

# State of the Art: Challenges in analytical modelling of multi-storey shear wall buildings

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**ABSTRACT:** Shear walls have been used extensively as the main lateral force resisting systems in multi-storey buildings. Recent development in performance based design urges practicing engineers to conduct nonlinear static or dynamic analysis to evaluate seismic performance of multi-storey shear wall buildings by employing distinct analytical models suggested in literature. These models mainly incorporate macro models as against the micro models often employed for isolated walls or experimental specimens. Reliable and robust analytical models for large scale multi-story shear wall buildings should ideally exhibit reasonable global seismic behaviour of the whole structure as well as important local characteristics of structural walls under seismic actions while minimising the required computational effort.

Many challenging issues still remain in modelling nonlinear behaviour of shear walls in multi-storey buildings. Experimental observations have demonstrated that when shear walls undergo large drifts, the *plane section remains plane* hypothesis is not true. This raises question marks over the commonly used approaches of treating shear walls as equivalent wide columns because this analogy uses a moment curvature analysis with the Bernoulli hypothesis. An important issue is the significance of shear behaviour and its interaction with the axial and flexural behaviour which is commonly overlooked in the nonlinear analysis of multi-storey shear wall buildings largely because of lack of efficient analytical models. Further, shear walls interact with the floor slabs affecting the overall seismic behaviour of the building. This paper attempts to review existing analytical models for shear walls in multi-story buildings and their advantages and limitations.

## 1 INTRODUCTION

Shear walls are commonly used as a main lateral force resisting system in low, medium and high rise reinforced concrete (RC) buildings in seismically active countries. With development of performance based design or assessment, engineers require to conduct nonlinear static or dynamic analysis to accurately estimate local and global seismic demands in terms of inter storey drift ratios, element rotations, section curvatures or strains. While material strain or curvature demands seems to be more robust in seismic design or assessment, available commercially used software and assessment guidelines give acceptance criteria for ductile components in different limit states (or performances) in terms of rotations in beams, columns and slender walls (ASCE41-06). The New Zealand standard (NZS3101-06) evaluates the curvature and strain ductility demand in different limit states in RC building components.

A robust analytical shear wall model for nonlinear analysis of multi-storey buildings is essential for reliable seismic performance assessment. These models must be capable of estimating the global seismic demands with an acceptable accuracy and within a reasonable computational time. Moreover, they must be applicable in three dimensional analyses of multi storey buildings. There are many variables such as shear span ratio, interacting nonlinear axial, shear and flexural behaviour, boundary elements, connections to slabs and transverse girders, which affect the seismic behaviour of shear walls in buildings. Hence, accuracy of a model in the simulation of isolated wall specimen is not

necessarily sufficient to employ in real multi-storey shear wall building analyses. In the authors' knowledge, there is no current reliable macro element capable of capturing all the different failure patterns in multi-storey shear wall buildings.

In this paper, attention is focused on relatively simple and reasonably accurate wall analytical models based on a macroscopic approach. The main features of these models are discussed and an attempt is made to offer a practical model that is capable of predicting the behaviour of shear walls in three dimensional reinforced concrete structures.

## 2 REVIEW OF AVAILABLE NONLINEAR MODELS

### 2.1 Wide column analogy

Treating a shear wall as a wide column is a common approach. In this model (Fig. 1), rotation occurs around the wall centroidal axis and movement of the neutral axis and rocking (upward and downward movement of boundary elements) cannot be captured. However, when the vertical deformation of wall edges is of great importance, especially in the case of considering wall interaction with adjacent frames, this effect can be accounted for by adding horizontal rigid beams on either side of the vertical columns (Bertero et al. 1984). However, this approach cannot realistically model movement of the wall edges, especially with large axial tension, and the elongation of a wall under horizontal reversed cyclic demands.

In the one component model (Giberson 1967), the line elements aligned at the wall centroidal axis require the elastic flexural stiffness and strength (based on section moment curvature analysis or code recommendations) in the middle segment and also the post-elastic stiffness for nonlinear rotational spring at the ends (Fig. 1). The end springs have an infinite stiffness before the occurrence of flexural yielding and all plastic deformation is lumped in these springs. The one component model has been modified to include inelastic shear springs at its end in series with the flexural springs (Satyarno et al. 2000) (Fig. 1). The most commonly used moment rotation hysteresis rule for the end rotational springs are the Takeda or modified Takeda hysteresis (Otani 1974, and Saiidi and Sozen 1979) having a trilinear backbone curve to account for cracking, yielding and strain hardening of the concrete elements and with stiffness degradation. In a linear analysis, design codes commonly recommend a constant flexural stiffness reduction factor over the entire height of multi-storey buildings to account for concrete cracking, reinforcement yielding and axial forces. However, a wide range of recommendations is found for these flexural stiffness values in different codes. This model may not be appropriate in shear wall buildings with high axial gravity forces, high shear force demand or walls with varying axial forces during the analysis. Moreover, shear wall buildings with high axial forces and a light longitudinal reinforcement restrict cracking only to a small portion of the walls; and the significant un-cracked part is commonly neglected in the moment curvature idealization or equivalent stiffness method.

