Effect of in-structure damping models on the performance of linear frames with optimal distribution of dampers

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ABSTRACT The effectiveness of control strategies in achieving the objectives of a performance based design is well accepted in the earthquake engineering community. Consequently, various methods have been proposed for the optimal design of dampers and their distribution along the height of a building. These methods usually assume a Rayleigh damping matrix to model the in-structure (inherent) damping. In this paper we attempt to investigate the effect of in-structure damping modeling on the performance of optimal designs attained by different methods. The performance of each method is measured in accordance with the optimization problem used for its design. For this purpose the present study focuses on optimally designing the damper distribution using the following design methods - the simplified sequential search algorithm (SSSA), Takewaki's method based on minimizing drift transfer function, and the analysis/redesign approach. The different in-structure damping models used for the study are classical Rayleigh damping and Russell's elemental damping model. Uniform distribution of dampers is also used in this study as it is the most popular and common way of damper distribution prevailing in the industry. The quantity of added damping is assumed to be sufficient not to induce any inelastic excursions in the parent frame. The effect of the in-structure damping model in a controlled frame is found to be a function of the amount of added damping.

1 INTRODUCTION

Conventional capacity design strategy relies on the "evasion" of seismic forces by enduring inelastic deformations. This philosophy could also be observed as "dissipation with damage" as seismic energy is dissipated by inelastic deformation. Due to the reliance of this philosophy on inelastic deformations, it incurs heavy damages to the parent structure making it non-functional after a major seismic event. So in order to reduce damage, from a dynamic perspective, a more rational approach would be to rely on "dissipation without damage" rather than "evasion/dissipation" of seismic forces by damage. One way to achieve this is by increasing the amount of damping in the system by adding dissipation dampers. So the resultant net damping in the system would be a combination of the inherent in-structure damping in the system (mainly due to the material or structural damping) and damping due to added dampers. This net damping would be responsible for the reduction of unwanted response during a seismic event.

Earlier studies have shown that in order to achieve a reliable performance, an optimal distribution of added damping devices is required (Takewaki 1997, Takewaki 2009, Garcia 2001, Levy and Lavan 2006). Previous studies have also highlighted the fact that in the case of bare frames an erroneous in-structure damping model can have a disastrous effect on the overall response prediction of the system especially when the parent frame becomes nonlinear (Val and Segal 2005). The optimal distribution process of the added devices also depends on the inherent in-structure damping in the system as the net response reduction is a function of the net damping. In analytical terms this means, there can be a possibility that if the in-structure damping model fails to capture the realistic damping in the system, then what seems optimal in analysis might not be optimal in reality. Now the main question to be answered is how much effect the difference in choice of in-structure damping models would have on the optimal response of the controlled frame? Focusing on the uncertainty prevalent in the choice of the in-structure damping model, this paper mainly illustrates the effect of different choices of in-structure damping models on the optimal response of the controlled frame. The present study illustrates the sensitivity of both classical Rayleigh damping model and the advanced Russell's spatial hysteresis model on the performance of the controlled linear frames. Three optimisation schemes, the simplified sequential search algorithm (SSSA) (Garcia 2001), Takewaki's method based on minimizing drift transfer function (Takewaki 1997), and the analysis/redesign approach (Levy and Lavan 2006)as well as a uniform distribution of dampers is used to design the control frame. Comparative simulation studies using the two in-structure damping models described above are presented. Also a comparative study is presented on the uncontrolled frame to illustrate the effect of the in-structure damping model on the overall response.

2 BRIEF OVERVIEW OF THE MODELS OF DAMPING USED IN THE PRESENT STUDY

This section gives a brief overview of classical and non-classical elemental spatial hysteresis damping models which are used in the numerical studies presented in this paper. To get a detailed review on all other models of damping interested readers should refer to Banks and Inman (1991), Adhikari (2000), Puthanpurayil et al (2011), Smyrou et al (2011). Banks and Inman (1991) deals with damping mainly in continuous system whereas the other papers deal with damping in discrete systems. A full state of the art description on damping is given in DeSilva (2007).

