity, the damping model exhibited an anomalous behaviour; when the stiffness reduced due to excursion of inelasticity and the damping matrix remained constant, huge unrealistic actions (damping moments) were exhibited by the initial stiffness based Rayleigh damping model.

Figure 1.0 depicts a damping moment history plot. It can clearly be seen from figure 1.0 that the damping moments are instantaneous, velocity based, force impulses which are almost like adding instantaneous viscous dampers to equilibrate the equation of motion. These forces affects the local higher order modes, which occurs because of the inelasticity, by damping them out indiscreetly. The cumulative effect of this was reflected in the global computed responses of the structure. This is a very important point especially when considerable computational effort is being expended to get the IDA responses. *An IDA is a result of large number of nonlinear time history analyses: so, if the model used itself predicts the responses in error, no statistical treatment of such results generated by many analyses would give a reliable estimate invalidating the whole effort of IDA.*

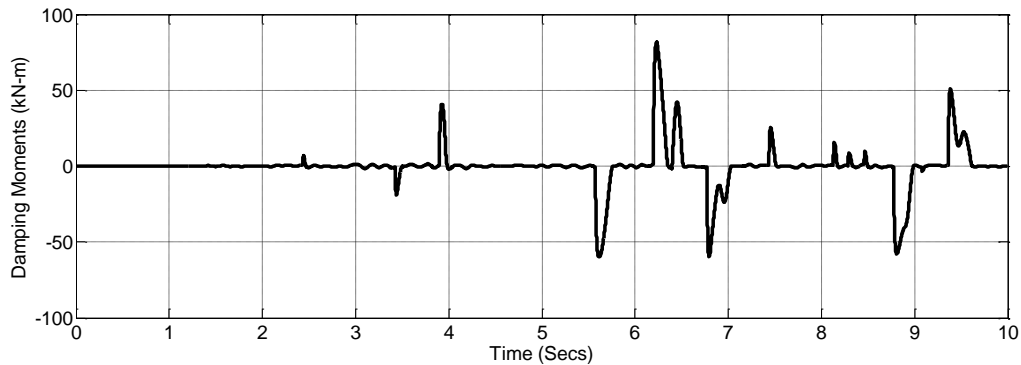


Figure 1. Damping moment plot for the four storey frame (adopted from Carr et al. 2017)

#### 2.1.1.1 Suggested remedies for the Rayleigh damping model. (Puthanpurayil et al. 2016)

A considerable amount of research effort had been expended in fixing the issues of unrealistic forces exhibited by Rayleigh damping. One of the earliest attempt in this direction was the use of tangent stiffness based Rayleigh damping (Sharpe 1974). Instead of using the initial stiffness, the tangent stiffness was used for forming the damping matrix. This alleviated the effect of these damping moments to a certain extent. But as evidenced in Section 3, there was still the presence of a considerable large damping moments. Ruaumoko (Carr 1980, 2007) and PERFORM 3D (Powell 2007) provide tangent stiffness based Rayleigh damping models.

Leger and Dussault (1992), further modified this model and proposed to form the damping matrix using instantaneous weighting factors along with tangent stiffness. The proposed form of damping matrix is as follows,

$$\mathbf{C} = \alpha_{\tan gent} \mathbf{M} + \beta_{\tan gent} \mathbf{K}_{\tan gent} \quad (1)$$

Though conceptually this was elegant, the main impediment in this formulation was the need to compute the instantaneous weighting factor which required instantaneous estimation of the Eigen parameters. For

very large structural systems, this is impracticable.

Bernal (1994) suggested an alternative methodology by assembling damping matrix using condensed stiffness matrix in such a way that no damping was attached to the massless degrees of freedom. The method proposed requires the damping matrix to be assembled restricting the Caughey series to zero or negative. This might be a possible way of eliminating the spurious forces; but the requirement for condensation and use of Caughey series made this method computationally expensive.