The wide column analogy adopts the Bernoulli Hypothesis and it uses the *plane section remains plane* assumption in its formulation by enforcing a linear distribution of strain at the section level. Moreover, the shear strength and stiffness properties of the walls are commonly derived independently. The shear spring properties are assumed constant during the structural analysis. In other words, this model commonly overlooks shear-flexure interaction. This implies that shear strength and stiffness do not degrade by increasing flexural rotational or displacement ductility demands. Satyarno et al. (2000) implemented a shear spring in the finite element analysis program Rauamoko using the curvature ductility demands to reduce the shear strength of shear springs.

The advantage of this beam analytical model is its computational efficiency in a nonlinear response history analysis of large multi-storey shear wall buildings. It is also easy to calculate capacity in terms of rotation or inter storey drifts and to compare with available performance acceptance criteria in guidelines. Hence, this model is commonly used in exploring dynamic response of multi-storey shear wall buildings (Goodsir et al. 1983, Roudriguez et al. 2002).

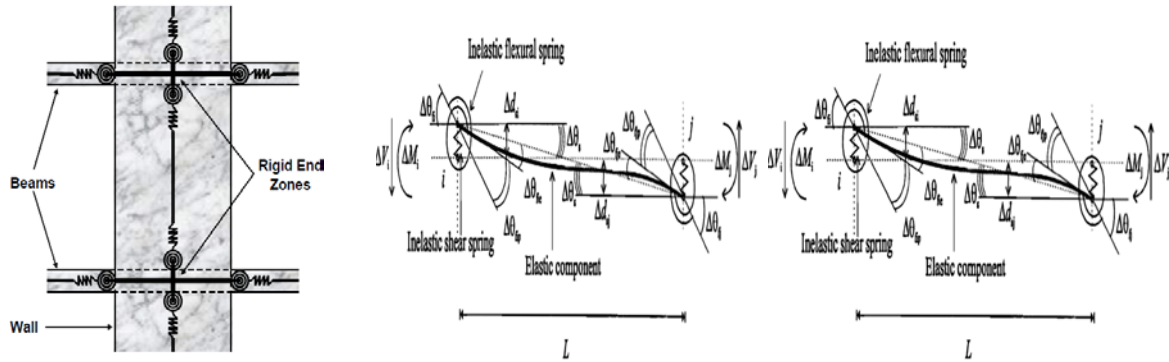


Figure 1. Wide column analogy and one component Giberson (with and without shear spring).

## 2.2 Line elements with fibre section

In this approach, sections are discretized to many uniaxial steel and concrete fibres with their own mechanical and geometric properties (Fig. 2). The basic idea was introduced by Park et al. (1972) to capture flexural cyclic behaviour of beams. Based on this concept, Taylor (1977) proposed a wall element using uniaxial cyclic behaviour of concrete and reinforcement at each fibre at each integration section over the length of the wall. This model, which was incorporated in Ruaumoko, allows for the shift in neutral axis which is very important in coupled shear walls.

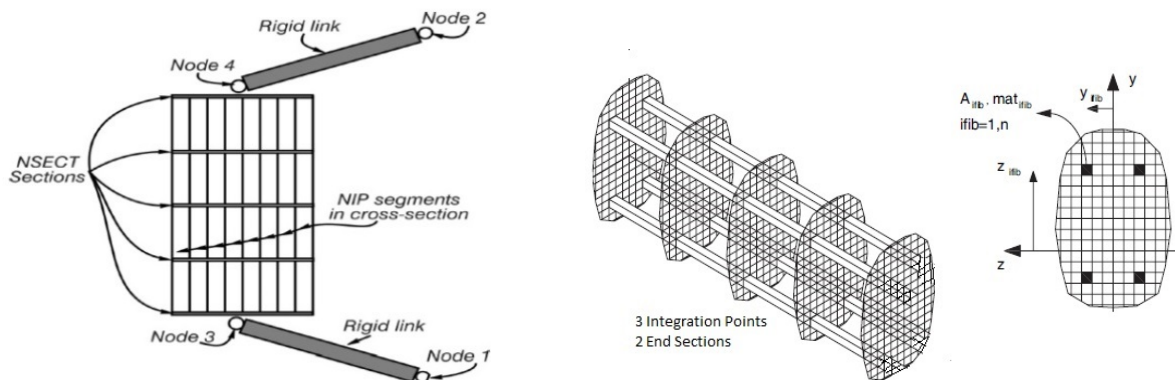


Figure 2. Wall element in Ruaumoko2D and a fibre section line element in OpenSees.

