The transition period $T_L$ in the recommended spectra of the draft *New Zealand Seismic Isolation Guidelines*


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**ABSTRACT:** The draft *New Zealand Seismic Isolation Guidelines* recommend an increase in the transition period $T_L$ of 3s for all of New Zealand, apart from Northland and Auckland. $T_L$ is the boundary between the constant-velocity and constant-displacement branches of the spectral shape factors of the structural design standard NZS1170.5. The recommended increased $T_L$ values are 5s for Waikato, Taranaki, Western Bay of Plenty, Tauranga and Rotorua, and 10s elsewhere. These $T_L$ values are less than the IBC values of 12s for California, and 16s for Alaska where large magnitude subduction earthquakes dominate the hazard. The changes increase the long-period spectral components that are often important for seismically-isolated structures.

The recommended increases are based on a combination of a literature review, studies of the response spectra of some of the larger recent New Zealand earthquakes, Fourier spectra from the Canterbury earthquake sequence, and the NGA-West ground-motion prediction equations. Although often contradictory in their results, these studies led to the recommendation for New Zealand of a magnitude-dependent relationship that produces $T_L=3s$ for magnitude 6.5 and $T_L=10s$ at magnitude 7.5. These values conform at magnitude 6.5 to those from the well-known Calvi (2008) relationship that is often used in Europe for displacement-based design, and with the IBC values from the U.S. at magnitude 7.5. There are large differences in the relationships between $T_L$ and moment magnitude $M_w$ used in the U.S. and Europe, and between the results of engineering studies, based on analyses of recorded response spectra, and seismological studies, based on theoretical or recorded Fourier spectra.

1 **THE TRANSITION PERIOD $T_L$**

1.1 $T_L$ and NZS1170.5 spectral shape factors

Development of *New Zealand Seismic Isolation Guidelines* are nearing completion by an NZSEE study group. It was recognised that design of long-period seismic isolation systems may require the specification of response spectra to periods longer than the maximum of 4.5s for which the acceleration spectral shape factors (normalised 5% damped acceleration response spectra) are defined in the New Zealand structural design standard NZS1170.5. Accordingly, in the draft Guidelines, the NZS1170.5 spectra are modified and their period range extended up to 10s. In extending the period range, it was necessary to consider the transition period $T_L$ between the constant-velocity and constant-displacement branches of the response spectra. NZS1170.5 has an assumed value of $T_L=3s$, with constant-displacement behaviour assigned between 3s and 4.5s.

In modifying the NZS1170 spectra, it may seem obvious to simply extend the constant spectral displacement behaviour out to 10s. However, seismological theory (e.g. Brune, 1970; Boore, 1983) and modern ground-motion prediction equations suggest that a transition period of 3s is too short for larger magnitude earthquakes, such as may govern design in the higher seismicity regions of New Zealand. The effect is to make the NZS1170.5 spectra too low at periods longer than 3s. Current US design standards have $T_L$ values as large as 12s in parts of California and 16s in parts of Alaska. Accordingly, the transition period $T_L$ at which the spectral shapes change from constant-velocity to constant-displacement behaviour has been increased in the draft Guidelines for higher hazard parts of the country, as discussed in this paper.
1.2 Approaches for determining $T_L$

Several approaches were taken to produce recommendations for the transition period $T_L$ as a function of moment magnitude $M_w$. These included reviews of the engineering and seismological literature on the corner periods of response and Fourier spectra; $T_L-M_w$ expressions recommended in international seismic design standards; published corner frequencies of Fourier source spectra from the Canterbury earthquake sequence; and the determination of $T_L$ from response spectra of recent large New Zealand earthquakes, and from spectra predicted by the NGA-West earthquake ground-motion prediction equations.

2 LITERATURE SURVEY

Both engineering and seismological studies were considered in the literature survey. Seismological studies are usually concerned with the corner period $f_c$ of the Fourier spectrum of the source motions, which depends on the magnitude $M_w$ and the Brune stress parameter $\Delta \sigma$. Engineering studies based directly on response spectra tend to produce transition periods $T_L$ that are shorter than $1/f_c$, perhaps from path and site effects increasing the short period parts of the spectrum relative to the long-period parts. The Brune stress parameter is not usually considered as a parameter in studies based on response spectra.