Hall (2006) proposed a capped viscous damping model comprising only of the stiffness component of global Rayleigh model with certain bounds imposed in accordance with the actual physical mechanism. Charney (2008) presented a set of recommendations aimed at eliminating the issues associated with global Rayleigh damping. *At the outset, the best strategy as described by Charney (2008) was the overall elimination of the use of the global Rayleigh damping in inelastic dynamic analysis and suggested the use of nonlinear frictional or hysteretic damping with smooth hysteretic rules such as the Bouc hysteretic (1967).* According to Charney (2008) if only the Rayleigh damping model is available the best option might be to use the Leger and Dussault model (1992). If this is not possible, then Charney suggested the use of the tangent stiffness based global Rayleigh damping. Charney (2008) also gave a warning that, if the initial stiffness based global Rayleigh model is used, where stiffness is based on the elastic stiffness of the structure, extreme caution need to be exercised especially when explicit springs are used to represent nonlinearity. This would be very critical when one uses link elements as available in some commercial software packages to represent nonlinearity as before the spring goes nonlinear there will be a very large stiffness. Thus, special care would be needed to explicitly avoid very large stiffness creeping into the damping matrix which may produce very unrealistic damping moments. Charney (2008) also presented an excellent review of the damping procedures used by some of the current commercial software.

Zareian and Medina (2010) using an equivalent 8 degree of freedom element proposed an alternative approach by reformulating the damping matrix with a Rayleigh type approach using a time invariant stiffness matrix assembled by assigning zero stiffness proportional damping to the degrees of freedom that have the potential to experience inelastic deformation. In the computation of the damping matrix, only the stiffness proportional components of the elastic beam element stiffness were accounted for and no damping is assigned to the semi-rigid springs. This actually takes into account the observation made by Charney (2008) of not including the pseudo penalty stiffness term in the damping matrix computation. This provides a means of reducing the spurious damping actions and maintaining numerical stability in the inelastic dynamic analysis. The main impediment in this method is the reliance on the 8 degree of freedom beam element formulation which inherently has computational issues.

Jehel et al. (2014) developed analytical formulas for both initial stiffness based global Rayleigh damping and tangent stiffness based global Rayleigh damping for controlling the modal damping ratio. Though there exists a simple formula for controlling the modal damping when tangent stiffness based global Rayleigh damping is used; no such direct formula exists when initial stiffness is used.

## **2.2 Existing viscous damping models not commonly used in practice (Puthanpurayil et al. 2016)**

Though not popular in practice, there are other models existing in the literature. The main reason for their lesser popularity when compared to the Rayleigh damping model is mainly due to the mathematical rigor involved in implementing them, the computational demand these models pose on the analysis and the unfamiliarity of the analysts in their usage.

First of these models which falls into this category is the Caughey series damping model. Caughey (1960) derived a general form of the viscous damping matrix with orthogonal properties. Albeit generic in its mathematical format, implementing this model is highly cumbersome due to requirement of deriving the terms in the series (Charney 2008). The classical Rayleigh damping model is a special case of the Caughey damping model using only the two terms in the series.

Wilson and Penzien (1972) derived a more direct and efficient procedure for direct evaluation of the Caughey type damping matrix as compared to the original Caughey's formulation. The damping ratio could be independently assigned to all modes in this model. The main limitation of this model was that it required the estimation of complete Eigen parameters of the system and the model also produces a

fully populated damping matrix violating the efficient skyline storage format used by all commercial software packages. A few commercial software packages such as Ruaumoko and PERFORM 3D implement this model; (Carr 1980, 2007; PERFORM 3D 2006). In these packages, Perform3D applies this method only for a maximum of 50 modes whereas Ruaumoko applies the method for all modes (Carr 1980, 2007; PERFORM 3D 2006). The application of Wilson-Penzien model in nonlinear dynamics has shown that it produces damping moments which are much less than any of the Rayleigh models and enables the analyst to have more control by the way of specifying damping ratios individually for each of the modes.