2.1 Classical viscous damping

Viscous damping is mainly achieved by the incorporation of Rayleigh's dissipation function given as

$$F = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\dot{q}_i\dot{q}_j = \frac{1}{2}\dot{q}_j C\dot{q}' \quad (1)$$

where 'C' represents a non-negative definite symmetric matrix. Rayleigh further demonstrated that one way of obtaining the **'**C**'** matrix is by a linear combination of the mass and stiffness matrices, which is given as,

$$C = \alpha M + \beta K \quad (2)$$

where α and β are calculated as functions of frequency using a preconceived damping ratio.

This model is commonly used to model damping in MDOF (Multi-Degree of Freedom) systems in practice and its popularity is mainly due to the fact that it uses the already computed mass and stiffness matrices and demands only the calculation of the constants α and β (Carr 2007).

2.2 Elemental spatial hysteresis model

Elemental spatial hysteresis model was first proposed by Russell (1991). The Russell model incorporated Euler beam formulation is given as

$$\rho A\ddot{w} + Q_i + Q_e + EIw^4 - f(x,t) = 0 \quad (3)$$

Over here the internal damping term is given as (Russell 1991),

$$Q_i = -2\frac{\partial}{\partial x}\left\{\int_0^L h(x,\xi)\left[\frac{\partial^2 w(x,t)}{\partial t\partial x} - \frac{\partial^2 w(\xi,t)}{\partial t\partial x}\right]d\xi\right\} \quad (4)$$

and the external damping term is (Banks and Inman 1992)

$$Q_e = \gamma \frac{\partial w(x,t)}{\partial t} \quad (5)$$

The boundary terms assuming a cantilever beam is given as,

$w(0,t) = 0$, (Dirichlet's condition) (6)

$w_x(0,t) = 0$ (Neumann Condition) (7)

$-(EIw_{xx}(x,t)\big|_{x=L} = \mathrm{M}$ (Neumann Condition) (8)

$$\frac{\partial}{\partial x}\left(-(EIw_{xx}(x,t))\right|_{x=L} + 2\int_0^L h(L,\xi)\left(w_{tx}(L,t) - w_{tx}(\xi,t)\right)d\xi = V \text{ (Neumann Condition)} \quad (9)$$

$'w'$ respresents the spatio-temporal variation of deflection. The internal damping term is described as a torque acting on the beam at point $'x'$ due to the differential rotation of the beam at points ξ "near" x (Russell 1991). The $h(x,\xi)$ is called the interaction kernel and is a function of both $'x'$ and $'\xi'$. The kernel function $h(x,\xi)$ in eq. (4) can take any mathematical causal model. For the present study in the following numerical sections $h(x,\xi)$ adopts the Gaussian error function given as follows

$$h(x,\xi) = \frac{a}{b\sqrt{2\pi}} e^{-(x-\xi)^2/2b^2} \quad (10)$$

Over here *'a' and 'b'* are damping coefficient constants.
For developing the in-structure damping matrix in the present study, classical Galerkin implementation is used and Hermitian shape functions are used for interpolation.

2 METHODS CHOSEN FOR THE STUDY

Four methods are chosen for the present study. A very brief overview of all the methods are given in the following subsections. Interested readers should refer to the relevant papers for a detail understanding of the optimization methods.

2.1.1 *Uniform Distribution*

The total added damping ***C*** is added uniformly to all the storeys. This can be mathematically expressed as,

$$c_i = C/n \,, \quad (11)$$

where *'n'* is the number of storeys and *i=1……n* and c_i is the damping coefficient per storey.

2.1.2 *Takewaki's method*

The aim of Takewaki's method (1997, 2009) is to minimize the sum of the amplitudes of the transfer function of inter-storey drifts evaluated at the undamped fundamental natural circular frequency ω_1 subjected to a constraint on total damping coefficients. Since the method fully relies on the structures dynamic behavior it is independent of the ground motion.