The Bernoulli' hypothesis is commonly used in fibre section formulation. This type of beam assumes that the cross-sections remain plane and normal to the reference axis during the deformation history. This assumption implies a perfect bond between reinforcement and concrete. The force based fibre beam column element assumes the linear moment and axial force distribution over the length of element. Based on the flexibility approach, the section forces are determined (moment and axial forces) from interpolation of element end forces. Nonlinear behaviour of the materials is tracked in three cross sections along the length of the elements. In these models distribution of plasticity is induced through the member cross section and along the member length by numerical integration. The fibre elements report the seismic demands as strains in reinforcement and concrete. Calculation of plastic hinge rotations in these models requires the user to post process outputs (strains). The strain demands in this approach are quite sensitive to moment gradient, element length, integration method and strain hardening. Likewise, displacement based elements enforce linear curvature along the member. Therefore, more elements are required in regions of high curvature variations (like plastic hinge zones). These two elements are implemented in OpenSees.

Martinelli and Filippou (2009) employed this approach to simulate the shaking table test of a seven storey shear wall building. He recommended application of this model for walls of medium to high slenderness undergoing primarily flexural response with negligible shear effects. More recently, Pugh (2012) demonstrated the incapability of force based fibre element in capturing ductility demands even

with increasing the number of integration points. A material regularization method was proposed to adjust the uniaxial behaviour of concrete and steel based on the number of integration points (mesh dependent behaviour) in force based fibre elements.

The big advantage of this approach is that axial flexural interaction can be explicitly captured and there is no need to define controversial values of effective elastic stiffness for members. This flexural stiffness is a serious concern for engineers especially in coupled walls when the axial forces can significantly change the flexural stiffness during the analysis. On the other hand, this approach has some unanswered questions about their robustness to predict nonlinear shear strains and their degradation with axial strains.

**2.3 Truss analogy**

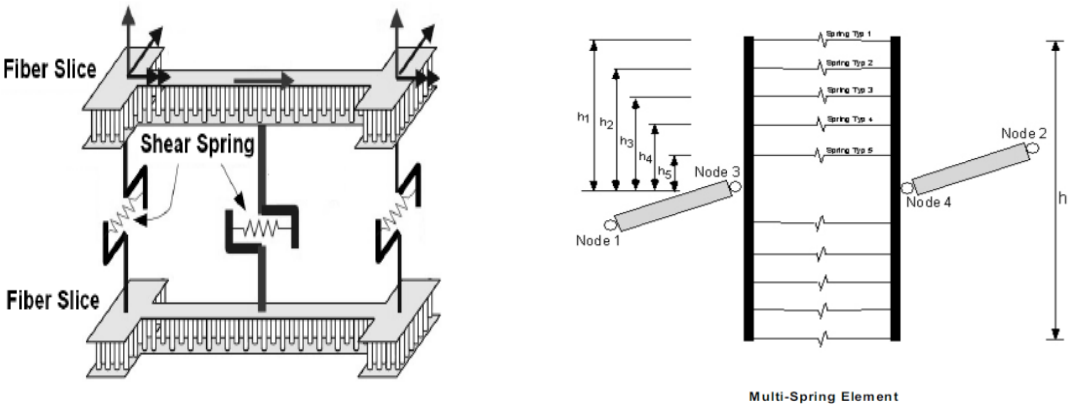
An equivalent truss model was employed to predict monotonic strength capacity of walls in the experimental tests carried out by Hiraishi (1984) and Oesteler et al. (1984). Truss members are used in this macro model which consists of two vertical and one diagonal truss member connected to each other by a rigid horizontal beam or tie. The diagonal truss is used to represent the diagonal compression strut in the web providing shear resistance. The assumption in this analogy is that the truss elements are statically determinate. Non-prismatic truss elements can be employed in plastic hinge regions to avoid the *plane section remains plane* assumption.

The applicability of this approach is usually limited only to monotonic loading mainly because assigning appropriate properties of the truss members under cyclic loading is very challenging (Vallenas et al. 1979). Moreover, it is not easy to decide a suitable number of truss members for whole shear wall, and realistic prediction of deformation due to gravity load and lateral force is not easy to achieve in this analogy. However, if carefully calibrated, this model may give useful results under small gravity load and static monotonic force (Linde and Bachmann, 1994).

Recently, some researchers attempted to improve the truss analogy of Hrennikoff (1941) to simulate the cyclic response of shear wall specimens (Panagiotou et al. 2012). In the enhanced lattice models, implemented into Ruaumoko, longitudinal, transverse and diagonal truss elements in a finite element type mesh were used to represent concrete and reinforcement steel. This model captures stiffness and strength degradation, the strain histories in the reinforcement and concrete, and accounts for the dependency of the concrete stress-strain relationship in compression on the transverse strains.

**2.4 Multi spring elements**

The multi spring element concept (Fig. 3) was initially introduced for analytical modelling of columns (Lai et al. 1984), and it was used to simulate column sections under biaxial bending. A similar concept was employed to simulate a three dimensional shear wall building by Fu et al. (1992). This model has been further enhanced by Li (2006) to account for cracking, tension stiffening and confinement effect in compression and non-linearity in shear.