### 2.3 Existing Non-viscous damping models not commonly available in seismic dynamic analysis (Puthanpurayil et al. 2016)

Non-viscous damping models are models in which damping force depends not only on the instantaneous responses, but also on the history of responses through a causal dissipative kernel function (Woodhouse 1998). They are called non-viscous mainly because on integration by parts the damping force becomes a function of displacement. To implement this model in the nonlinear domain special algorithms are required to solve the resulting integro-differential equation. One such method is the AAR method (Puthanpurayil et al. 2014). Except for Ruaumoko, till date, to the knowledge of authors, no other commercial package has implemented this model; mainly this could be attributed to the level of mathematical rigour and parametrization required by this model.

There are also other models like frequency independent coulomb friction model, modified hysteretic model (Muravski 2004) and continuum damping models. Due to the requirement of a special implementation platform and the associated parametrization required, they are still not used in seismic nonlinear analysis.

### 2.4 Recently proposed damping models (Puthanpurayil et al. 2016; Carr et al. 2017)

Reviewing all the shortcomings described in the above sections, required attributes for an ideal empirical mathematical model representing damping might be listed as follows: (a) no appearance of unrealistic forces/moments associated with the damping phenomenon as the analysis progresses (b) ease of implementation in an existing commercial software framework (c) no explicit increase in the computational time due to the choice of the damping model. The first attribute, the presence of the unrealistic damping forces, may result in considerable inaccuracies in displacements and internal forces whereas the other two attributes are more related to the practical utility of the model from a commercial implementation point of view.

To cater to the above factors, Puthanpurayil et al. (2016) proposed a new paradigm of modelling damping by formulating the dissipation phenomenon at elemental level. It has also been demonstrated that at the elemental level the models were devoid of all the untoward effects of the global damping models. An over view of the damping model classification is given in figure 2.0.

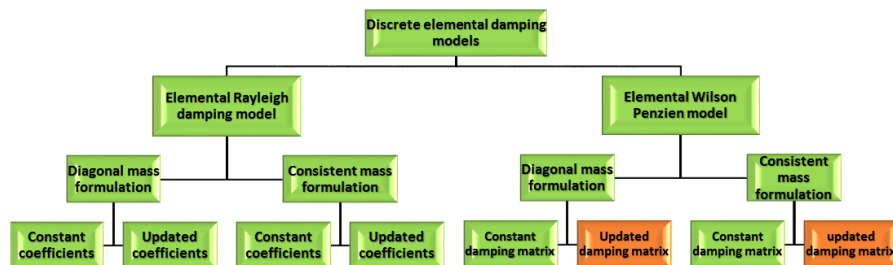


Figure 2. Overview of the elemental damping models

## 3 NUMERICAL RESULT COMPILATION

This section consolidates the performance of the elemental damping models as compared to the global damping models. The results for a four storey RC frame as described in Puthanpurayil et al. (2016) has

been used for the study. The results from a single ground motion study is presented first and the next section compiles previously published IDA results and discusses the effect of the damping model on the IDA responses.

### 3.1 Single ground motion study

The main aim of this study is to illustrate the variability in the responses obtained because of different choices of damping models even for a single ground motion analysis. Two classical global damping models and one elemental damping model is used for the present study. It has to be noted that elemental model is used as a constant damping matrix throughout the nonlinear analysis.

