Moment capacity of shallow foundations on clay under fixed vertical load

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ABSTRACT: Shallow foundations during the earthquakes are subjected to moment and shear in addition to the fixed vertical load. Large ground motions may also cause foundation rocking (cyclic uplift at the edges). Algie (2011) proposed a design method based on a hyperbolic relationship between applied moment and rotation for shallow foundations. This paper examines the stress distribution underneath the footings subjected to a moment using 3D finite element modelling. It also emphasizes on the concept of effective width of the footing subjected to moment. The FEM analysis is carried out using PLAXIS 3D 2011 software program. Studies were carried out to determine the appropriate mesh size for the model.

The results indicated that stresses are higher along the edges of the footing than at the center when footing is subjected to moment. Also, partial width of the footing experience zero stresses even when there is no actual loss of contact between the soil and footing. It was also found that the existing solutions for effective footing width and stress distribution for eccentrically loaded footings are incorrect and do not consider the effects of soil nonlinearity and shear strength. This paper exhibits a better understanding of footing behaviour and the effective width concept when subjected to moment, which is vital for estimation of the moment capacity of the footing.

1 INTRODUCTION

Large ground motions during earthquakes induce moment and horizontal shear on existing foundations. Shallow foundations undergo differential settlement and rotation under the action of moment. There is also loss of contact between the underside of the footing and the underlying soil. In such cases the stress distribution profile underneath the footing changes and part of the width of the footing becomes ineffective. The correct evaluation of effective width of footing is essential to estimate moment capacity of shallow foundations. Various researchers such as Meyerhof (1953) have proposed theories to evaluate effective width of the footing under eccentric loading. Most of the theories are empirical and are analysed as a 2D problem. With recent advances in numerical modelling and availability of sophisticated 3D finite element programs like PLAXIS 3D, complex geotechnical problems can be modelled. This paper presents a better understanding of effective width concept using 3D finite element analysis.

2 THEORY

Meyerhof (1953) proposed effective area method for eccentrically loaded footing of width ‘B’ and length ‘L’ and eccentricity ‘e’ as shown in Figure 1.a. The effective area A’ was given by:

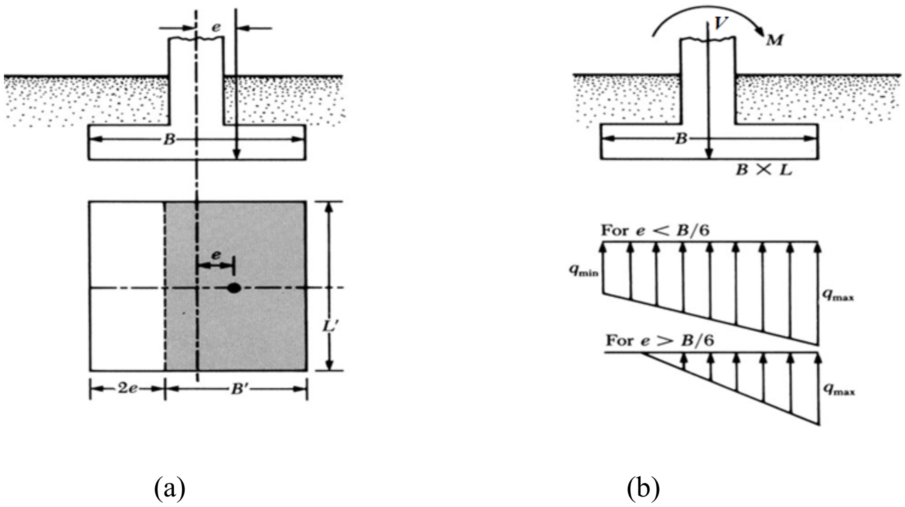


Figure 1. (a) Meyerhof (1953) effective area concept (b) Stress distribution based on linear pressure distribution

$$A = B'L', \quad B' = B - 2e, L' = L - 2e \tag{1}$$

The stress distribution under the footing, based on linear distribution theory, is as shown by Figure 1.b

$$q_{\max} = \frac{Q}{BL}\left(1 + \frac{6e}{B}\right) \tag{2}$$

$$q_{\min} = \frac{Q}{BL}\left(1 - \frac{6e}{B}\right) \tag{3}$$

Where, A = area of the footing, B = footing width, L = footing length, Q = Vertical load and e = eccentricity = M/V.