**Figure 3. Multi spring model for wall in Finite Element Programs Canny and Ruaumoko.**

Another version of multi-spring element was developed by Speith et al. (2004) (Fig. 3). This element comprises of 10 concrete and 10 steel springs with cyclic behaviour which can be employed to simulate behaviour of critical sections in shear walls. It has been implemented in Ruaumoko and has been used to capture seismic performance and modelling of post-tensioned precast reinforced concrete frame structures with rocking beam-column connections and also shear wall structures.

One of the main turning points in understanding hysteresis behaviour of shear wall buildings was when Kabeyasawa et al. (1983) conducted shaking table test of a seven storey RC building with shear wall and frames. Three vertical line element model (TVLEM) was proposed as a reliable tool to predict the shear wall response by Kabeyasawa et al. (1983). TVLEM approach idealized a wall member under uniform bending (constant curvature in each storey) as three vertical springs with infinitely rigid beams at the top and bottom (Fig. 4). Two outside truss like elements represented the axial stiffness of the boundary columns and their axial stiffness varied with the sign and level of axial stress, and degraded with tensile stress history. This was modelled using the Axial-Stiffness Hysteresis Model (ASHM) shown in Figure 4. The central vertical element was a one component model in which vertical, horizontal and rotational springs were concentrated at the base. The effect of strain gradient across the wall section was represented by the rotational spring in the centre and shear deformation was controlled by the deformation of horizontal spring with Origin Oriented Hysteresis Model (OOHM) (Fig. 5). Most of the important aspects of nonlinear global behaviour of a wall could be simulated by this model quite well except for the shear deformations. However, this mode was developed based on specific test data and it requires many empirical parameters for calibration.

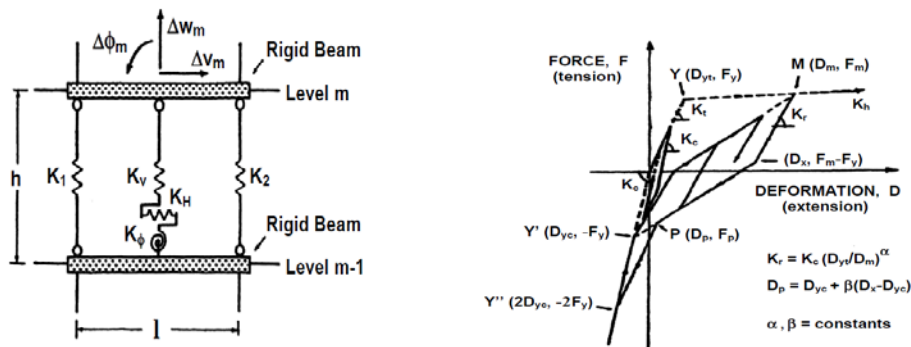


Figure 4. TVLEM model and hysteresis rule for axial springs (ASHM).

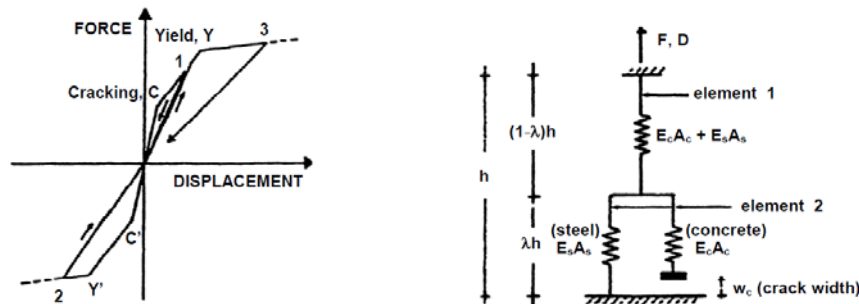


Figure 5. OOHM model for shear and parallel springs to capture tension stiffening in TVLEM.

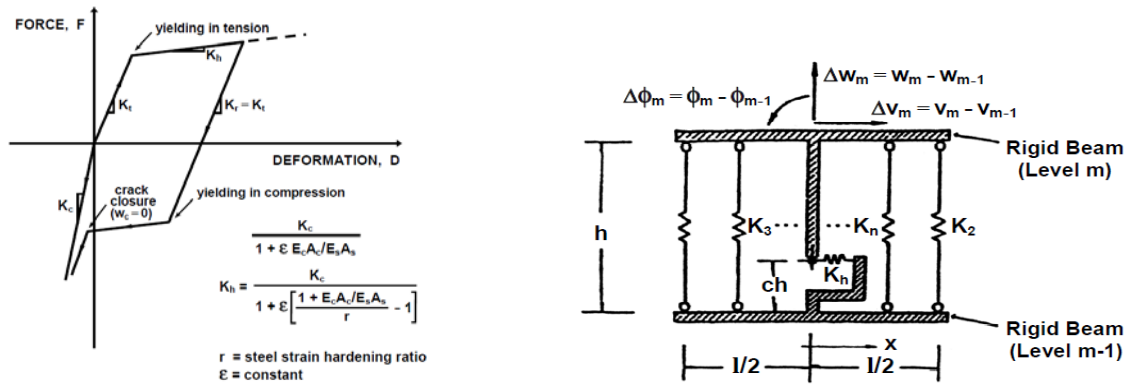


Figure 6. MVLEM model based on material law.