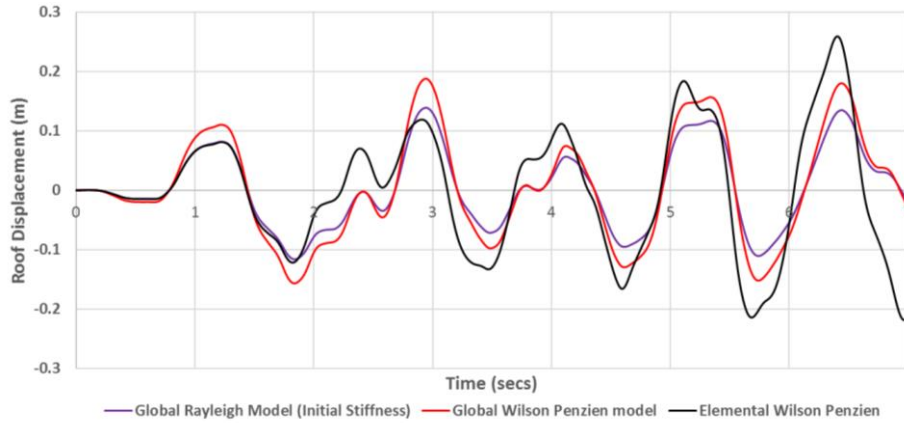


Figure 3. Represents the roof displacement plot for a single ground motion study.

This is very significant mainly because IDA estimates are obtained by a series of this type of analyses; so if the choice of damping model is not reliable from an engineering perspective there is a high chance that the performance assessment deduced from IDA might not be a realistic reflection of reality. Figure 3.0 represents the roof displacement for 3 different damping models. It can be clearly seen that different damping models give different responses and the elemental damping model gives a possibly more conservative response in terms of the roof displacement. Note that 5% global damping ratio is used for all the models in this study.

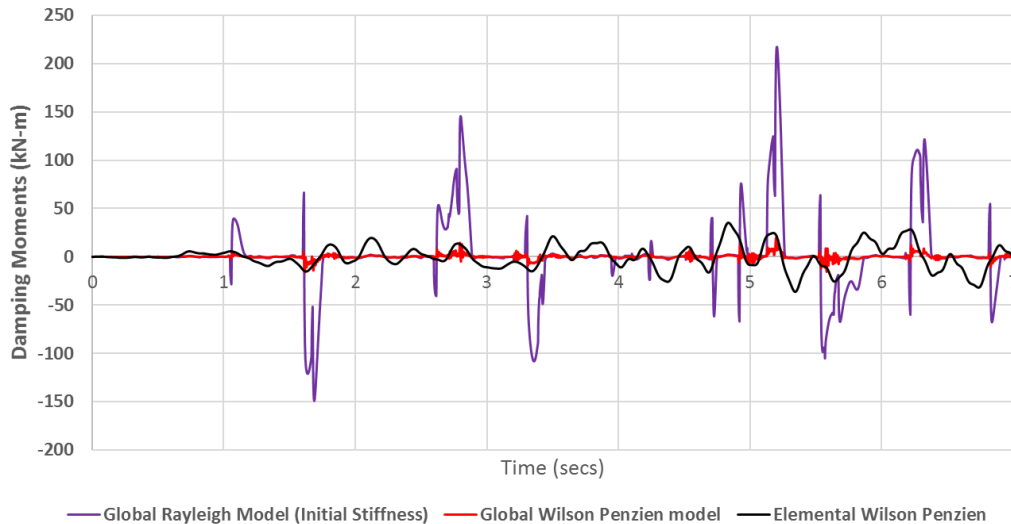


Figure 4. Damping moment plots

Figure 4.0 represents the damping moment plot. It could be clearly seen that, compared to the global Rayleigh damping, the Elemental Wilson Penzien gives much reduced damping moments whereas in comparison to the Global Wilson Penzien, the model exhibits higher damping moments; This is as expected; Puthanpurayil et al. (2016) have shown that the implementation of this model as a constant

damping matrix always produces higher damping moments than the global Wilson Penzien model. But the biggest advantage of this model is that the model produces the skyline pattern of the damping matrix and also gives higher estimate for engineering demand parameters once the system incurs inelastic excursions. To improve the model Carr et al. (2017) implemented the updated tangent version of the model; this yields both reduced damping moments as well as a more conservative engineering demand parameters.

