

# Semi-active control of structure with MR damper using wavelet-based LQR

N. Khanmohammadi Hazaveh, S. Pampanin, G. Chase & G. Rodgers,

*Department of Civil Engineering, University of Canterbury, Christchurch.*

P. Ghaderi

*Department of Civil Engineering, University of IUST, Iran.*



2014 NZSEE  
Conference

**ABSTRACT:** This study presents a new method to find the optimal control forces for magnetorheological (MR) dampers. The method uses three algorithms: discrete wavelet transform (DWT), linear quadratic regulator (LQR), and clipped-optimal control algorithm. DWT is used to obtain the local energy distribution of the motivation over the frequency bands in order to modify of the conventional LQR. Clipped-optimal control algorithm is used in order to approach the MR damper control force to the desired optimal force that is obtained from modified LQR. Moreover, Bouc-Wen phenomenological model is utilized to investigate the nonlinear behaviour of the MR dampers. The method is applied on single-degree-of-freedom (SDOF) systems subjected to a Next Generation Attenuation (NGA) projects near fault earthquake. The results indicate that the proposed method is more effective at reducing the displacement response of the structure in real time than conventional LQR controllers.

## 1 INTRODUCTION

With the development of construction techniques, it is possible to build large-span bridges, pipelines, dams, high-rise buildings. However, this achievement also generates new problems; specifically, how these structures can be protected from external excitation such as strong winds and severe earthquakes. One of the solutions to reduce tragic consequences of natural hazards is using supplemental control devices that can reduce the response of civil engineering structures and protect them from damage under external loadings.

The structural control systems can be classified as active, passive or semi-active. Active systems are complex and expensive because they require force actuators. On the other extreme hand, passive control systems do not require an external power source to control of structure have been shown to be effective, robust, economical solution. An interesting and appealing improvement of passive control is given by semi-active control systems which require only a small external power source for operation (e.g. a battery). The semi-active devices cannot destabilize the structure because they do not input the energy to the system and just absorb or store vibratory energy (chase et al., 2006). They only need a small amount of external power to be operated. Because of this low dependence on external power sources and the removal of instability concerns, semi-active systems may become an attractive solution for the improvement of reliability of low-damage system, regardless of the uncertainties on the input ground motion.

Among many other semi-active devices that could be used as dampers in the structures, the MR damper achieves high-level adaptive performance (Fig 1). Mechanical simplicity, high dynamic range, low power requirements, large force capacity, high stability, robustness, and reliability are among desirable features of the MR dampers. MR dampers are capable of generating controllable damping forces by using MR fluids. MR fluids are composed of magnetized tiny particles that are scattered in a mineral liquid such as silicon oil. When a magnetic field is applied to this liquid, particle chains form just in a few milliseconds, and the fluid becomes a semi-solid which exhibit plastic behaviour.

Although the MR damper is promising in control applications, its major drawback lies in the inherent non-linear behaviour of the MR dampers and modelling the dynamic behaviour of them. There are

two types of dynamic models for the MR dampers: non-parametric models and parametric models. Many non-parametric models have been used to control the dynamic behaviour of the MR dampers such as neural network-based models (Wang and Liao, 2005) and fuzzy logic-based models (Kim et al., 2008). The Bingham model (Lee and Wereley, 2002), non-linear hysteretic bi-viscous model (Kamath and Wereley, 1997), hyperbolic tangent model (Christenson et al., 2008) and Bouc-Wen hysteresis model (Jansen and Dyke, 2000) are some of the parametric models that have been used to model the behaviour of MR dampers.

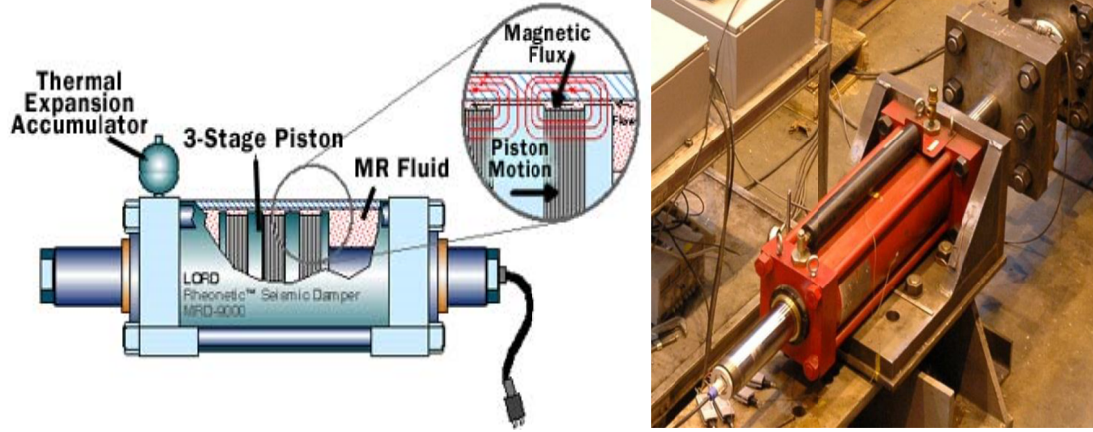


Figure 1. Large-scale semi-active damper schematic (Yang et al., 2002).