Many researchers tried to reduce empirical assumptions in the TVLEM model and increase the reliability of the model by employing material based hysteresis rules. Vulcano et al. (1988) improved the TVLEM by replacing the rotational spring with a number of longitudinal springs (Fig. 6). In their multiple vertical line element model (MVLEM), a structural wall is represented as a stack of springs which are placed on top of each other. While the two outside springs model the axial stiffness of the boundary columns, the interior springs with axial stiffness, represent globally the axial and flexural stiffness values of the central panel. They introduced the centre of rotation for a wall panel based on the assumption of constant or linear curvature distribution in each wall panel. However, suitable values for centre of rotation in each wall need calibration. Furthermore, they attempted to use hysteresis response of steel springs and concrete springs in parallel and in series to account for the tension stiffening effect (Fig. 5).

The main limitations of such multi spring models are the adoption of *plane section remains plane* in their formulation which is poor assumption in deep beams and shear walls. Moreover, nonlinear shear flexure interaction cannot be reproduced by these models and under high shear stress demands the shear response cannot be realistically predicted. Colotti (1993) modified the above model to enable it to capture shear flexure interaction in monotonic loading, especially when shear span ratios are less than 2.5. He employed the modified compression field theory to account for shear flexure interaction in each panel of a wall.

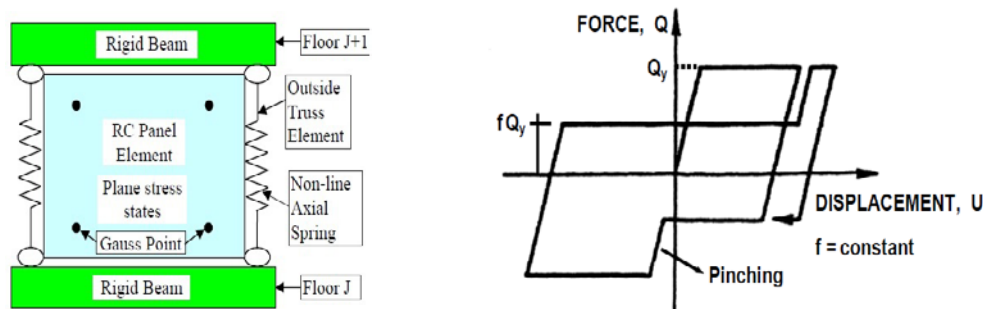


Figure 7. Improved models for shear behaviour.

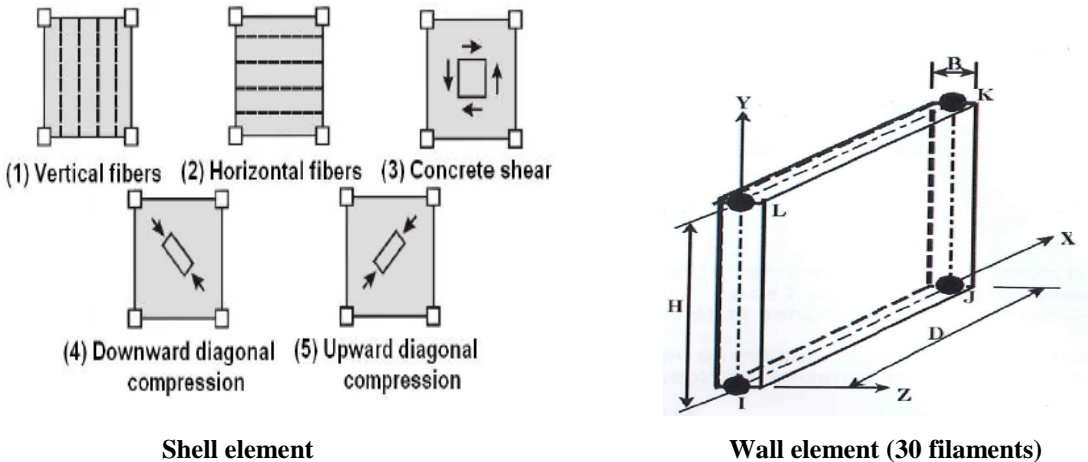
Linde and Bachmann (1994) modified the basic TVLEM by adding additional degrees of freedom at each node of the infinite rigid beam to eliminate the middle rotational spring. They attempted to generate the wall stiffness matrix based on elastic section properties and wall kinematic behaviour. However, in the proposed element, like initial element, an origin oriented hysteresis model (OOHM) was employed for shear behaviour to capture the dynamic curvature ductility demands in multi-storey shear wall buildings. Ghobarrah and Youssef (1999) attempted to enhance the TVLEM model to capture shear flexure interaction in cyclic analysis. They adopted modified compression field theory to consider the axial and shear strain interaction in each wall panel element. Chen and Kabeyasawa (2000) improved the TVLEM shear model by replacing the shear spring with an isoparametric panel element with biaxial behaviour in the middle of the wall (Fig. 7).