2.1.3 *The analysis/redesign method*

For linear scenario discussed in this paper, the analysis/redesign method (Levy and Lavan, 2006) selects an active ground motion based on its maximum spectral displacement characteristics. Once the active ground motion is selected a response analysis is performed and the objective function which is the damping vector and the performance index p_i is computed. The expression for p_i is given as,

$$p_i = \frac{\delta_i^{peak}}{\delta_{allowable}} \tag{12}$$

where δ_i^{peak} is the peak interstorey drift and $\delta_{allowable}$ is the allowable interstorey drift. A fully stressed design is achieved when p_i tends to unity. If the target performance index of unity is not achieved then the damping coefficients are updated using a recurrence relationship given as,

$$c_i^{(k+1)} = c_i^{(k)} \left(pi_i^{(k)} \right)^{1/q} \tag{13}$$

where *'q'* is 0.5 for linear analysis.

2.1.4 *Simplified Sequential search algorithm*

This is a simple iterative approach which seeks to optimize the damper placement procedure by sequentially placing the devices in locations where they can generate the maximum force. This method was proposed by Garcia (2001) as a simplification for the classical method of Sequential search algorithm proposed by Zhang and Soong (1992). The total added damping is divided into equally sized discrete devices and the devices are sequentially placed governed by an optimal location index given as

$$\gamma_l = \alpha_1 \delta_i + \alpha_2 \dot{\delta}_i \tag{14}$$

where γ_l is the location index and δ_i and $\dot{\delta}_i$ are interstorey drift and interstorey velocity. For pure linear viscous dampers $\alpha_1 = 0$ and $\alpha_2 = 1$. The bare frame is subjected to a time history analysis and γ_l is computed for each storeys. The first device is placed with the highest γ_l value. The process is repeated for the second device and on until all devices are placed.

3 NUMERICAL STUDY

This section presents a numerical study performed to assess the sensitivity of the choice of in-structure damping models on the optimal response of the linear control frame. A two bay seven storey reinforced concrete frame as shown in fig.1 is used for the present study with a constant storey height of 3m and bay width of 5m. The geometric dimensions of the frame members used are given in table.1. The beam geometric dimensions of the frame members presented in table 1 is an equivalent geometric property derived considering the effect of slab and beam together to correctly represent the lumped mass. In this study beams and columns are modelled by using elastic 2D Euler beam elements with 3 degrees of freedom per node. The Young's modulus is adopted as per NZS 3101 with $f_c' = 30MPa$. The Young's modulus adopted is reduced to 80% of the value to reflect the concrete cracking due to durability considerations. A density of $2300 \, kg/m^3$ is used for the present study.

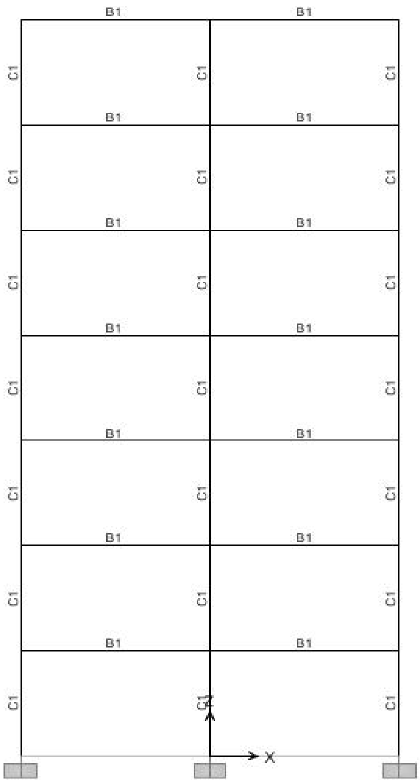


Figure 1. 2D frame used for the study

Table 1 Geometric dimensions of the frame members

Structural Element	Width (B) in mms	Depth (D) in mms
Column (C1)	600	600
Beam (B1)	1300	400