To characterize the behaviour of a MR fluid damper, Spencer et al. (1997) introduced the simple Bouc-Wen model. This model can predict the force-displacement and force-velocity behaviour well, and results obtained from this model are similar to the experimental data (Spencer et al., 1997). The simple Bouc-Wen Model cannot capture the force roll-off when the acceleration and velocity have opposite signs and the magnitude of the velocities is small. Therefore, to overcome this drawback, Spencer et al. (1997) proposed the modified version of the Bouc-Wen model with high level accuracy.

Using an appropriate control algorithm is very important in order to achieve the desirable control performance via reforming the magnitude of applied magnetic field according to a defined algorithm. Comprehensive studies have been done to determine the optimal actuator force for the active vibration control systems. The most widespread methods are LQR, LQG, H<sub>2</sub>, H<sub>∞</sub>. The LQR is used widely to determine the appropriate control force by many researchers. However, classical control algorithms such as LQR suffer from some inherent shortcomings for structural applications. For instance, one of the major shortcomings of the LQR algorithm for application to forced vibration control of structures is its inability to explicitly account for the excitation. To simulate realistic circumstances, the excitation must be known prior to determining the optimal control force to achieve more reliable solutions. The effect of the specific earthquakes has been accounted for in a few studies (Wu et al., 1998, Wu et al., 1994). For example Panarillo et al. (1997) introduced a method based on updating weighting matrices from a database of earthquakes. Nonetheless, in these studies, offline databases were still required. Biswajit Basu et al. (2008) and Amini et al. (2013) proposed a wavelet-based adaptive LQR and PSO-wavelet-LQR control to design the controller by updating the weighting matrices, respectively. These methods determine the time-varying gain matrices by updating the weighting matrices online, through the Ricatti equation. Therefore, these methods do not need prior information about external excitation, hence eliminating the need for an offline database.

In this article we use modified Bouc-wen model to model the MR damper behaviour. Moreover, Clipped-optimal control algorithm based on Wavelet-LQR is employed to find optimal control force of MR damper. The application of the proposed approach to a number of pulse-like near-fault ground motions is presented, and the efficiency of using MR dampers is evaluated.

## 2 MODIFIED BOUC-WEN MODEL

The schematic of the MR damper mechanical model for the modified Bouc-Wen model is shown in Figure 2b. In this case, nonlinear force of MR damper is calculated by (Yang et al., 2002):

$$\begin{aligned} F &= \alpha z + c_0 (\dot{x} + \dot{y}) + k_0 (x - y) + k_1 (x - x_0) \\ &= c_1 \dot{y} + k_1 (x - x_0) \end{aligned} \quad (1)$$

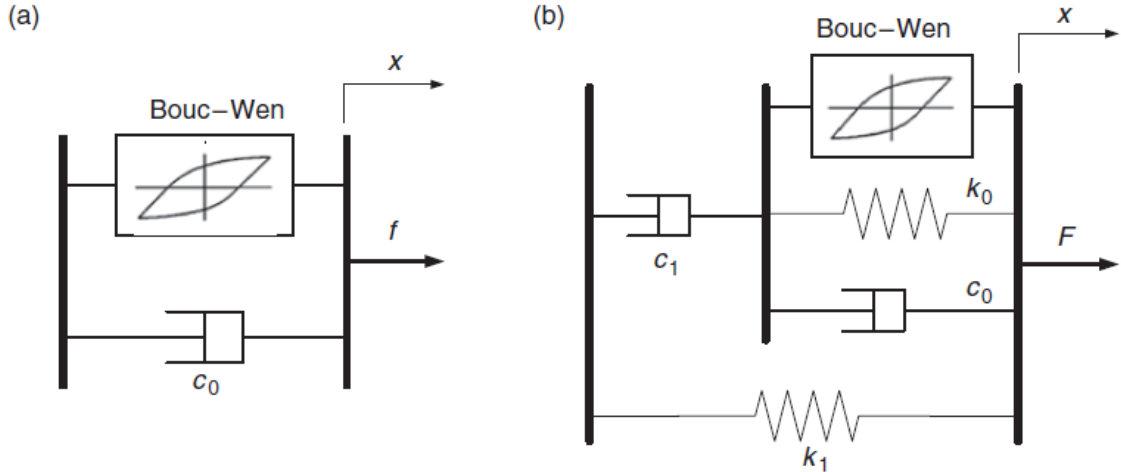
Where  $\alpha$  is Bouc-wen model parameter related to the MR material yield stress and  $z$  is hysteretic displacement of model given by:

$$\dot{z} = -\gamma |\dot{x} - \dot{y}| z |z|^{n-1} - \beta (\dot{x} - \dot{y}) |z|^n + A(\dot{x} - \dot{y}) \quad (2)$$

$\dot{y}$  is defined as:

$$\dot{y} = \frac{1}{c_0 + c_1} \{ \alpha z + c_0 \dot{x} + k_0 (x - y) \} \quad (3)$$

Where  $c_0$  is the viscous damping parameter at high velocities;  $c_1$  is the viscous damping parameter for the force roll-off at low velocities;  $k_0$  controls the stiffness at large velocities;  $k_1$  represents the accumulator stiffness;  $x_0$  is the initial displacement of the spring stiffness  $k_0$ ;  $\gamma$ ,  $\beta$  and  $A$  are adjustable shape parameters of the hysteresis loops, i.e., the linearity in the unloading and the transition between pre-yielding and post-yielding regions.



**Figure 2. Mechanical model for MR damper: (a) Simple Bouc-Wen model, (b) modified Bouc-wen model.**

Optimal performance for MR damper control systems is gained by varying applied voltage to the current driver according to the measured feedback at any moment. Thus, to determine a comprehensive model that is valid for fluctuating magnetic fields, parameters  $\alpha$ ,  $c_0$ ,  $c_1$  and  $k_0$  in Equations 1-3 are defined as a linear function of the efficient voltage  $u$  as given in Equation 4 to Equation 7.

$$\alpha(u) = a_a + a_b u \quad (4)$$

$$k_0(u) = k_{0a} + a_b u \quad (5)$$

$$c_0(u) = c_{0a} + c_{0b} u \quad (6)$$

$$c_1(u) = c_{1a} + c_{1b} u \quad (7)$$

To accommodate the dynamics involved in the MR fluid reaching rheological equilibrium, the following first order filter is employed to calculate efficient voltage,  $u$ .

$$u = -\eta(\dot{u} - v) \quad (8)$$

Where,  $v$  and  $u$  are input and output voltages of a first-order filter, respectively; and  $\eta$  is the time constant of the first-order filter.

Figure 3 illustrates the comparison between the response of this model and the experimental results for a 3kN MR damper in a real control condition that a damper would face during the control time. It is obvious that this model is capable of predicting MR damper nonlinear behaviour very well.