If eccentricity, e = B/6, q_{min} reduces to zero and footing loses contact with soil. In such case, maximum stress under footing is given by

$$q_{\max} = \frac{4V}{3L(B - 2e)} \tag{4}$$

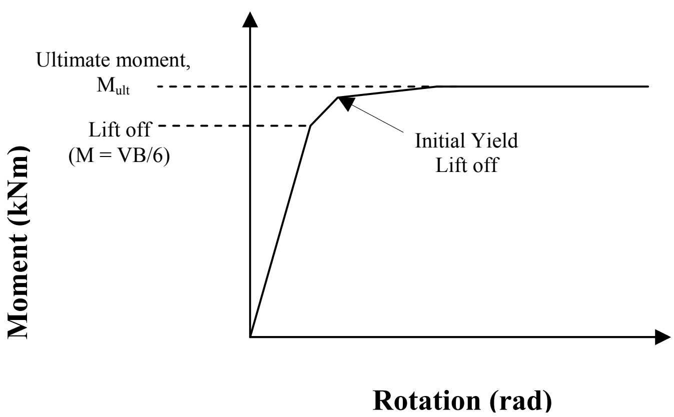


Figure 2. Moment rotation curve based on linear spring model

Figure 2 shows the moment rotation curve based on the spring model with elastic spring (Pender 2010a and 2010b). The springs are detachable once the uplift occurs. The ultimate moment capacity is evaluated using effective width given in Eq. 1. The above equations are based on the assumption that the pressure distribution beneath the footing is linear and do not take into account the limitations on the maximum pressure imposed by the soil shear strength. A detailed 3D study needs to be carried out for better understanding and an evaluation of effective width of the footing.

3 FINITE ELEMENT MODELLING

For better simulation of field conditions, 3D finite element analysis was carried out using the PLAXIS 3D program. PLAXIS 3D is 3 dimensional finite element analysis program with advanced soil models to simulate nonlinear soil structure interaction. The model geometry consists of a square foundation of dimensions 6mx6m resting on ground surface with interface elements between the underside and the underlying soil. A fixed vertical load is applied at the centre and subsequently moment is applied about a centroidal axis. As soil behaviour is nonlinear, the hardening soil model (Obrzud, R. and Truty, A. 2013, PLAXIS B.V, 2011) was used for analysis. A localized fine mesh was concentrated in the vicinity of that footing as ~~is~~ shown in Figure 3. The soil parameters used are shown in Table 1.

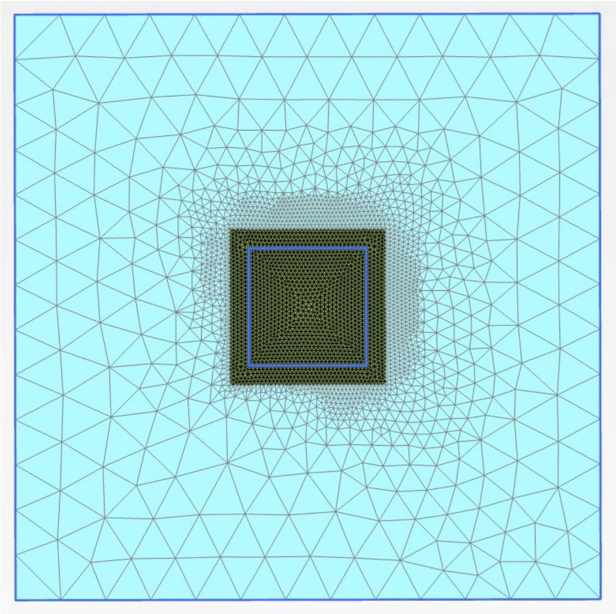


Figure 3. Top view of foundation model with interface elements

Table 1. Soil Parameters

Sl No	Shear strength	Modulus of elasticity of soil			Unit Weight	Interface friction factor, Rinter	Ultimate vertical load, V_{uo}**** (kN)
	s_u (kPa)	E50* (kPa)	Eoed** (kPa)	Eur*** (kPa)	γ (kN/m3)		
I	35	4481	4481	4481	19	0.99	8216
II	85	7932	7932	23794	19.50	0.99	18870
III	157	14247	14247	42740	21.00	0.99	34860
IV	278	24490	23103	69308	22.00	0.99	61730