The MVLEM model was recently improved by implementing refined hysteretic uniaxial, cyclic material constitutive models instead of simplified force-deformation rules to predict the inelastic response of slender RC walls (Orakcal et al. 2006; Massone et al. 2006, Kolozvari et al. 2012). They attempted to reproduce monotonic and cyclic nonlinear shear flexure interaction in this element by adopting modified compression field theory and this element has been implemented in the nonlinear finite element analysis program OpenSees. However, experimental verification for some specimens with high shear stress demands was not convincing.

**2.5 Shell elements with fibre section**

In order to model bending, shear and diagonal compression behaviour, a wall element consisting of five layers acting in parallel was developed and implemented in Perform3D (Fig. 8). In the vertical and horizontal directions, axial and bending modes of wall behaviour were reproduced by some layers or springs based on vertical and horizontal fibre cross section properties including steel and concrete fibres. One shear layer assumes constant shear stress uniformly distributed over the wall length and its properties are defined from the shear resistance of concrete. Diagonal strut layers assume constant diagonal compression stress over the wall length. Through interaction with the axial-bending deformation in horizontal and vertical layers (springs), the diagonal compression layer can transfer the shear and account for the contribution of horizontal reinforcement in the shear resistance of the wall. However, the stiffness matrix derivation for this element is not clear in the manual and the model does not appear to predict the behaviour accurately in case of high shear force demands in squat walls. Moreover, no experimental verification is available to assess the robustness of the proposed element. This element has been used in analytical investigation of shear walls by several researchers (Kim and Wallace 2014, Tuna 2012).



**Figure 8. Shell element and Wall element.**

**2.6 New wall macro element in Ruaumoko3D**

A new macro element based on uniaxial behaviour of many filaments has been incorporated in the nonlinear analysis program Ruaumoko3D (Fig. 8). The Taylor Wall element (Fig. 2) was fine in a two dimensional model as the shear-centre is always in the plane of the wall. In three dimensions this is not possible as locating the shear-centre when parts of the concrete section are cracked or the steel is yielding is difficult. As an alternative approach, taking a near-rectangular finite element, the wall geometry with flanges and closed cells is built up and the shear centre is dealt with automatically. The element has 24 degrees of freedom including the drilling degrees of freedom (or a rotational degree of freedom perpendicular to element) at the nodes which make it easy to connect with beam and slab elements. The wall cross-section element includes 10 concrete and 10 steel filaments in the vertical direction, 4 concrete and 4 steel filaments in the horizontal direction and either 2 diagonal springs representing the shear action similar to that proposed by Peng et al. (2013) or as a single shear spring similar to that shown for the MVLEM model. The diagonal shear spring model introduces an axial-flexure-shear interaction whereas the single shear spring model has only the axial-flexure interaction. Many hysteresis options are available for the concrete and reinforcement. One of the main features of

this element which makes it distinct from others is the use of cubic functions for in-plane edge displacements avoiding the *plane section remains plane* constraint. The out-of-plane behaviour is modelled with a hybrid-stress plate bending finite element though a full two-dimensional cross section is under development. Initial results are promising but the element needs extensive verification and further research to enhance the shear-flexure interaction.

### 3 GLOBAL CHALLENGES

#### 3.1 Effect of shear force and shear deformation

Flexural stresses are distributed in wider lengths in walls compared to columns and beams. The simultaneous presence of shear force and moment results in shear cracking before yielding of transverse reinforcement over the length of a wall. These shear flexure cracks can affect the overall behaviour of walls in plastic hinge regions even in slender walls with flexure dominant behaviour.

Most engineers use the shear modulus of concrete based on elastic theory while using a wide column analogy for shear walls in low and mid-rise buildings. The elastic shear modulus of concrete is calculated as  $0.4E_c$  and keeping it constant during nonlinear response history analysis gives very small shear deformations in plastic hinge regions. Thus, its effect is commonly assumed negligible in the overall response of walls.