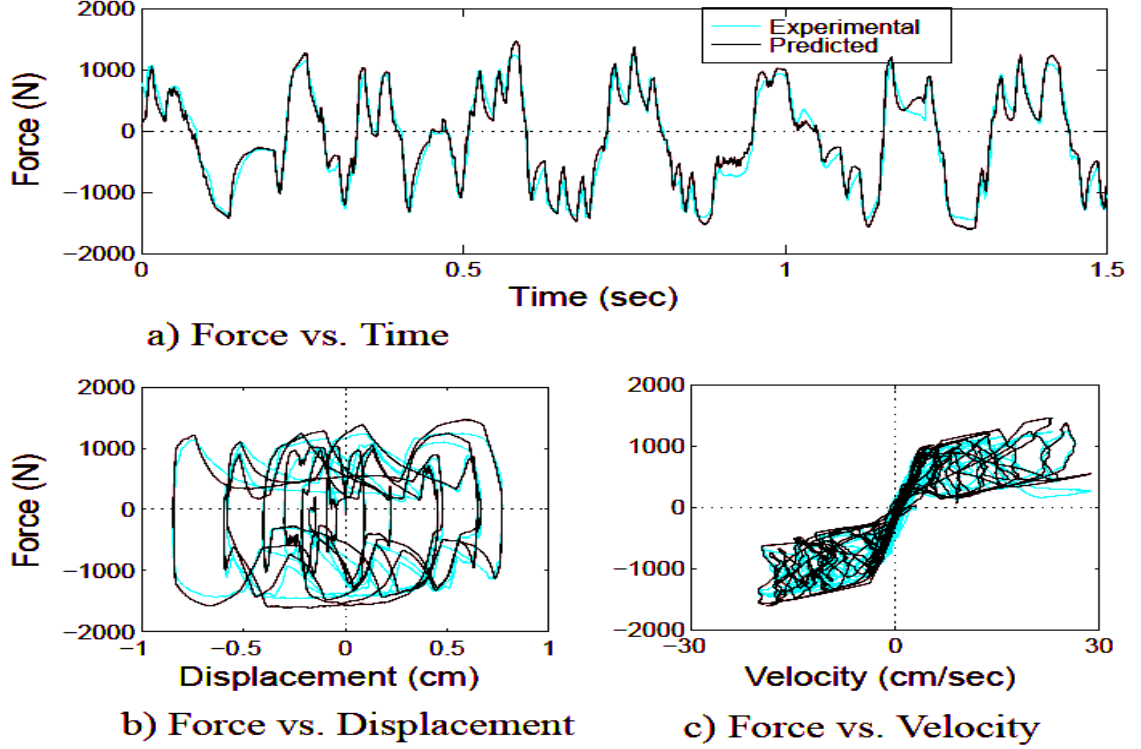


Figure 3. Predicted response by the Bouc-Wen phenomenological model in comparison with the experimental data for a 3kN MR damper in a control simulation test (Spencer et al., 1997).

### 3 INTEGRATED STRUCTURE-MR DAMPER SYSTEM

When  $n$ -degree-of-freedom ( $N$ -DOF) systems with  $r$  MR dampers are subjected to external excitation and control forces, they govern equations of motion and can be written as:

$$M \ddot{q}(t) + C \dot{q}(t) + Kq(t) = L.u(t) + H.f_e(t) \quad (9)$$

where  $C$ , and  $K$  are the mass, damping, and stiffness matrices of the structure without dampers, respectively. If “ $q$ ” in Equation 9 is taken as the relative displacement with respect to the ground, then mass matrix  $M$  is considered to be diagonal. Damping matrix  $C$  takes a form similar to  $K$ .

$$q = [q_1 q_2 q_3 \dots q_n] \quad (10)$$

Displacement vector is defined as  $q(t) = n \times 1$  and  $q_i$  is the displacement of  $i$ th floor relative to ground ( $i = 1, 2, \dots, N$ ), control force vector,  $u(t)$ , is of the order  $l \times 1$ , and  $f_e(t)$  is the external dynamic force vector of dimension  $r \times 1$ ,  $L$  and  $H$  are  $n \times l$  and  $n \times r$  location matrices, which define locations of the control forces and the external excitations, respectively. A state-space representation of Equation 9 can be written as:

$$\{\dot{x}\} = [A]\{x\} + [B]\{u\} + [E]f_e \quad (11)$$

Where

$$\{x\} = \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} \quad (12)$$

$\{x\}$  is the state vector of dimension  $2n \times 1$ , and

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \quad (13)$$

$$B = \begin{bmatrix} 0 \\ M^{-1}L \end{bmatrix} \quad (14)$$

$$E = \begin{bmatrix} 0 \\ M^{-1}H \end{bmatrix} \quad (15)$$

$2n \times 2n$ ,  $2n \times 1$ , and  $2n \times r$  are the system matrix, control location, and external excitation location matrices, respectively. The matrices “0” and “I” in Equations 13 to 15 denote the zero and identity matrices of size  $n \times n$ , respectively. The LQR algorithm can determine the optimal control forces for the system with the aim of minimizing the cost function. The cost function is a quadratic function of the control effort and the state. The cost function is defined as

$$J = \int_0^{t_f} [\{x\}^T [Q] \{x\} + \{u\}^T [R] \{u\}] dt \quad (16)$$

The matrices  $Q$  and  $R$  are called the response and control energy weighting matrices, respectively. The optimal control force vector at each time step can be given as

$$f_{opt} = -R^{-1}B^T P S \quad (17)$$

where  $P$  is the Riccati matrix and  $S$  is the state feedback of the system at each time step.

Semi-active control systems are typically highly non-linear. One algorithm that has been shown to be effective for use with the MR damper is a clipped-optimal control approach, proposed by Dyke, et al. (1996). The clipped-optimal control approach is to design a linear optimal controller that calculates a vector of desired control forces based on the measured structural responses and the measured control force vector applied to the structure. If the magnitude of the force produced by the damper is smaller than the magnitude of the desired optimal force and the two forces have the same sign, the voltage applied to the current driver is increased to the maximum level so as to increase the force produced by the damper to match the desired control force. Otherwise, the commanded voltage is set to zero. The algorithm for selecting the command signal for the MR damper is stated as

$$v_i = V_{max} H(\{f_{ci} - f_i\} f_i) \quad (18)$$

Although a variety of approaches may be used to design the optimal controller, LQR methods are advocated because of their successful application in previous studies. The approach to optimal control design is discussed in detail in (Mohajer Rahbari 2013).

