

Simplified vs. optimal techniques for viscous damper design: some preliminary observations

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ABSTRACT: The effectiveness of control strategies in achieving the objectives of a seismic design is well accepted in the earthquake engineering community. Consequently, various methods have been proposed for the optimal design of dampers and their distribution up the building height. A majority of these methods rely on highly-computational expensive optimisation frameworks and often rely on frequency or time-domain methods for analysis. From a commercial perspective, in the initial concept design stage, this is a major impediment as simple methods are needed to assess the feasibility of adopting viscous dampers - both in terms of seismic performance and financial viability. In this paper a preliminary investigation is made into the recently-developed direct displacement-based damper design methodology. The preliminary results obtained strongly give a positive indication that the direct displacement-based methodology may be used as a practical tool for design in an initial concept design stage. The paper also highlights the benefits of using a more mathematically sophisticated optimisation algorithm in the detailed design stage as it gives huge financial benefits in terms of minimising the damping needed to achieve a targeted drift.

INTRODUCTION

The goal of earthquake engineering is to reduce seismic risk in urban and rural areas to socio-economically-acceptable limits (i.e., to provide both life safety and economic protection). The conventional capacity design strategy relies on the “evasion” of seismic forces by enduring inelastic deformations. This philosophy could also be described as “dissipation with degradation” as seismic energy is dissipated by inelastic deformation. Due to the reliance of this philosophy on inelastic deformations, this may result in heavy damage to the parent structure which, in turn, results in heavy economic losses. Some of the best examples of this type of loss include claims for the 2016 Kaikoura and 2010/2011 Canterbury earthquakes. Although the Kaikoura earthquake did not cause any major collapse or life-safety issues, so many commercial buildings in Wellington and nearby places were shut down - causing society a loss estimated to be ~US\$1 billion (NBR website, 2016). Similarly, for the Canterbury sequence of earthquakes, a total loss of NZD \$ 40 billion to date is being estimated (3 News NZ, February 22, 2017). Similar observations could be made of events in other parts of the world. To cite some: in 1989 the M6.9 Loma Prieta earthquake caused more than US \$8 billion in direct damage (several buildings and bridges suffering total or partial collapse) although no major loss of life occurred. Similar observations were made of the 1995 Kobe earthquake (US \$102.5 billion in damage, 2.5% of Japan's GDP at the time) and the 1999 Chi-Chi earthquake causing US \$10 billion worth of damage. These observations raise a huge concern on the present-day design philosophy, and call for innovative strategies which reduce damage and ensure both “life safety” and “economic safety”.

Considering all these aspects, from a dynamic perspective, a more rational approach to reduce damage might be to rely on the philosophy of “dissipation without degradation” rather than “evasion/dissipation” of seismic forces by degradation. One of the ways to achieve this is by increasing the amount of damping in the system by using either active, semi-active or passive control. In this paper the focus is on pure viscous dampers. Viscous dampers are mechanical devices inserted in the parent structure to increase the damping of the whole structural system. The force in a viscous damper is a function of the velocity of the extension/compression in the damper and is related as follows:

$$\mathbf{f}_{damp}(t) = \mathbf{c}\{\dot{\mathbf{x}}(t)\}^\eta \quad (1)$$

Here, $\mathbf{f}_{damp}(t)$ represents the damper force, \mathbf{c} is the damper coefficient, $\dot{\mathbf{x}}(t)$ is the velocity and the exponential η varies from ~ 0.15 to 2.0 . For seismic purposes, the effective range of η is ~ 0.15 - 1.0 (Constantinou et al. 1993). This dependency of the force on the relative velocity is the main reason why dampers are effective in reducing seismic responses.

A typical installation is as shown in Fig. 1.0



Figure 1.0. A typical installation of viscous dampers (Courtesy of Miyamoto Ltd. website)

Present state of the art approach to damper design

The typical design of a system incorporating viscous dampers requires addressing a highly-coupled dynamic two-fold problem; the first one is determining how much damping is required, and the second is where to position individual dampers. These aspects are coupled, and a realistic optimum/economic solution might not be achieved if they are treated as uncoupled.