3.1 Uncontrolled frame

Table 2 gives the first seven periods of the structure used for the study and fig.2a presents the first three undamped mode shapes for illustration purpose. Figs.2b and 2c compares the peak interstorey drifts computed using classical Rayleigh damping with $\xi = 5\%$, $\xi = 2\%$ and elemental damping models with appropriate parameters *(a, b and γ)* corresponding to $\xi = 5\%$ and $\xi = 2\%$. The appropriate parameters of the elemental damping models are identified by matching the roof top displacement obtained by the free vibration of the seven storey frame. The identified parameters are given in table 3. A 5% damping ratio is adopted solely based on the most common practice prevalent in the industry whereas a 2% damping ratio is assumed as it is an intuitive accepted norm that the in-structure damping in controlled frames can be considerably less as compared to the uncontrolled frame due lesser deformation of the frame. Figs.2b and 2c illustrates the response of both elemental damping model and Rayleigh damping model. In the case of Rayleigh damping with $\xi = 5\%$ a difference of approximately about ~11%is obtained in the peak drift whereas in Rayleigh damping with $\xi = 2\%$ a difference of about only ~0.8 % is obtained in the peak responses.

Table 2. Period Summary

Mode number	Period (Sec)
1	0.75
2	0.23
3	0.12
4	0.07
5	0.05
6	0.049
7	0.04

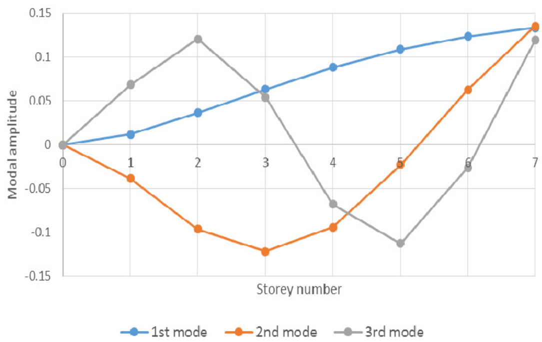
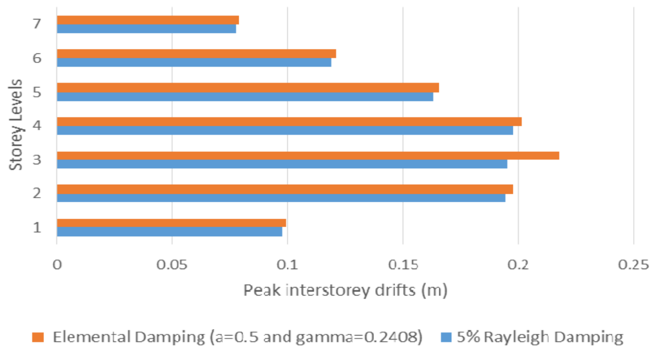


Table 3. Parameters of elemental damping model

	$\xi = 5\%$	$\xi = 2\%$
a	0.5	0.2
b	$L_{element}$	$L_{element}$
γ	0.248	0.102



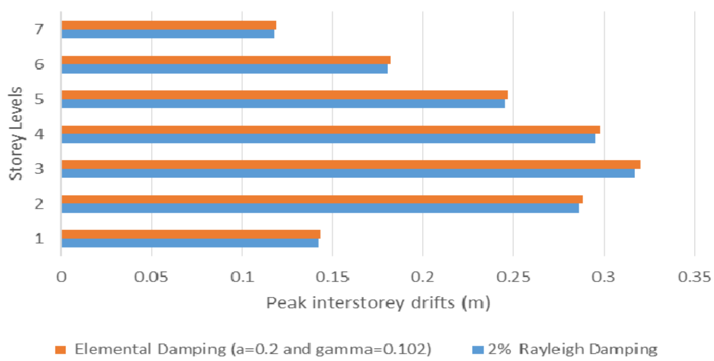


Figure 2. a) Mode shapes 2b) peak inter-storey drifts of uncontrolled frame with both 5% Rayleigh damping and Elemental damping model 2c) peak inter-storey drifts of uncontrolled frame with both 2% Rayleigh damping and Elemental damping model

3.2 Controlled Frame

Following strategy is adopted for assessing the sensitivity of the choice of in-structure damping models on the optimal response of the controlled frame. First the optimal damper distribution and total added damping for the control frame is computed using the analysis/redesign method. The total added damping thus obtained is then distributed using all the other methods with in-structure damping model represented by classical Rayleigh damping. The optimally controlled frame thus obtained from all the four methods is then subjected to a time history analysis with elemental model with appropriate parameters (refer table 3) representing the in-structure damping and the peak drift responses are compared.