* soil modulus at 50% strain,

** odeometer modulus

*** unloading reloading modulus

**** V_{uo} is the ultimate bearing strength of the foundation under vertical loading with no moment and shear.

For each value of undrained shear strength, s_u, two cases of vertical loads were considered

1. 6mx6m foundation with static vertical load, $V = 0.1\ V_{uo}$
2. 6mx6m foundation with static vertical load, $V = 0.4V_{uo}$

Moment rotation curves were developed for each of the above load cases and stress distribution under the footing and effective width, B' were examined at different values of moment.

4 RESULTS AND DISCUSSION

The moment rotation curves obtained from linear spring model (Figure 2) and PLAXIS 3D analysis are shown in Figure 4 for 6mx6m footing resting on clay with s_u = 85 kPa, with constant vertical loads of 1887kN and 7550kN respectively. The difference in results between the two models is because the spring model only considers stiffness and M/V ratio while ignoring soil shear strength. Therefore the effective width estimated from the spring model is lesser than the PLAXIS 3D analysis and hence the lower ultimate moment capacity.

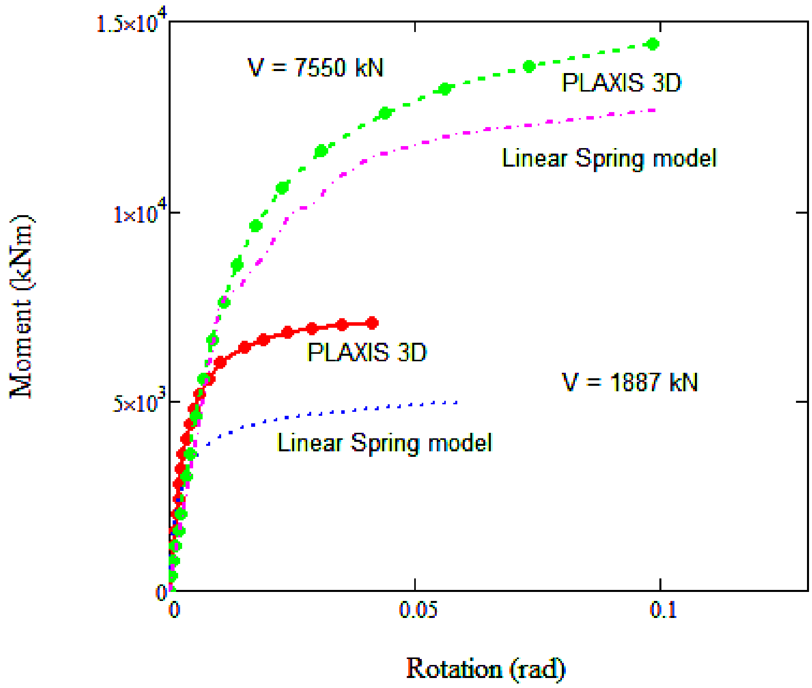


Figure 4. Moment rotation curves obtained from PLAXIS 3D analysis and linear spring model for 6mx6m footing for Su = 85kPa case with constant vertical loads of V = 1887kN and V = 7550kN

The figures 5 and 6 show the plot of stress distribution from PLAXIS 3D analysis under the footing in the direction of applied moment along the edge and at the center for various moment values with fixed vertical loads of 1887kN and 7550kN respectively. The maximum stress the under the footing is restricted to $6s_u$ kPa in Figure 6.c.