Considerable advancements in tackling this coupled problem have been reported. One school of thought attacks the problem in a de-coupled manner in which the total damping is pre-determined and optimal techniques based on classical optimisation theory are used to place the dampers optimally to achieve a target performance; relevant works in this direction are: Zhang & Soong 1992, Tsuji & Nakamura 1996, Takewaki, 1997a, 1997b, 1998, 1999, Singh & Moreschi, 2001, 2002, Garcia 2001. In some of these works, approaches to estimate a reasonable total-added damping are also proposed.

Direct schemes for the coupled problem were first presented by Lavan and Levy (2005, 2006, 2010), and Lavan (2012, 2015a, 2015b). They also presented a practical analysis/redesign procedure for arriving at the optimal designs by exploiting the advantage of the fully-stressed characteristics of the optimal solution. While this formulation lends itself to a design framework based on allowable inter-story drifts, a considerable amount of mathematical implementation and upfront detailed modelling is required to compute the quantity and distribution of dampers

From a practical engineering perspective, especially at a concept stage, most of these methods described above might be non-viable options - mainly because of the frequency/time domain framework in which they are formulated. It has to be noted emphatically that, in a detailed-design stage, these approaches give very reliable and financially viable results as they target the prescribed drifts.

In order to cater for the practical aspects of considering viscous dampers as a feasible solution for improving seismic performance, Lin et al (2003) proposed direct displacement-based design (DDBD) for quantifying and distributing the dampers. This method was further improved by Sullivan and Lago

(2012). Although the obtained quantity or distribution may or may not be optimal, the advantage of this method is still that it is elegantly simple, computationally inexpensive (mainly because it does not require frequency-domain or time-history analysis), and very closely follows the very familiar conventional displacement design of a frame. In this paper results of a very preliminary study are presented to review the possible upper boundedness of the DDBD approach in quantifying the dampers. The upper-bound nature of the method is very important as it would be easy for a designer to quantify the dampers based on this approach with some confidence in a concept design stage mainly for costing purposes when investigating the possibility of using dampers. This paper also highlights the fact that, in a detailed design stage, considerable financial benefit is achieved by the use of more rigorous mathematically-sophisticated algorithms.

DESIGN METHODOLOGIES INVESTIGATED

This section mainly outlines the design methodologies investigated in this study.

DDBD-based damper design

A full stepwise description of the DDBD approach is given in Sullivan and Lago (2012). The DDBD approach for damper design basically relies on the classical framework of DDBD proposed by Priestley et al. (2007). The relevant steps are consolidated as follows with a very brief description (Sullivan and Lago 2012):

- a. Define an empirical design displacement profile based on a target drift.
- b. Assume the proportion of the damper design base shear; normally, it is taken as follows:

$$f_{damper,i} = \beta V_i \quad (2)$$

Where $f_{damper,i}$ is the design damper force at the i^{th} level, β is a factor which varies from 0.3-0.6, and V_i represents the storey shear at level i . In the present study, β is given uniformly over the height of the building, and the two values adopted for β are 0.3 and 0.5. The recommended value is 0.3 (Sullivan and Lago 2012). A value of 0.5 might be a little high as will be evidenced in Section 3.0 which is still used mainly to check the sensitivity of the factor.

- c. Calculate the SDOF system damping.
- d. Scale the 5% target displacement spectrum to account for the damping.
- e. Compute the effective period of the SDOF.
- f. Compute the effective stiffness and base shear.
- g. Compute member forces and damper design forces as outlined in step b.
- h. Compute damper coefficients.

Problem formulation for classical optimisation schemes

This section describes the formal optimisation problem. The performance index is selected as the maximum of the inter-storey drifts below a selected target drift.