#### 4 MODIFIED LQR METHOD

In this study, the real time DWT controller is updated at regular time steps from the initial time ( $t_0$ ) until the current time ( $t_c$ ) to achieve the local energy distribution of the motivation over frequency bands. The time interval under consideration  $[t_0, t_c]$  is sub-divided into time window bands. The time of  $i_{th}$  window is  $[t_{i-1}, t_i]$  of which the signal can be decomposed into time frequency bands by wavelet. Through discrete wavelet transform (DWT) with multi-resolution analysis (MRA) algorithm the exact decomposition of signals over a time window bands are obtained in real time. The local energy content at different frequency bands over the considered time window are given by the MRA. It is obvious, the frequency contains maximum energy is domain frequency of that window. When the domain frequency of each window closes to the natural frequency of the system, the resonances occurred in the structure. This causes high displacement response in system. To mitigate the displacement responses of structure, the high control force is needed. In order to mitigate the responses of structure, it is suitable to decrease the value of the control energy weighting matrix  $[R]$ . The advantage of this local optimal solution is that it has the ability to change the value of the matrix  $R$  on especial frequency in contrast to the classical LQR which is a global optimal solution. To achieve this, the control energy weighting matrices are updated for every time window by a scalar multiplier and can be defined as:

$$R = \delta[I] \quad (19)$$

where  $\delta$  is a scalar parameter used to scale the weighting matrix and is obtained based on the time-frequency analysis of a response state. Hence, the scalar parameter of gain matrix can be written as:

$\delta \neq 1$                       if the frequency of excitation is close to the natural frequency of system,  
 $\delta = 1$                       Otherwise.

The value of the  $\delta$  has been proposed as less than one when the resonance happens. This makes it possible to change the weighting matrices for different frequency bands. The control energy weighting matrices are reduced when the structure has a significant high value of displacement response. This reduction of weighting matrices sets off the lesser displacement without penalty. Therefore, the positive aspect of proposed method is that the gain matrices are calculated adaptively by using the time-varying weighting matrices depending on online response characteristics instead of a priori (offline) choice of the weights as in the classical case (Amini et al., 2013). Figure 4 shows the block diagram of semi-active device and flowchart of the LQR method and proposed method.