3.2.1 *Analysis /Redesign Method (Levy and Lavan,2006)*

Spectral displacements curves are generated for ensemble of ground motions using a single degree of freedom structure with the same fundamental frequency as the structure under consideration. Maximum spectral displacement is obtained for Loma-Prieta earthquake. So the optimisation is done for the Loma-Prieta ground motion. The $\delta_{allowable}$ in eq.(12) is taken as 0.01m. Figs. 3a and 3b represent the optimal damping obtained and the corresponding p_i values.

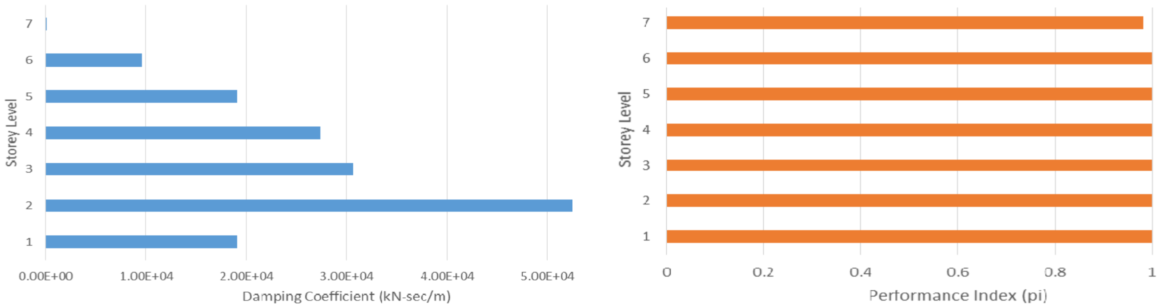


Figure 3. a) Optimal damping of the control frame; 3b) The storey level performance index

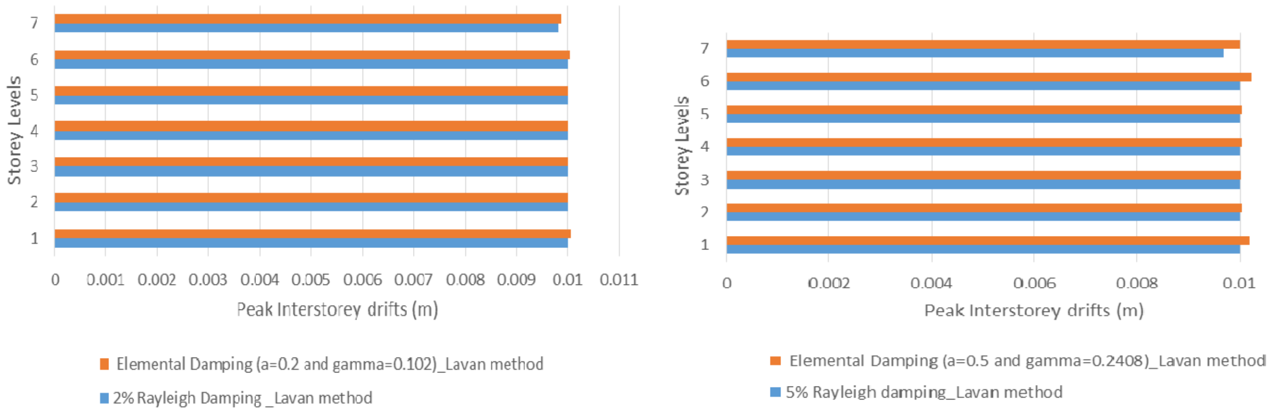


Figure 4. a) Sensitivity of the peak interstory drift with $\xi = 2\%$.4b) Sensitivity of the peak interstory drift with $\xi = 5\%$.