2.1 Seismological theory of source spectra and the corner frequency $f_c$ and transition period $T_L$

In seismological approaches to representing earthquake spectra, the source moment rate Fourier amplitude spectrum $M(f)$, which has the same dependence on frequency as the source displacement spectrum, is represented in terms of the seismic moment $M_0$ and the corner frequency $f_c$ by

$$M(f) = M_0 / [(1 + ff_c)3]$$

There is not a sharp transition in the spectrum $M(f)$ at frequency $f_c$, but rather $f_c$ corresponds to the frequency at which low-frequency asymptote $M_0$ and high-frequency asymptote $M_0(f_c/f)^2$ intersect.

In the Brune model, the corner-frequency depends on the stress parameter $\Delta \sigma$, the seismic moment $M_0$ and the shear-wave velocity $V_s$ at the source (e.g. Boore 1983, modified to MKS units) through

$$f_c = 0.49 V_s (\Delta \sigma/M_0)^{1/3}$$

Using the relation between seismic moment $M_0$ and moment magnitude $M_w$ (equation (7) of Hanks & Kanamori, 1979, converted to MKS units) leads to

$$\log_{10} f_c = 0.5 \log_{10} \Delta \sigma - 0.5 M_w + \log_{10} V_s - 3.325$$

In the literature, it appears that the transition period $T_L$ of the response spectrum is often assumed to be the inverse of $f_c$. Using this assumption and the notation $M_{w'}$ for the magnitude for which $f_c$ is 1 Hz (or $1/f_c$ is 1 s) for a stress drop $\Delta \sigma_{ref}$ and reference source velocity $V_{sref}$, the equation for $T_L$ becomes

$$T_L = (\Delta \sigma/\Delta \sigma_{ref})^{1/3} (V_{sref}/V_s) 10^{0.5 (M_{w'}-M_w)}$$

2.2 Summary of $T_L$ results from seismological theory and used in engineering design

Table 1 and Figure 1 compare $T_L$-$M_w$ results used in engineering design with those from seismological studies. The engineering and seismological expressions are different, and both give contrasting values between the US and Europe. In Figure 1, the results of the US studies are shown as the solid lines in log-normal space, with longer corner periods $T_L$ than from the European studies. The engineering studies produced slower increases of $T_L$ with magnitude than the seismological studies, at least for larger magnitudes.

The American IBC engineering design standard combines a function for $T_L$ in terms of $M_w$ with magnitudes derived from deaggregation of seismic hazard results to form the basis for maps of $T_L$ (Crouse et al, 2006). The IBC standard gives discrete values of $T_L$ for various magnitude bands that are plotted at the mid-range magnitudes in Figure 1, together with a closely-corresponding expression given by Abrahamson & Silva (2008):

$$\log_{10} T_L = -1.25 + 0.3 M_w$$

Calvi et al. (2008) reported a simple response-spectrum based relationship between $T_L$ and $M_w$ that was
used in formulating displacement-based design approaches in Europe (Priestley et al., 2007):

$$T_L = 1.0 + 2.5 (M_W - 5.7)$$  

(6)

Table 1. Transition Period $T_L$ as a function of Moment Magnitude $M_w$.

<table>
<thead>
<tr>
<th>Moment Magnitude $M_w$</th>
<th>Corner Period $T_L$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Engineering design</td>
</tr>
<tr>
<td>6.0-6.5</td>
<td>4</td>
</tr>
<tr>
<td>6.5-7.0</td>
<td>6</td>
</tr>
<tr>
<td>7.0-7.5</td>
<td>8</td>
</tr>
<tr>
<td>7.5-8.0</td>
<td>12</td>
</tr>
<tr>
<td>8.0-8.5</td>
<td>16</td>
</tr>
<tr>
<td>8.5-9.0</td>
<td>20</td>
</tr>
</tbody>
</table>

Figure 1. Corner periods $T_L$ as a function of magnitude $M_w$ from expressions in the literature. Solid lines are from US studies, and dashed curves are from European studies. The two lines with steep slopes at all magnitudes are from seismological studies, while the two relations with flatter slopes are from engineering studies.

The two seismologically-based results both used Brune-type expressions of the form of equation (4), but with very different values of the magnitude $M_w'$ at which $T_L$ is 1 second, namely $M_w' = 5$ in Joyner and Boore (1988), and $M_w' = 6.4$ in Erdik et al. (2011). Calvi et al. (2008) noted these “unresolved differences in predictions of corner period between US seismologists (based mainly in seismological theory) and European seismologists (mainly empirically based)”.