Many experimental tests (Oesteler et al. 1976 and Hines et al. 1999) demonstrated that in most walls designed for yielding in flexure, shear cracking induces considerable shear deformation in plastic hinge regions and consequently it affects the overall deflection of walls. This implies that shear cracking decreases the elastic shear modulus of concrete even in flexure dominant walls which is often overlooked in nonlinear response history. Lack of reliable experimental data for nonlinear behaviour of shear spring (shear force versus shear strains) in slender walls and appropriate analytical tools are some of the main issues. Shear deformation is more pronounced in slender walls when shear transfer mechanisms start to deteriorate because of high shear stress in plastic hinge regions. This phenomenon decreases the shear strength and shear stiffness of a wall which is called shear-flexure interaction (Beyer et al. 2011).

Krolicki et al. (2011) enhanced the shear strength and stiffness prediction equations for shear walls originally proposed by Kowalsky and Priestley (2000). The shear strength formula was improved to reliably estimate the cyclic shear capacity of walls considering the effect of axial force and displacement ductility. Shear failure is one of the major concerns even in capacity designed walls because of the inherent large uncertainty in shear strength and its deterioration mechanism. However, it is not uncommon for researchers to keep the shear strength constant during analysis and to assume that shear strength degradation or failure is controlled solely by curvature ductility demands.

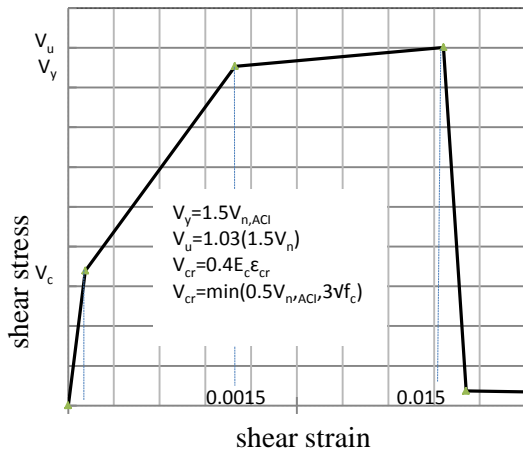
Mergos and Beyer (2011) introduced a beam element to include shear stiffness degradation during analysis. Based on some experimental results, they assumed that flexural to shear displacement ratio remains more or less constant during the whole range of inelastic cycles. The equation below was proposed to calculate the shear deformation based on wall geometry ( $H_n$  is the wall shear span) and assuming constant neutral axis depth ( $c$ ) after yielding. The above analytical method is implemented in the IDARC program.

$$\frac{\Delta_s}{\Delta_f} = \frac{\left(\frac{L_w}{2} - c\right)}{H_n \cdot \tan\beta}$$

The shear backbone curve displayed in Figure 9 was proposed by Kelly (2002) using experimental data available in the literature. Another model proposed in ASCE/SEI 41-06 for shear behaviour of slender walls, where flexural yielding limits the wall shear demand, is shown in Figure 10. The shear force in this model is calculated based on  $V=M/H_{eff}$ , where  $H_{eff}$  is the effective height of the lateral resultant forces, and the shear strain at yield is taken as 0.0015.



shear behavior(ASCE/SEI 41-06) backbone curve



shear behavior(Kelly(2002))

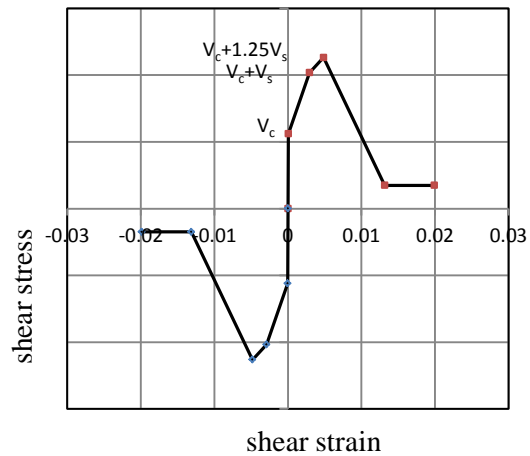
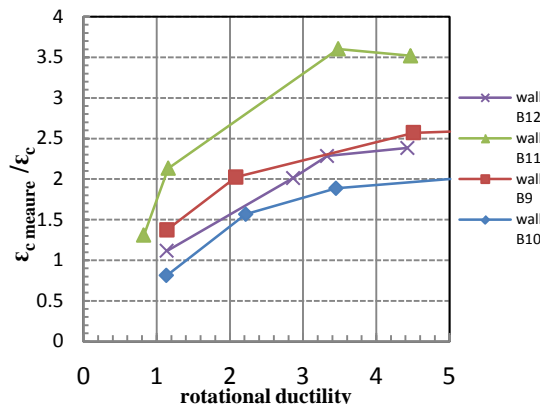


Figure 9. Recommendations for shear behaviour of walls.