### 3.2 Effect of damping models on IDA (Carr et al. 2017)

As the focus of the present paper is to highlight the effect of damping models on the IDA, results published in Carr et al. (2017) is compiled in this section. The following abbreviations are used to identify different damping models included in the plots:

#### **Global:**

ISRD	Initial stiffness based global Rayleigh damping
TSRD1	Tangent stiffness based global Rayleigh damping with constant coefficients (Sharpe 1974)
TSRD2	Tangent stiffness based global Rayleigh damping with updated coefficients
GWP	Global Wilson-Penzien model

#### **Elemental:**

ELRD1:	Elemental Rayleigh damping with constant elemental proportionality coefficients
ELRD2:	Elemental Rayleigh damping with updated proportionality coefficients
EWP:	Elemental Wilson-Penzien model implemented as a constant damping matrix
UEWP:	Elemental Wilson-Penzien model implemented as a tangent matrix using secant formulation to avoid damping hysteresis.

Figure 5 illustrates the the mean IDA curves for location-independent peak inter-storey drift ratio as the engineering demand parameter (EDP). The variability exhibited by different damping models is very evident in the plots presented in figure 5.0. The plots also exhibit the fact that the different damping models even affect the yielding point estimate of the system; all the elemental models tend to exhibit clearly defined yielding at a PGA of 0.5g whereas the global models tend to show yielding around a PGA of 0.6g. This is a very important observation; if inappropriate choice of damping model is adopted, then the analysis might not recognise the fact that the estimate of the yield point of the back bone of the structure might be erroneous.

The pressing question would be this “which of these curves represent the most realistic scenario?” The unfortunate answer is that, “we do not really know” because damping is an “observed phenomenon” and the models used are only nothing better than mathematical approximations which try to reflect what happens in reality. From a conservative engineering perspective, till the physics or material science identifies the exact process or phenomenon of damping and the mathematical representation of it, structural engineers will need to use those models which give reliable results with no adverse side effects such as the appearance of unrealistic damping actions. From that perspective it may be deduced that the elemental damping models ,viscous or non-viscous,give better conservative estimates and should be used for IDA analysis. Although not shown here, the damping moments obtained by the elemental models were also very small. Puthanpurayil et al. (2016) also presented the dispersion plots which quantify the variability of these models.

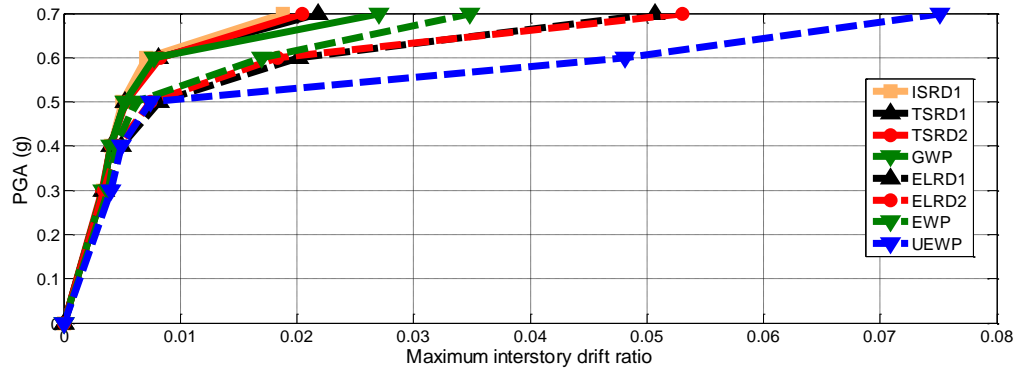


Figure 5. IDA curves for location independent peak drift ratio

#### 4 CONCLUSION

The effects of choice of different damping models on the computed IDA responses have been highlighted. It has been shown that global classical Rayleigh damping model produces very unconservative estimate for IDA curves whereas all the elemental damping models seems to produce more conservative estimate. The effect of damping models on the yield state estimate of the back bone is also presented. It has been emphasised that right choice of damping model is imperative for reliable estimate of IDA curves. From the preliminary studies presented in this paper, authors believe that updated elemental Wilson Penzien model with secant estimate of the damping matrix employed in secant formulation of implementation would be a possible direction of inherent damping modelling in nonlinear time history analysis.

#### 5 ACKNOWLEDGEMENTS

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