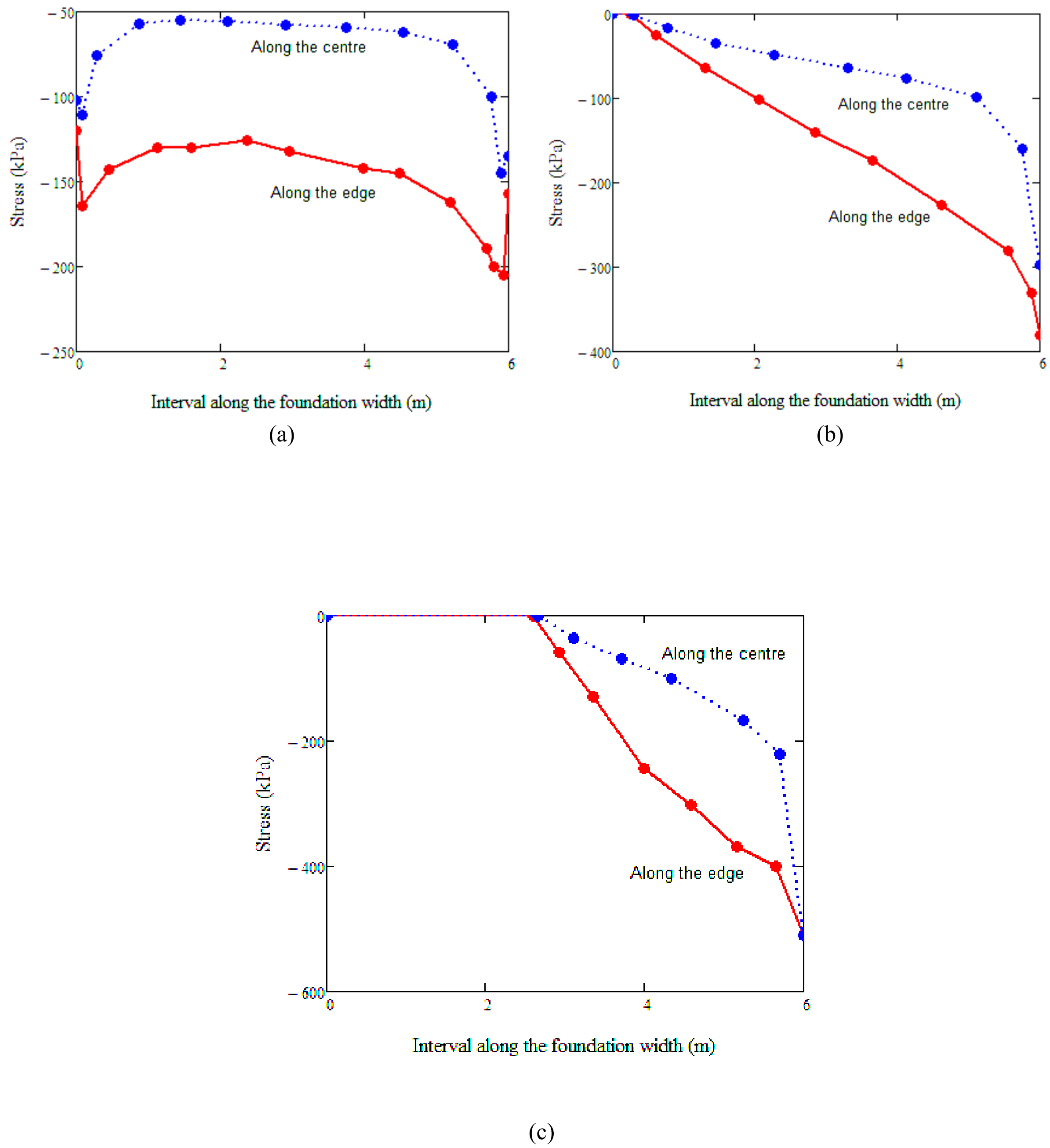
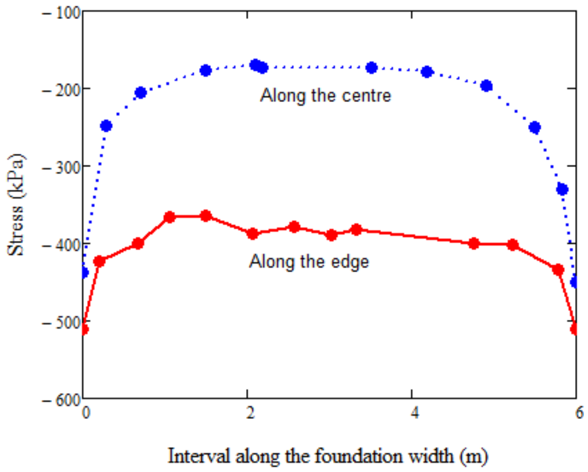
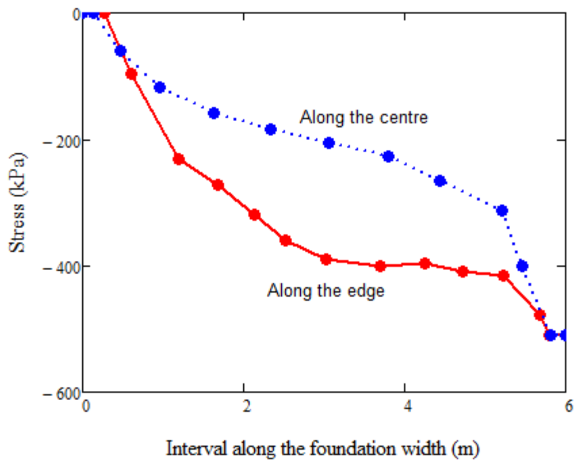