Equations of motion

The equations of motion of the nonlinear frame with added dampers are given as:

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{u}}(t) + [\mathbf{C} + \mathbf{C}_{damper}(\mathbf{c}_d)]\dot{\mathbf{u}}(t) + \mathbf{f}_s(\mathbf{u}(t), \dot{\mathbf{u}}(t)) &= -\mathbf{M}\ddot{\mathbf{u}}_g(t) \\ \mathbf{u}(0) &= 0; \dot{\mathbf{u}}(0) = 0 \end{aligned} \quad (3)$$

In Eq. (3), \mathbf{M} represents the mass matrix and $\mathbf{f}_s(\mathbf{u}(t), \dot{\mathbf{u}}(t))$ represents the restoring forces' vector at time t . Similarly, \mathbf{C} represents the inherent damping matrix. $\mathbf{C}_{damper}(\mathbf{c}_d)$ is the added supplemental damping matrix and \mathbf{c}_d is the added damping vector. \mathbf{i} represents the ground motion directional vector, and $\ddot{\mathbf{u}}(t), \dot{\mathbf{u}}(t)$ and $\mathbf{u}(t)$ are the relative accelerations, relative velocities and relative displacements, respectively. $\ddot{\mathbf{u}}_g(t)$ is the ground acceleration.

Optimisation problem formulation

The optimisation problem is formulated as,

$$\min J(\mathbf{c}_d) = \mathbf{c}_d^T \mathbf{1} \quad (4)$$

Subject to:

$$\Xi = \frac{\Phi}{\Phi_{allowable}} \leq 1.0 \quad (5)$$

Here, Φ refers to the maximum drift computed - based on the maximum peak response. Mathematically, Φ is given as:

$$\left. \begin{aligned} \Phi &= \max_i \left(\max_t (abs(\mathbf{d}_i(t))) \right) \\ \text{where } \mathbf{d}_i(t) &\text{ is the response vector computed using transformation matrix } \mathbf{H} \text{ as,} \\ \mathbf{d}_i(t) &= \mathbf{H}_i \mathbf{u}(t), \\ \text{where "i" refers to the storey level and satisfies the following equation,} \\ \mathbf{M}\ddot{\mathbf{u}}(t) + [\mathbf{C} + \mathbf{C}_{damper}(\mathbf{c}_d)]\dot{\mathbf{u}}(t) + \mathbf{f}_s(\mathbf{u}(t), \dot{\mathbf{u}}(t)) &= -\mathbf{M}\ddot{\mathbf{u}}_g(t) \\ \mathbf{u}(0) &= 0; \dot{\mathbf{u}}(0) = 0 \\ 0 &\leq \mathbf{c}_d \end{aligned} \right\} \begin{array}{l} \text{whole} \\ \text{ensemble} \end{array} \quad (6)$$

Classical optimisation scheme

In order to make sure that the computed optimal number of dampers is correct, two methods of optimal design are implemented. One is a gradient-based optimisation scheme in which the gradients are computed analytically by the Adjoint Variable method (Lavan 2006, Puthanpurayil et al, 2015), and the other one is more of a sub-optimal method called the Analysis/redesign scheme (Lavan 2006, Lavan and Levy 2006). In both schemes, the problem formulation remains the same as outlined in Section 2.0. Only a brief description of the gradient-based formal optimisation scheme is presented in this Section. The Analysis/redesign method was mainly used to ensure the robustness of the formal optimisation scheme, and no further description of the method is given.

Gradient-based optimisation scheme

The damper-optimisation problem, as formulated in eqs. (4-6) is highly nonlinear in nature. Conceptually, the gradient-based scheme can be looked upon as an incremental, first-order Taylor series method in which the whole damper coefficient continuum domain is discretised into a set of piecewise-

linear optimisation problems which are solved iteratively. Only a very brief review of the steps is given in this Section. For more details interested readers should refer to Puthanpurayil et al (2015), Lavan and Levy (2006) and Lavan and Levy (2005).