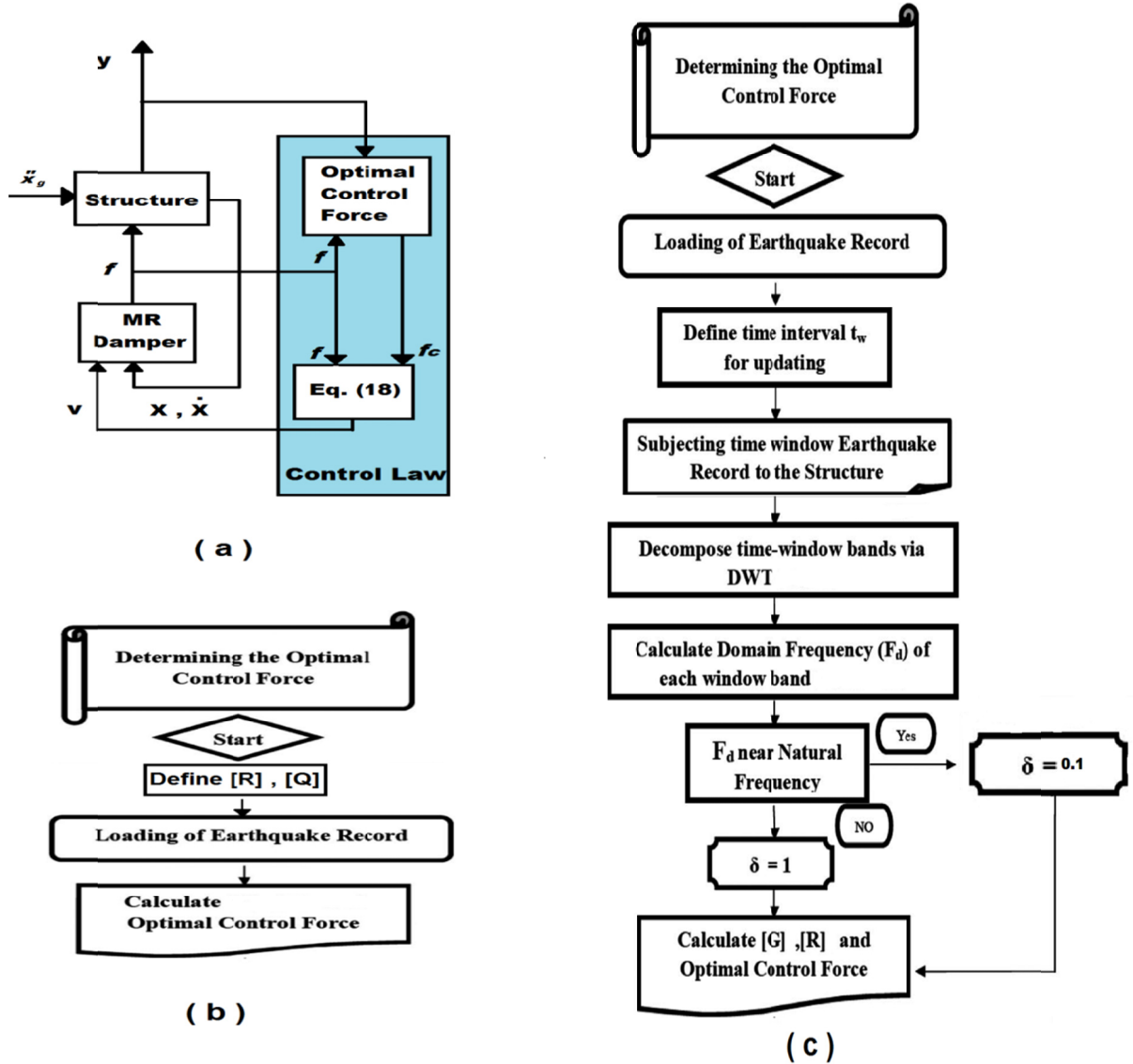


Figure 4. (a) Block diagram of semi-active control system (b) Flowchart of the classical LQR method (c) Flowchart of the Wavelet-LQR method.

## 5 CASE STUDY

In this section, to investigate the potential application of proposed method, the results of dynamic analysis of the typical SDOF with only one MR damper which has been excited by a Next Generation Attenuation (NGA) projects near fault earthquake are discussed (Baker 2007). The ground motion considered is the fault normal component of the 1994 Northridge at Sylmar-Olive View Med FF site. The SDOF system considered is assumed to have natural period of 1 second and a mode damping ratio of 2%. Moreover a MR damper with the capacity of 3 kN is installed to control the seismic responses of system. Optimal values for Bouc-Wen phenomenological model parameters for this damper are given in Table 1 and the maximum input voltage for this damper is equal to 2.25V.

**Table 1. parameters for MR damper model.**

Parameter	value	Parameter	value
$C_{0a}$	$21.0 \text{ N s cm}^{-1}$	$\alpha_a$	$140 \text{ N cm}^{-1}$
$C_{0b}$	$3.50 \text{ N s cm}^{-1} \text{ V}^{-1}$	$\alpha_b$	$695 \text{ N cm}^{-1} \text{ V}^{-1}$
$K_0$	$46.9 \text{ N cm}^{-1}$	$\gamma$	$363 \text{ cm}^{-2}$
$C_{1a}$	$283 \text{ N s cm}^{-1}$	$\beta$	$363 \text{ cm}^{-2}$
$C_{1b}$	$2.95 \text{ N s cm}^{-1} \text{ V}^{-1}$	$A$	301
$K_1$	$5.00 \text{ N cm}^{-1}$	$n$	2
$X_0$	14.3 cm	$\eta$	$190 \text{ s}^{-1}$

The parameter  $\delta$  used for scaling the weighting matrices is 0.1 when the central frequency of each window band is close to the natural frequency of the SDOF system, and for others frequencies is assumed to be 1. Hence, the weighting matrix component  $[R]$ , equals to 0.1  $[I]$  for resonance frequency bands and for the rest of the frequency bands it is kept as  $[I]$ . In addition, the matrix  $Q$  is chosen as identity for each band. Daubechies wavelet of order 4 (db4), is used as a mother wavelet to decompose the time history of acceleration for different window bands, to determine the frequency distribution of each band. The Daubechies wavelets have reasonably good localization in time and frequency to capture the effects of local frequency content in a time signal, and allow for fast decomposition by using MRA. The signals recorded in real time are decomposed for each interval window, which is considered as 1 second for updating. The gain matrices are updated for each window by solving the Riccati equation. Therefore, the control forces and controlled responses are calculated. The MATLAB software is utilized to calculate all computation.