The total quantity of added damping obtained using Rayleigh damping with $\xi = 2\%$ is $C = 158124 \, {kN-s}/{m}$ and with $\xi = 5\%$ is $C = 156738 \, {kN-s}/{m}$. Figs.4a and 4b represents the comparative plot of the peak interstorey drifts obtained by the classical Rayleigh model and the elemental damping model for Loma Prieta earthquake. In the case of Rayleigh damping with $\xi = 2\%$, the difference between both the models is negligible whereas a maximum of 2% discrepancy in peak drifts is obtained for Rayleigh damping with $\xi = 5\%$. This observation indicates the fact that the effect of the in-structure damping model in a controlled frame is a function of the amount of added damping.

3.2.2 *Uniform distribution*

The above obtained total damping *C* is uniformly distributed per storey as per eq.(11). and the controlled frame is reanalyzed using both Rayleigh damping and elemental damping models.

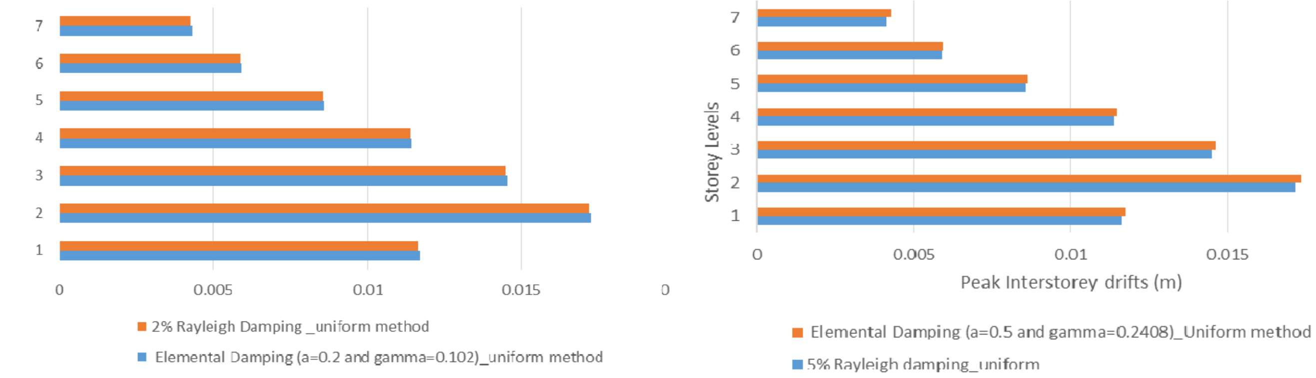


Figure 5. a) Sensitivity of the peak interstory drift with $\xi = 2\%$.5b) Sensitivity of the peak interstory drift with $\xi = 5\%$.

Figs.5a and 5b clearly outlines the fact that in the case of uniform distribution no significant effect of different choice of in-structure damping model is observed.

3.2.3 *Takewaki method (1997)*

Fig.6a depicts the optimal damping distribution obtained by Takewaki's method with constraint on the damping material given by the total added damping *C*. Fig.6b depicts the sensitivity of the peak drift response to the choice of different in-structure damping models using Loma Prieta earthquake. Only Rayleigh damping with $\xi = 2\%$ is presented in fig. 6b. Though not presented here same trend is exhibited by model with Rayleigh damping with $\xi = 5\%$.