The differences between the engineering-based and seismological expressions may indicate a difference between the transition period $T_L$ of the response spectra and the inverse $1/f_c$ of the corner frequency of the Brune spectrum. This remains to be investigated.
The IBC values, which are defined for magnitudes from 6 to 9, all exceed the NZS1170.5 value of 3s, while those of Calvi et al. exceed 3s for magnitudes larger than 6.5. Accordingly, the use of $T_L=3s$ in NZS1170.5 lags behind design practice in both the USA and Europe.

3 T\textsubscript{L} FROM FOURIER SPECTRA OF CANTERBURY EARTHQUAKE SEQUENCE

Oth and Kaiser (2014) determined the Brune corner frequencies $f_c$ from Fourier source spectra of events of the Canterbury earthquake sequence. The blue symbols in Figure 2 show the periods corresponding to $1/f_c$ in their study. The red symbols are discussed in the next section. The magnitudes range from 3.5 to 7.2, with only 3 of the events used in their study having magnitudes of 6 or greater.

![Corner Period versus Magnitude](image)

Figure 2. Corner periods $T_L$ as a function of moment magnitude $M_w$ for the Canterbury earthquake sequence (blue symbols), as provided by Anna Kaiser, and expressions conforming to the Brune-type spectrum with $T_L=1s$ at magnitudes of 4.3, 4.8, 5.0 and 5.5. The five red crosses are $T_L$ values based on response spectra (section 4).

A lower-bound Brune-type relation for $T_L$ as a function of $M_w$ for the data from Canterbury sequence, shown as the red line in Figure 2, corresponds to a high stress drop of about 50 MPa, and generally the magnitude associated with $T_L=1s$ is lower than its $M_w$ value of 5.5. The mid-range relations shown for the Canterbury sequence correspond to $T_L=1s$ at about $M_w$ 4.8 (green line in Figure 2) to 5 (the Joyner & Boore relation, orange line), with associated $\Delta\sigma$ of approximately 5 MPa and 10 MPa respectively.

4 T\textsubscript{L} FROM RESPONSE SPECTRA OF LARGE NEW ZEALAND EARTHQUAKES

As another assessment of $T_L$-$M_w$ relationships from a wider range of New Zealand earthquakes, and for response spectra rather than Fourier spectra, $T_L$ was determined from the spectral shapes (normalised by their values at 10s) of the 5% damped displacement response spectra of recorded motions from five New Zealand earthquakes of recent years (Figure 3 and Table 2).
Figure 3. Geometric mean shapes of the 5% damped displacement response spectra (normalised at 10s period) of records from five recent New Zealand earthquakes of magnitude 6.0 and larger. The largest magnitude Darfield earthquake (red curve) has a spectral shape distinctly different from the four smaller magnitude events.

In Figure 3, the spectral shape of the larger $M_w$ 7.08 Darfield earthquake (bold red curve) stands out from the rest, continuing to increase with period most of the way out to 10s, with an estimated $T_L$ value of 9.6s. From this small sample, it appears that the NZS1170.5 $T_L=3s$ value may be sufficiently long for magnitudes up to the mid sixes, but deficient for the larger magnitude Darfield earthquake.

Table 2. Transition periods $T_L$ determined from response spectral shapes of New Zealand earthquakes

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
<th>Magnitude $M_w$</th>
<th>Response Spectra</th>
<th>Transition Period $T_L$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darfield</td>
<td>4 September 2010</td>
<td>7.2</td>
<td></td>
<td>9.6</td>
</tr>
<tr>
<td>Christchurch</td>
<td>22 February 2011</td>
<td>6.2</td>
<td></td>
<td>2.9</td>
</tr>
<tr>
<td>Christchurch aftershock</td>
<td>13 June 2011</td>
<td>6.0</td>
<td></td>
<td>1.8</td>
</tr>
<tr>
<td>Cook Strait</td>
<td>21 July 2013</td>
<td>6.58</td>
<td></td>
<td>1.7</td>
</tr>
<tr>
<td>Cook Strait</td>
<td>16 August 2013</td>
<td>6.6</td>
<td></td>
<td>2.9</td>
</tr>
</tbody>
</table>

The response spectra-based transition periods listed in Table 2 correspond to the red crosses in Figure 2 in the previous section. These $T_L$ values appear to be less than those based on Fourier spectra. The three values at $M_w$ 7.2, 6.2 and 6.0 are from Canterbury sequence events, allowing direct comparison the $T_L$ values from the Fourier spectra (blue diamonds) and response spectra (red crosses) in Figure 2.