- *Selection of active ground motions*

As the main aim of the present study is to investigate the viability of the DDBD approach to be adopted in the scheming stage, a set of ground motions scaled to a specific displacement spectrum level is adopted. It is possible to use only scaled ground motions to compute the optimal quantity of dampers. However, in order to be computationally efficient, an active ground-motion methodology as described in Lavan and Levy (2006) is adopted in the present study. Active ground motion set can be considered as a subset of the selected suite of ground motions and comprises of those ground motions which maximises the responses of the structure. As per the methodology, spectral response curves of a linear single degree of freedom system having the same fundamental natural frequency as that of the parent structure vs. the damping coefficient are generated. Maximum spectral displacement is adopted in this study as the desired quantity of interest which determines whether the ground motion falls into an active set or passive set. The ground motion which produces the largest spectral response curve for a reasonable range of damping is taken as the critical active ground motion.

- *Computation of envelope drift responses for the inelastic frame*

Adopting an initial amount of damping vector \mathbf{c}_d , solve eq. (3) using any of the time integration schemes available in literature for the set of ground motions identified as *active ground motions*. In this study, the total equilibrium Newmark constant average acceleration method described in Carr (2007) and Puthanpurayil et al (2014) is used. Since the present study is predominantly to check the viability of the DDBD approach and to investigate whether there are any additional benefits for performing optimisation, the initial \mathbf{c}_d vector adopted is the same as that obtained from the DDBD approach. Although the framework itself does not require an informed selection of the initial \mathbf{c}_d vector, a judicious selection will reduce the computational time involved. Some of the other alternative ways to arrive at the initial amount of \mathbf{c}_d vector using approximate methods are available in the literature; for more details refer to Liang et al (2012).

- *Evaluation of performance index Ξ*

Evaluate Ξ using eqs. (5) and (6).

- *Gradient computation of the performance index Ξ and the objective function J*

Gradient for the objective function J is trivial and the sensitivity will return a vector $\mathbf{1}$. But the gradient of the performance index Ξ is not trivial. There are different ways in which the gradients can be computed; in this study the gradients are derived using the Adjoint Variable method (AVM) using a *differentiate and discretise approach* which is briefly outlined below:

The Lagrangian augmented equation is given as,

$$\Xi = \frac{\Phi}{\Phi_{allowable}} + \int_0^{t_f} \lambda^T(t) \{ \mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{f}_s(\mathbf{u}(t), \dot{\mathbf{u}}(t)) + \mathbf{M}\ddot{\mathbf{u}}_g(t) \} dt \quad (7)$$

On applying calculus of variations and solving the resultant Lagrangian enhanced equation using temporal discretisation schemes we get the gradient as,

$$\delta\Xi = \int_0^{t_f} \lambda^T(t) \delta\mathbf{C}\dot{\mathbf{u}}(t) dt \quad (8)$$

For more details refer Levy and Lavan (2006). In comparison to the classical computation of gradients using the finite difference method, an AVM method based on a *differentiate-and-discretise*

approach has high computational benefits.

- Estimate a new \mathbf{c}_d for the optimal design using Sequential Linear Programming (SLP)

The original optimisation problem is given in Equations (4) - (6). This is a highly nonlinear programming problem. One of the approaches to solve this problem is to discretise the nonlinear problem domain into piecewise-linear domains and then to solve the obtained linear domains by using SLP. Although other higher order schemes are available, SLP is still chosen mainly because other nonlinear schemes may require the computation of second-order sensitivity and hence necessitate the estimation of Hessian matrices which may pose serious difficulties in the vicinity of the optimum solution. Linearisation of the objective function given by Eq. (4) at the i^{th} iteration gives:

$$J^i(\mathbf{c}_d) = J(\mathbf{c}_d^i) + \{\nabla_{\mathbf{c}_d} J(\mathbf{c}_d^i)\}(\Delta\mathbf{c}_d^i) \quad (9)$$