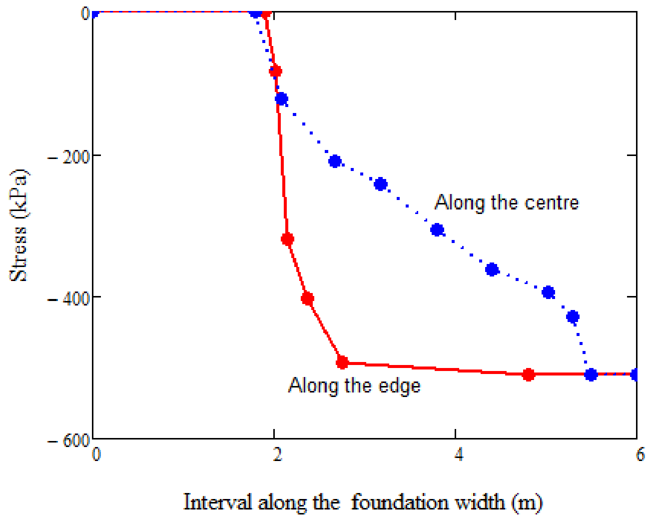
Figure 5. Stress distribution underneath the footing from PLAXIS 3D analysis along the direction of loading for s_u = 85kPa under (a) V = 1887kN, M = 400 kNm (b) V = 1887kN, M = 3200 kNm and (c) V = 1887kN, M = 5600 kNm



(a)



(b)



(c)

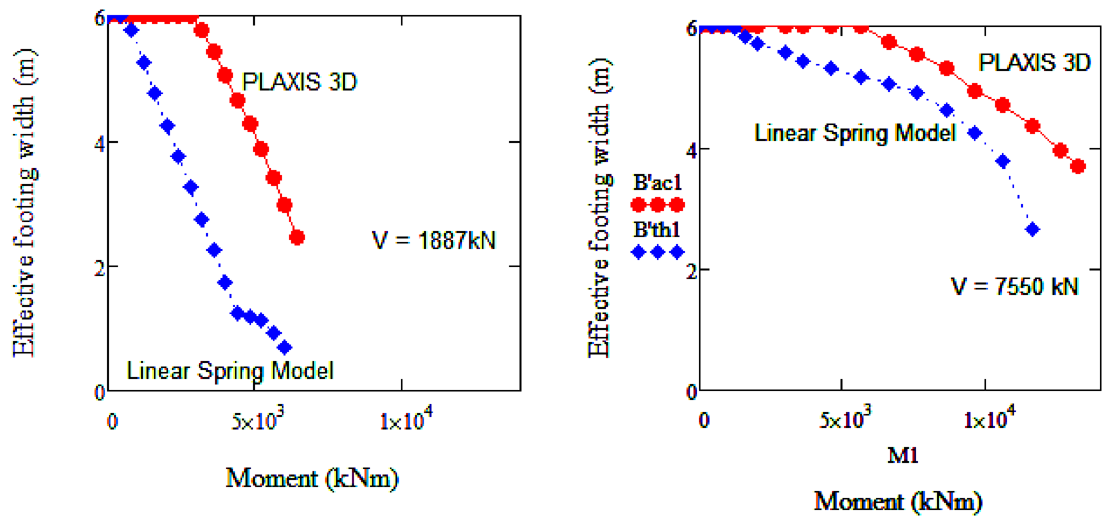
Figure 6. Stress distribution underneath the footing from PLAXIS 3D analysis along the direction of loading for s_u = 85kPa under (a) V = 7550kN, M = 400 kNm (b) V = 7550kN, M = 6600 kNm and (c) V = 7550kN, M = 12600 kNm