5 $T_L$ FROM NGA WEST (2014) GROUND-MOTION PREDICTION EQUATIONS

Table 3 lists the corner periods that have been estimated from the 50-percentile displacement response spectra produced for source distances of 20 km and magnitudes from 3 to 8 by four of the ground-motion prediction equations (GMPEs) of the NGA-West project (Gregor et al., 2014). These recent high-quality models of spectra for crustal earthquakes have been assumed to provide good approximations to actual
data, but there is a wide range of $T_L$ values between the different models. The estimated $T_L$ values for all but one model exceed 3s for magnitude 7 and greater, and for all models by magnitude 8. The $T_L$ values for two of the models are approximately 10s for magnitude 8.

Table 3. Transition periods $T_L$ (s) as a function of magnitude Mw for four of the NGA West (2014) GMPEs

<table>
<thead>
<tr>
<th>Model</th>
<th>Mw 3</th>
<th>Mw 4</th>
<th>Mw 5</th>
<th>Mw 6</th>
<th>Mw 7</th>
<th>Mw 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASK (2014)</td>
<td>0.12</td>
<td>0.32</td>
<td>0.53</td>
<td>2.4</td>
<td>6.7</td>
<td>10.0</td>
</tr>
<tr>
<td>Campbell &amp; Bozorgnia (2014)</td>
<td>0.24</td>
<td>0.40</td>
<td>0.60</td>
<td>1.8</td>
<td>3.9</td>
<td>5.6</td>
</tr>
<tr>
<td>Chiou &amp; Youngs (2014)</td>
<td>0.25</td>
<td>0.43</td>
<td>0.59</td>
<td>1.3</td>
<td>2.6</td>
<td>4.3</td>
</tr>
<tr>
<td>Boore (2014)</td>
<td>0.19</td>
<td>0.33</td>
<td>0.78</td>
<td>2.0</td>
<td>6.1</td>
<td>9.9</td>
</tr>
</tbody>
</table>

6 RECOMMENDED $T_L$-Mw RELATION FROM THE ABOVE STUDIES

Figure 4 compares the two engineering response spectra-based $T_L$-Mw relations considered in the literature review with the response-spectra based $T_L$-Mw pairs presented in the previous two sections, from recent New Zealand earthquakes and modern US-based GMPEs. The sparse New Zealand data appear to lie near the Calvi et al. (2008) relation (dashed purple curve) for magnitude 6 to 6.6, but the magnitude 7.2 point from the Darfield earthquake lies slightly above the Abrahamson & Silva (2008) line, corresponding to the IBC design values used in the US.

![Figure 4. Comparison of the $T_L$-Mw relation recommended for the Seismic Isolation Guidelines (bold red line) and other relations, as discussed in the text. Individual symbols are $T_L$ values estimated from NGA West 2014 GMPEs, and from recent New Zealand earthquakes.](image)

For simplicity in application, it was decided to base the magnitudes $M_w$ for calculating $T_L$ on the largest magnitudes on a regional basis that are likely to significantly affect the spectra estimated for return periods associated with collapse avoidance motions, nominally of about 2500 years for IL2 structures, to about 5000 to 10,000 years, for higher importance levels. Figure 6.3 of the NZTA Bridge Manual
(Third edition, Amendment 2, effective May 2016) provides a map showing these magnitudes, which range from magnitude 6.5 to 7.8. Thus the $T_L$-$M_w$ relation is required for magnitudes of 6.5 and greater. This is a proxy for the more formal approach used for US codes, where the $T_L$ values were based on the modal magnitudes determined from deaggregations of the estimated 2% in 50 years hazard at 2s period, at which the GMPEs were thought to be reasonably reliable (Crouse et al., 2006).