Ratio of measured to calculated(M-phi) concrete compressive strain



Concrete strain gradient in rectangular wall section

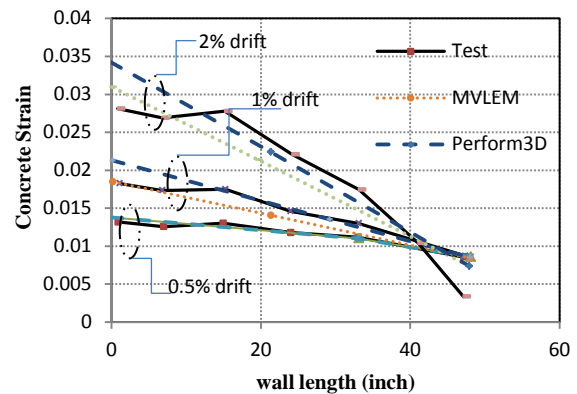


Figure 10. Concrete compressive strain variation in wall section (and with ductility).

### 3.2 Three dimensional effects

Practicing engineers need a computationally efficient wall model to use in seismic performance assessment/prediction of multi-storey shear wall buildings with different configurations. Such models must be three dimensional and be able to interact with beams and slabs which can induce additional actions on walls and it can result in considerable change in the assumed shear span ratio and the shear demands on the wall.

To explore more on this issue, one of the specimens from the PCA test program (R2 specimen) was selected to conduct nonlinear pushover analysis to understand the effect of slab in the predicted wall response. This specimen was modelled based on the material properties specified in the test report. The new wall element based on concrete and steel filaments in a wall section was employed in analysis. Results indicate that the yielding moment and moment capacity of the wall agrees well with experimental results. In the next step, a slab of dimensions 2000x1905x60 mm was assumed in each transverse direction. Gravity columns at slab corners were defined to remain elastic, which constrained vertical movement of slab corner nodes. The out-of-plane stiffness of the slabs was activated assuming infinite in-plane stiffness (rigid diaphragm).

The results (Fig. 11) demonstrate that before yielding of the wall in flexure, the effect of slabs on walls was negligible. However, as soon as the wall yielded, the contribution of stiffness from the slabs and the axial stiffness of columns increased the post yield stiffness of the wall by up to 30 percent. The

restrained slabs (even without transverse girders common in monolithic construction) intensify the seismic base shear demand in multi-storey shear walls. However, results show that the slab out-of-plane stiffness does not change the flexural capacity of the wall as much.

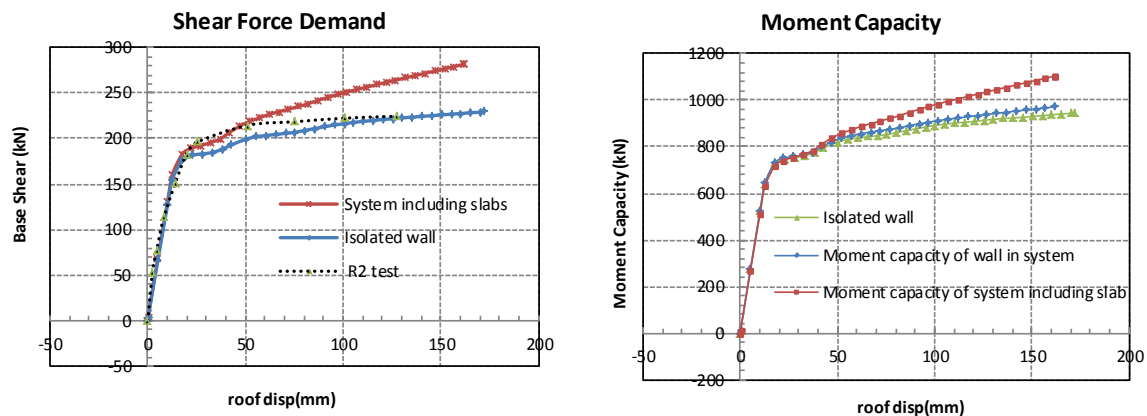


Figure 11. Contribution of out of plane stiffness of slab and gravity columns.

#### 4 CONCLUSIONS

Common approaches to modelling RC shear walls available in literature are scrutinised in this paper. The advantages and drawbacks of available models/elements for analytical modelling of multi-storey shear wall buildings are discussed. Although a wide variety of analytical modelling approaches have been proposed/developed/employed by different researchers; most of these are aimed to simulate the experimental response of isolated walls and are not easily/readily applicable in practice where engineers have to deal with 3D response of shear wall buildings in which walls invariably interact with other elements of the building. Three main drawbacks of several existing models are: (1) inability to capture the nonlinear strain profile due to the plane section remains plane assumption; (2) inability to convincingly account of shear-axial-flexural interaction; and (3) inability to allow consideration of the effect of slabs on the wall response. To facilitate a realistic performance assessment of shear wall buildings by practicing engineers, a macro wall element is needed which offers a reliable approach to modelling nonlinear behaviour of shear wall buildings without the abovementioned drawbacks. Currently, an attempt is being made at University of Canterbury to develop a new macro wall element comprising of a number of filaments analysed based on material behaviour. Although the new wall element appears promising and lacks any major obvious limitations, this is still under development and more research is needed to verify the model for different applications.

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