From the Figures 5 and 6, it is clear that the stress distribution under the footing is not uniform across the whole area of the footing. The stresses are considerably larger along the edges than at the centre. The value of moment at which loss of contact between footing and soil occurs for various undrained shear strength values is presented in Table 2 below.

Table 2. Moment at which loss of contact occurs

s_u (kPa)	V (kN)	V/Vuo	Moment at which loss of contact occurs in	
			PLAXIS 3D (kNm)	Linear Spring model, VB/6 (kNm)
37	822	0.1	2000	822
	3287	0.4	3000	3287
85	1887	0.1	3200	1887
	7550	0.4	6600	7550
157	3486	0.1	4800	3486
	13900	0.4	12000	13900
278	6173	0.1	8400	6173
	24692	0.4	22800	24692

Table 2 clearly shows that the moment at which lift off occurs is different for PLAXIS 3D analysis and linear spring model solution especially for the case of V/Vuo = 0.1. It is clear that for the larger vertical loads there is a better match between the moments at which loss of contact is initiated for the spring bed model and the data obtained from PLAXIS (which is also apparent from the moment-rotation curves in Figure 4). From figures 5.b and 6.b we can see that loss of contact between the footing and soil occurs at moments of 3200kNm and 6600kNm respectively when the stress at the left edge reduces to zero. These moment values represent points of first nonlinearity in moment rotation curves presented in Figure 4. Figure 7 presents the effective width vs moment plot for the footing obtained from PLAXIS 3D analysis and linear spring model (Figure 2) for the case of s_u = 85kPa.



(a) (b)

Figure 7. Effective width (B') vs Applied moment (M) plot for footing for s_u = 85kPa case (a) V = 1887kN and (b) V = 7550kN

From the plot shown in Figure 7, it is clear that effective width of the footing obtained from PLAXIS 3D analysis is much larger than the value obtained from linear spring model (Fig.2). The reason is the linear spring model does not take into account soil nonlinearity and shear strength. Hence the ultimate moment capacity of the footing estimated using linear spring model is less as the effective width is largely underestimated.

5 CONCLUSIONS

This paper presents a better understanding of the principle of stress distribution under a footing and footing effective width when subjected to moment. From the results presented in Section 4 it is clear that stress distribution profile under the footing is not same across the entire footing area. The stresses along the edge of the footing are much higher than stresses at the centre. The loss of contact between foundation and underlying soil occurs at a value of moment corresponding to a point beyond which moment rotation relationship is nonlinear (Fig.4). The effective width of the footing is not just a function of vertical load (V) and applied moment (M) but also soil shear strength. The effective width of the footing and ultimate moment capacity obtained from finite element modelling (PLAXIS 3D) is much higher than linear spring model. This clearly shows that soil nonlinearity and shear strength play a significant role in evaluation of ultimate moment capacity of the footing. Theoretical solutions ignore the effect of soil nonlinearity, not just shear strength resulting in underestimation of the moment capacity of shallow foundations leading to uneconomical design. Further study will be undertaken to obtain solutions to evaluate the moment at which footing lifts off under fixed vertical load and estimate footing effective width incorporating soil shear strength and nonlinear soil behaviour. Similar analyses have to be carried out on cohesionless soils in order to facilitate better understanding of effective footing area concept and evaluation of moment capacity of foundation.

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