Linearisation of the constraint at the i^{th} iteration satisfying Eq. (5) results in,

$$\Xi^i(\mathbf{c}_d) = \Xi(\mathbf{c}_d^i) + \{\nabla_{\mathbf{c}_d} \Xi(\mathbf{c}_d^i)\}(\Delta\mathbf{c}_d^i) \quad (10)$$

Equations (9) and (10) are solved and the incremental $\Delta\mathbf{c}_d$ vector required for the next iteration is obtained by solving a modified linear programming problem. Once $\Delta\mathbf{c}_d$ is obtained, the damping vector \mathbf{c}_d is updated as:

$$\mathbf{c}_d + \Delta\mathbf{c}_d \quad (11)$$

- Check for termination condition

The iteration is terminated if the change in the added damper vector $\Delta\mathbf{c}_d$ is less than the tolerance or the maximum number of iterations has been reached. Otherwise, update the iteration number as $i=i+1$ and proceed back to Step 1.0. Check for all other ground motions in the ensemble to see whether the present optimal damper distribution violates the constraint on the drift. If it violates it, then the ground motion gets added into the active set of ground motions and the optimisation steps are repeated.

NUMERICAL STUDY

A four-storey reinforced concrete frame designed in accordance with Eurocode 8 (EC8) and Eurocode 2 (EC2) is used to illustrate the proposed optimisation procedure (Arede 1997). The frame is designed for high seismicity assuming a PGA of 0.3 g. The geometric details of the frame and the arrangement of the dampers are given in Fig. 2. Material and section properties, along with the nonlinear parameters, are given in Appendix A.

A target drift of 1.8 % was identified as the performance limit and DDBD design was performed using this target drift. The obtained final damper vector is adopted as the initial vector for both the optimisation schemes, and the obtained distribution of dampers per floor level is given in Fig. 3. As described in Section 2.1, the β factor in DDBD is varied and the damper quantities are given for $\beta = 0.3$ and $\beta = 0.5$. The results reinforce the recommendation of a $\beta = 0.3$ by Sullivan and Lago (2012).

The damper distribution from the formal optimisation scheme is obtained by running the active *ground motion set* which is scaled to the respective 5 % damping displacement spectrum used for the DDBD design. The algorithm described in the previous section is used for formal optimization and the obtained results (damper quantities and distribution) are checked in parallel using the Analysis/Redesign approach (Levy and Lavan 2006).