**Table 2. Peak Responses due to the Northridge Earthquake.**

Control Method	Displacement(mm)
Uncontrolled	237.36
Pass-off	185
Reduction%	22.05%
Pass-on	142.42
Reduction%	39.74%
LQR	149.15
Reduction%	37.16%
Wavelet-LQR	142.40
Reduction%	40.1%



Also, to illustrate the potential application of the proposed method, the response of the semi-active clipped optimal controller based on wavelet-LQR is compared with conventional LQR, two passive case and uncontrolled structure. The two passive cases are termed Passive-off and passive-on, which refers to the cases in which the voltage to the MR fluid damper is held at a constant value of  $V=0$  and  $V=V_{\max}$ , respectively. At  $V=0$  the Mr damper primarily exhibits the characteristics of viscous device (i.e., the force-displacement relationship is nearly linear). However, as the voltage increase, the force required to yield the fluid increase and produces behaviour associated with a plastic material in parallel with a viscous damper. Maximum responses of system for the case of uncontrolled, passive-on, passive-off, traditional LQR and the proposed method are presented in Table 2 for a prescribed ground motion.

Figure 4 shows the displacement of system for the three earthquakes. As can be seen from the table 2 and figure 5, the advised LQR algorithm reduces the peak displacement of system often more than using conventional LQR method. Therefore, it is seen that the proposed adaptive LQR is more efficient than the classical LQR. Interestingly, the forces applied by the MR damper operating in semi-active mode that using wavelet-LQR are often smaller than those corresponding to the damper operating in passive-on mode, indicating that larger damper forces do not always produce better result.

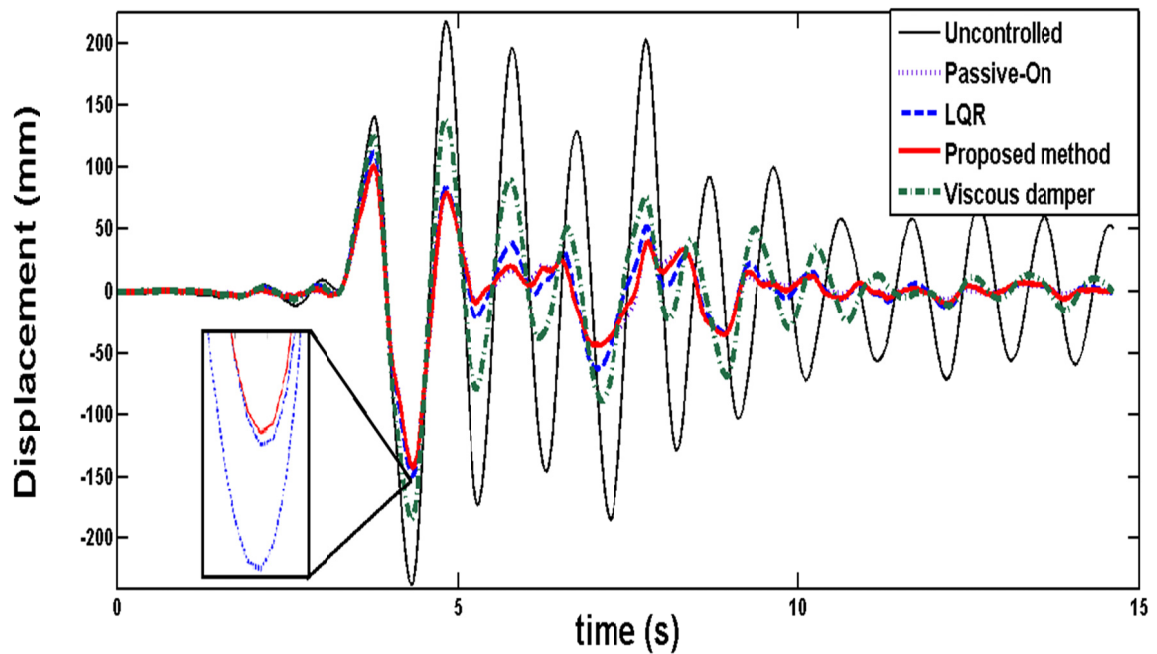


Figure 5. Displacement of system due to 1994 Northridge case.