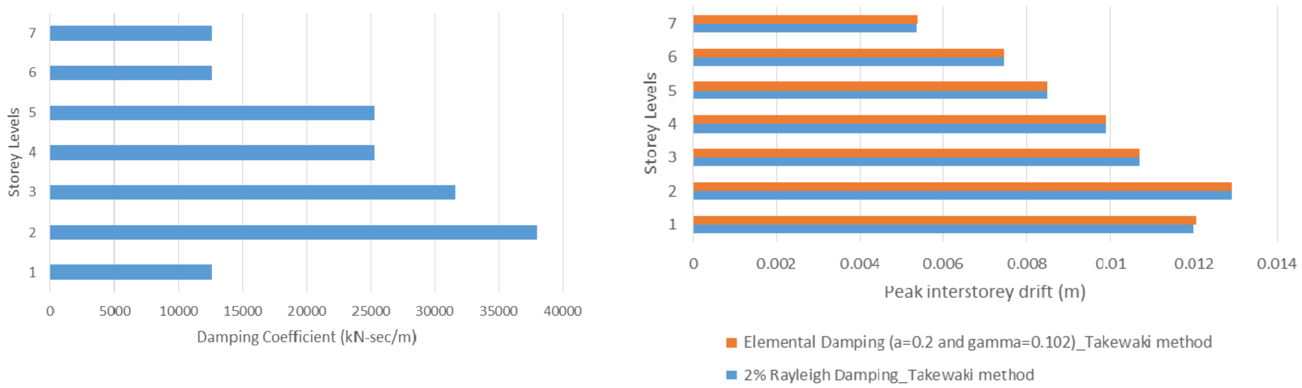


Figure 6. a) Optimal damping of the control frame 6b). Sensitivity of the peak interstory drift to different choice of in-structure damping models

In contrast to the uncontrolled frame, no significant discrepancy in the response is observed as a choice of different damping models.

3.2.4 *Simplified Sequential Search Method (Garcia, 2001)*

The total damping *C* obtained with $\xi = 2\%$ is optimally distributed using SSSA method.

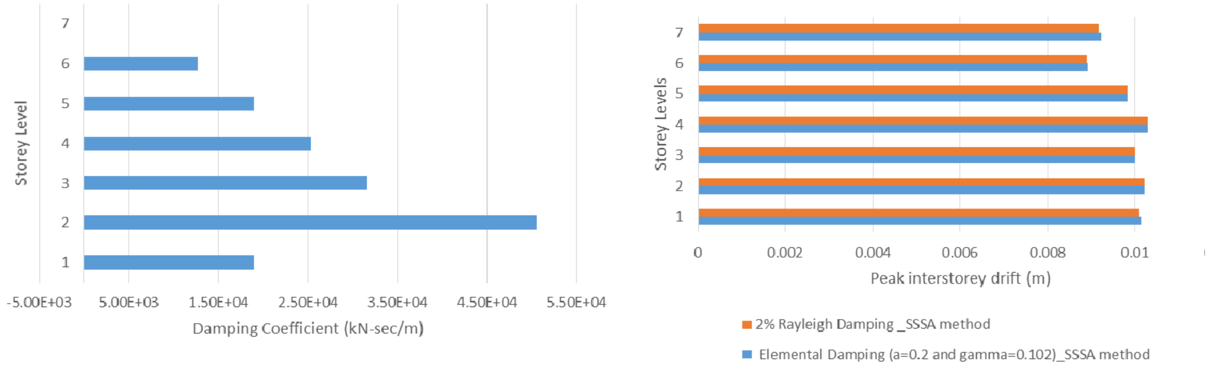


Figure 7. a) Optimal damping of the control frame 7b). Sensitivity of the peak interstory drift to different choice of in-structure damping models

Fig.7a represents the optimal damping distribution and fig.7b depicts the sensitivity of the peak drift response to the choice of different damping models using Loma Prieta earthquake. The discrepancies observed is negligible signifying the fact that with more added damping, the effect of in-structure damping remains negligible for linear parent frames.

4 CONCLUSION

Sensitivity of the choice of in-structure damping model on the optimal response of control frame is investigated. Russell's elemental spatial hysteresis model and classical Rayleigh damping model are compared in the study. Discrepancies are observed in the response of the uncontrolled frame signifying the fact that a correct representation of the in-structure damping model is imperative in the bare frame analysis. In the case of optimally added damping, the in-structure damping sensitivity in linear scenario is found to be very low. Though not presented here, the above study indicates that the sensitivity increases when the amount of added damping decreases.

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