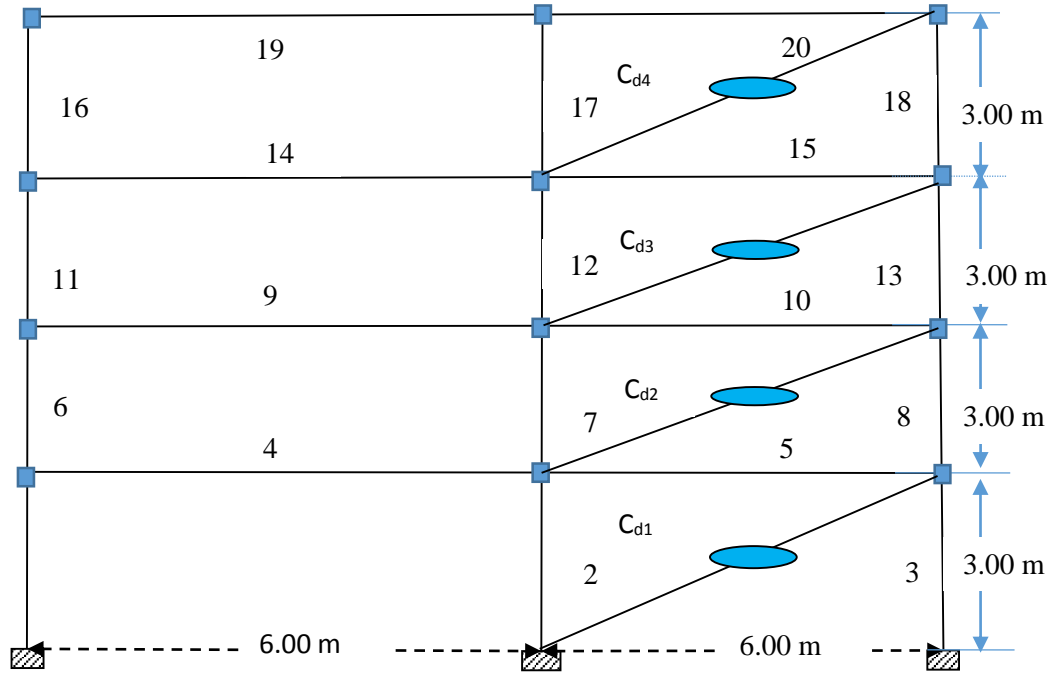


Figure 2.0. C_{di} refers to added dampers and $i=1 \dots 4$

An approximate difference of only 3.5 % in the quantity of dampers is obtained between the formal optimisation scheme and the analysis/redesign methodology. This is highlighted in Fig. 3.0.

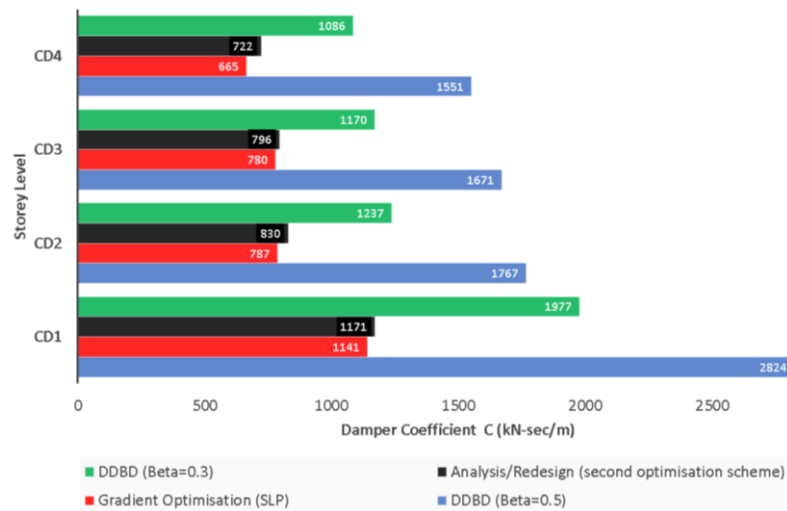


Figure 3.0. Damper distribution per floor level obtained from different methods.

Obviously, Fig. 3.0 illustrates that, on optimisation, a better economical distribution is achieved. For this specific frame, a reduction of ~40 % in the total number of dampers is obtained from the formal optimisation methodology as compared to the quantity obtained by DDBD with a $\beta = 0.3$; when , $\beta = 0.5$ this difference can go up to ~57 %. **This clearly shows that, in the detailed-design stage, the adoption of formal optimisation schemes is an imperative and would give a huge financial benefit.** Although no concrete conclusions on the upper boundedness of DDBD can be obtained at this stage, the results obtained for this specific frame give a clear indication that it may be used as a useful tool at the concept design stage for sizing dampers. Computing a reliable upper bound is an important aspect as this will dictate the financial feasibility of the damper design in a concept design stage and will give enough confidence for the design engineer to proceed to the final detailed design. The other obvious advantage of DDBD is that it is extremely fast as it only requires static analysis - as compared to the nonlinear time-history analysis required by the formal optimisation schemes.