## 6 CONCLUSION

A wavelet-LQR algorithm based on modified Bouc-Wen model has been implemented in the present study to control the seismic vibrations of structure with MR dampers. To lead the MR damper force close to the optimal control force during the control time, wavelet-LQR algorithm is used. In this method, the optimal control force is obtained by modifying the conventional LQR controller by updating the weighting matrices applied to the response energy and the control effort, over time intervals. The efficiency of the proposed modified LQR controller is evaluated in terms of the reduction of the response when the SDOF system, with one MR damper, is subjected to a Next Generation Attenuation (NGA) projects near fault earthquake, and results are compared with conventional LQR, passive-on, passive-off and uncontrolled system. The proposed modified LQR controller performs better than the classical LQR controller in reducing the displacement response of the structure. Remarkably, this method could be more efficient than passive-on control system.



Because the proposed method has the ability to vary its properties according to the external load to more effectively control the structure, this method performed better than both the passive-off and passive-on control systems. Based on these results, it is concluded that the proposed semi-active control system might be the right choice for reduction responses of structures.

## REFERENCES

- Amini, F., Khanmohammadi Hazaveh, N. & Abdolahirad, A. 2013. Wavelet PSO-Based LQR Algorithm for Optimal Structural Control Using Active Tuned Mass Dampers. *Copmputer-Aided Civil and Infrastructure Engineering*, 28: 542-557.
- Baker, J. W. 2007, Quantitative classification of near-fault ground motions using wavelet analysis, *Bulletin of the Seismological Society of America*, 97(5), 1486–1501.
- Basu, B. & Nagarajaiah, S. 2008. A wavelet-based time-varying adaptive LQR algorithm for structural control, *Engineering structures*, 30, 2470-2477.
- Chase, J.G., Mulligana, K.J., Guea, A., Alnot, T., Rodgers, G., Mander, J.B., Elliott, R., Deam, B., Cleeve, L. & Heaton, D. 2006. Re-shaping Hysteretic Behaviour Using Semi-active Resettable Device Damper, *Journal of Engineering structures*, 28:1418-1429.
- Christenson, R., Lin, Y. Z., Emmons, A. & Bass, B. 2008. Large-scale experimental verification of semiactive control through real-time hybrid simulation, *ASCE Journal of Structural Engineering*, 134(4), 522–34.
- Dyke, S. J. & Spencer, B. F. 1996. Seismic response control using multiple MR dampers. in *Proceedings of the 2nd International Workshop on Structure Control*, Hong Kong University of Science and Technology Research Center, Hong Kong.
- Jansen, L. M. & Dyke, S. J. 2000. Semiactive control strategies for MR dampers: comparative study, *ASCE Journal of Engineering Mechanics*, 126(8), 795–803.
- Kamath, G. M. & Wereley, N. 1997. Nonlinear viscoelasticplastic mechanism-based model of an electroheolocal damper, *Journal of Guidance, Control Dynamics*, 20, 1125– 32.
- Kim, Y., Langari, R. & Hurlebaus, S. 2008. Semiactive nonlinear control of a building with a magnetorheological damper system, *Mechanical Systems and Signal Processing* 23(2), 300–15.
- Lee, D. Y. & Wereley, N. M. 2002. Analysis of electro- and magneto-rheological flow mode dampers using Herschel- Bulkley model. In *Proceeding of the SPIE Smart Structure and Materials Conference*, Newport Beach, CA.
- Mohajer Rahbari, N., Farahmand Azar, B., Talatahari, S. & Safari, H. 2012. Semi-active direct control method for seismic alleviation of structures using MR damper. *Structural Control and Health Monitoring*, 20:1021-1042.
- Panariello GF, Betti R, Longman RW. 1997. Optimal structural control via training on ensemble of earthquakes. *Journal of Engineering Mechanics*, 123(11):1170-9.
- Spencer, B. F., Dyke, S. J., Sain, M. K. & Carlson, J. D. 1997. Phenomenological model of a magnetorheological damper, *ASCE Journal of Engineering Mechanics* 123(3), 230– 8.
- Wang, D. H. & Liao, W. H. (2005), Modeling and control of magnetorheological fluid dampers using neural networks, *Smart Materials and Structures*, 14, 111–26.
- Wu W & Nagarajaiah, S. 1996. Application of partitioned predictor corrector approach in nonlinear dynamic structural analysis and optimal control. Report 974. Missouri (Columbia, MO): Dept of Civil Engineering.
- Wu, W-H., Chase, JG., & Smith, HA. 1994. Inclusion of forcing function effects in optimal structural control. In: *Proc. first world conf. on struct. control*. IASC, TP2-22-TP2-31.
- Yang, G., Spencer, BF Jr., Carlson, JD. & Sain, MK. 2002. Large-scale MR Fluid Dampers: Modelling and Dynamic Performance Considerations. *Engineering Structures*, 24(3):309–323.