On the basis of these considerations, the recommended relation is the solid red line in Figure 4. At $M_w$ 6.5, it has a $T_L$ value of 3s, corresponding to the Calvi et al. (2008) relation (dashed purple line) that is used in displacement-based design in Europe. At larger magnitudes, it increases more rapidly than the Calvi relation, to reach the Abrahamson & Silva (2008) IBC relation (dashed black line) that is used in US codes at $M_w$ 7.5 where $T_L$ is 10s, the maximum period considered in the draft Guidelines. $T_L$ is likely to increase beyond 10s for magnitudes larger than 7.5. Fitting a log-linear expression for $T_L$ versus $M_w$ through these points produces

$$\log_{10}(T_L/3) = 0.5229(M_w-6.5)$$

(7)

This expression is similar the lower-bound curve for the Canterbury source-spectra shown in Figure 2.

7 RECOMMENDED REGIONAL $T_L$ VALUES

The $M_w$ values to be used to determine the transition period $T_L$ at a given location are based on a simplification of those recommended for consideration in determining collapse-avoidance motions in Figure 6.3 of the NZTA Bridge Manual (Third edition, Amendment 2, effective May 2016). The region of application of each magnitude value and its associated $T_L$ is defined by regional or district councils (Table 4). The values of Table 4 increase the transition period $T_L$ of the acceleration spectral shape factors for all but the lowest seismicity part of the country (Auckland and Northland). These $T_L$ values are less than the IBC values of 12s for California, and 16s for Alaska where large magnitude subduction earthquakes dominate the hazard. The changes increase the long-period spectral components that are often important for seismically-isolated structures.

<table>
<thead>
<tr>
<th>Regional/district council</th>
<th>Assigned $M_w$</th>
<th>Corner-period $T_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northland/Auckland</td>
<td>6.5</td>
<td>3s</td>
</tr>
<tr>
<td>Waikato, Taranaki, Western BOP,</td>
<td>6.9</td>
<td>5s</td>
</tr>
<tr>
<td>Tauranga, Rotorua</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elsewhere in New Zealand</td>
<td>7.5</td>
<td>$\geq10s$</td>
</tr>
</tbody>
</table>

8 CONCLUSIONS

This paper describes the basis for the regionally-defined transition periods $T_L$ between constant-velocity and constant-displacement branches of the spectra which are recommended in the draft New Zealand Seismic Isolation Guidelines.

A literature review revealed major differences in the relationships between $T_L$ and moment magnitude $M_w$ used in the U.S. and Europe. Major differences between the results of engineering-based studies, based on recorded response spectra, and seismological studies, based on the corner frequency of the Fourier spectra of the Brune source model, indicated that the common assumption that the corner period $T_L$ of response spectrum corresponds to the inverse $1/f_c$ of the corner frequency of the Brune spectrum may be incorrect. This observation was strengthened by the finding that transition periods derived in this study from the response spectra shapes for the three largest events of the Canterbury earthquake sequence were systematically lower than the periods corresponding to the inverse of Brune-spectra $f_c$ derived by Oth and Kaiser (2014). Accordingly, in this study the recommended magnitude-dependent relations for transition periods $T_L$ for response spectra are based largely on the results of engineering
rather than seismological studies.

The use of $T_1=3s$ in NZS1170.5 lags behind engineering design practice in both the USA and Europe, which use $T_1$ values larger than 3s at least for magnitudes larger than 6.5. This was reinforced by $T_1$ values for response spectra predicted by the 2014 NGA-West earthquake ground-motion equations generally exceeding 3s for magnitude 7 and larger.

Results of the response-spectra based studies led to the recommendation of a magnitude-dependent relationship that produces $T_1=3s$ for magnitude 6.5 and $T_1=10s$ at magnitude 7.5. These values conform at magnitude 6.5 to those from the well-known Calvi (2008) relationship that is often used in Europe for displacement-based design, and at magnitude 7.5 with the IBC values from the U.S.

$T_1$ values are recommended to be increased from the NZS1170.5 value of 3s for all of New Zealand apart from Northland and Auckland, up to a maximum of 10s in the most highly seismic parts of New Zealand. The maximum recommended $T_1$ value of 10s is less than the IBC values of 12s for California, and 16s for Alaska where large magnitude subduction earthquakes dominate the hazard.

9 ACKNOWLEDGEMENTS

Anna Kaiser kindly provided the data on corner frequencies calculated for the Canterbury sequence, and Chris Van Houtte produced the New Zealand response spectra data and the spectra calculated from the NGA West GMPEs.

REFERENCES