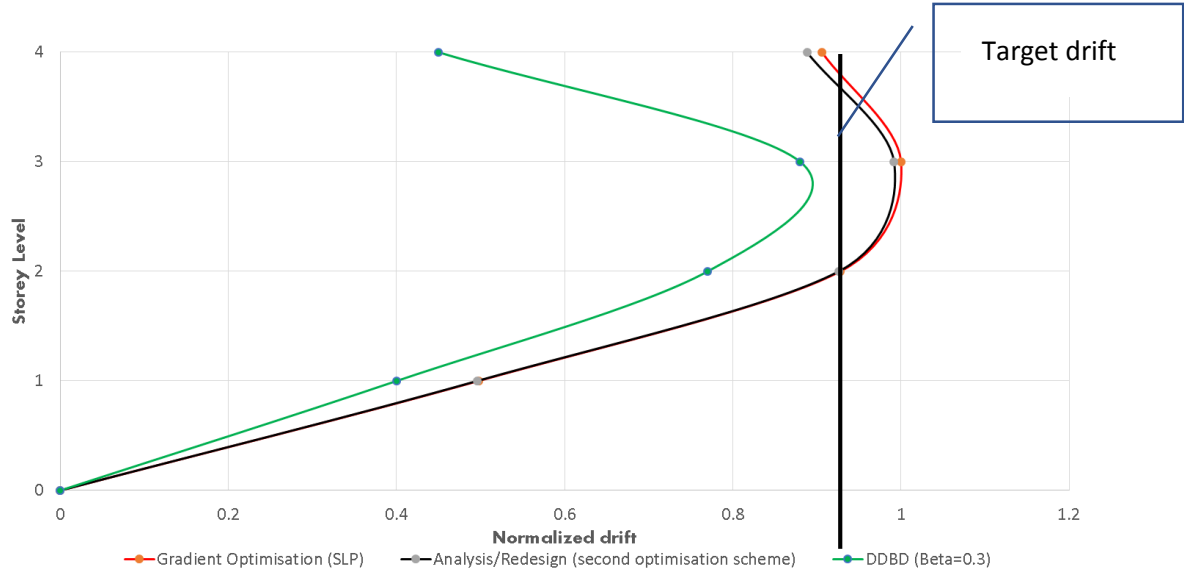


Figure 4.0. Normalised drift ratio plot.

In order to review the performance of the obtained design, the frame equipped with DDBD-designed dampers for $\beta = 0.3$ was subjected to the *active ground motion set*. Fig. 4.0 presents the normalized drift as exhibited by different schemes. The normalised drift is obtained as a ratio of the peak drift obtained per storey to the target drift. It clearly shows that the classical optimisation scheme design clearly achieves the drift objective whereas the DDBD gives conservative distribution of the drifts. **The more interesting aspect is that the conservative peak drift is not much less than the target drift (in this specific case only ~10% less than the target value) ; it has to be noted that for this specific frame, this reduction of further 10% is obtained at a higher cost of ~40 % more damping material. This reinforces the fact that an optimised scheme should be employed in the detailed design stage to increase the financial viability of the damper design.** On the other side this also clearly indicates, at least in the case of this frame, that DDBD gives an upper bound in terms of performance as well. Now a pressing question that arises is “would DDBD always give upper bound results?” This is a question which needs further research.

CONCLUSION

A preliminary investigation into the viability of using DDBD for the design of dampers in the schematic design stage is presented. The results obtained for this specific case study shows that DDBD may give conservative results for both damper numbers and distribution; it is also shown that DDBD satisfies the performance criteria conservatively; Whether DDBD based damper sizing would always give upper bound quantity of dampers need to be further investigated. Sufficient evidence is also presented to confirm that a considerable economic advantage in terms of reduced total quantity of damping required (directly proportional to the upfront cost) for satisfying the same performance would be achieved if formal, classical optimisation schemes are used in the detailed design stage.

APPENDIX A: MODELLING DETAILS OF THE FOUR-STOREY FRAME

Material Property

The dynamic Young's modulus of concrete $= 3.5 \times 10^{10} \text{ Nm}^{-2}$

Geometric Properties

Member number	Width (mm)	Depth (mm)
1,6,11,16,17,12,7,2,3,8,13,18	450	450
4,5,9,10,14,15,19,20	300	450

Nodal Mass

Floor level	Mass per node (kg)
1 st floor	29 800
2 nd -4 th floors	29 500

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