Rocking controlled design of shallow foundations

M.J. Pender  
*University of Auckland, Auckland*

T. B. Algie  
*Partners in Performance, Sydney, Australia.*

L. B. Storie and R. Salimath  
*Graduate students, University of Auckland, Auckland*

**ABSTRACT:** Kelly (2009) explains how low to medium-rise structures on shallow foundations may not have sufficient weight to prevent foundation rocking, that is cyclic uplifting and reattachment at the ends, during a design earthquake. Such rocking is not generally considered in foundation design but it is known that modest amounts can reduce foundation actions because of period lengthening.

The paper presents an approach to shallow foundation design that can incorporate rocking into both forced-based and displacement-based design of structure-foundation systems. The method assesses the small strain elastic rocking stiffness of the foundation and also the moment capacity at a fixed vertical load. These two soil-foundation parameters define the bounds on a nonlinear moment-rotation curve for the foundation; a hyperbolic curve provides a smooth transition between the bounds. A process of iteration, matching capacity with demand, is needed to determine the actions generated on the foundation by a given earthquake.

Field data on the rocking response of shallow foundations in Auckland residual clay, supplemented with 3D nonlinear finite element modelling of shallow foundations on cohesive and cohesionless soil, provide supporting evidence for the design approach. The finite element modelling includes nonlinear soil deformation as well as loss of contact, and reattachment, between parts of the foundation and the underlying soil. Insight into the relation between hysteretic damping and foundation rotation is also obtained from the finite element analysis.

1 **INTRODUCTION**

Classical soil structure interaction (SSI) studies show that for shallow foundations, designed in accord with LRFD requirements, the peak response including SSI is only marginally different from the fixed base response. To achieve gains from soil-structure interaction nonlinear soil behaviour must be mobilised, in this context the term Soil-Foundation-Structure-Interaction (SFSI) has recently come into vogue.

Kelly (2009) made proposals for the design of shallow foundations that rock during earthquake excitation. Rocking is understood as cyclic uplifting and reattachment at the ends of the foundation during the course of the earthquake excitation. Kelly explained that low to medium-rise structures on shallow foundations may not have sufficient weight to prevent foundation rocking; in which case the designer might want to take advantage of the real benefits that follow from accepting modest amounts of rocking. One of the topics Kelly recommended for further research was an appropriate method of handling soil nonlinearity; the intention of this paper, and the companion by Storie and Pender (2013), is to present current results of this work.
Field data gathered on the rocking response of shallow foundations on Auckland residual clay was reported and analysed by Algie et al (2010) and Algie (2011). This work, and related finite element modelling, provides the basis of a method of determining nonlinear moment-rotation curves for shallow foundations with uplift. Also required is information about the hysteretic damping associated with foundation rocking. Abaqus (Simulia 2010) and Plaxis 3D (Plaxis 2012) have been used to obtain this hysteretic damping information. Finally, the substitute structure method of Shibata and Sozen (1976), utilised recently by Priestley et al (2007), is used to obtain a single degree of freedom model of the structure - foundation system.

With these tools the paper proposes a method for estimating the earthquake response of shallow foundations for which uplift is permitted. The calculation process is an iterative one but relatively simple and complements the use of structural analysis software in which the foundation is modelled as a bed of springs followed in the companion paper by Storie and Pender (2013).

2 FIELD DATA AND FOUNDATION MOMENT-ROTATION CURVE

Field experiments have been conducted at a site in Auckland with shallow foundations subject to gradual pull-back followed by cyclic response after snap-back release; more details are given by Algie et al (2010) and Algie (2011). The set-up for the snap-back testing procedure is shown Figure 1. The response of the system to one impulsive excitation is obtained with each snap release. An added bonus is the static load-deflection curve obtained during the pull-back phase of the test. The pull-back response is the main input data used in this paper. Elsewhere, Algie (2011), the relations between the rocking period and the pull-back angular displacement and damping and the pull-back angular displacement are presented. A sequence of snaps from different initial loads shows how the nonlinear behaviour of the foundation develops as the applied load increases.

The site used for the tests, in Albany in the northern part of Auckland, consists of a profile of stiff cohesive soil formed by in situ weathering from tertiary age sandstone and siltstone (it is thus a residual soil profile). The ends of the steel frame shown in Figure 1 were supported on shallow foundations at each end; reinforced concrete 2.0 m in length and 0.4 m square. There were four sets of shallow foundations so the tests could be repeated at four different "sites". The steel frame structure was 2 m wide, 3.5 m high and 6 m long. Steel kentledge was strapped to the top of the frame to provide the required vertical foundation load.

The soil profile was investigated with 21 CPT tests between the surface and depth of 5 to 8 m; in some of these the shear wave velocity of the soil was measured which indicated a reasonably consistent shear wave velocity for the materials at the site equivalent to a small strain shear modulus for the soil of about 40 MPa.

Figure 1 Shallow foundation set-up for snap back testing. Simple structure loaded with kentledge and supported on shallow foundations at the right. Chains attached to a hydraulic ram with a quick release shackle connected to the crane to provide reaction.
Figure 2 Moment-rotation curves for the static pull-back phase of the shallow foundation snap-back tests. Note that for each successive test there has been a slight degradation in the moment-rotation curve.

Figure 3 Curve-fitted foundation moment-rotation relation along with some of the pull-back data plotted in Figure 2.

In Figure 2 are shown all the static moment-rotation curves obtained during the application of the pull-back forces to one of the foundation sets at the site. It is apparent that there is considerable nonlinearity in the moment-rotation curves and also that the stiffness is reduced from one snap-back to the next, in particular for those tests following the snap-back which applies the largest moment to the system.

A hyperbolic curve is fitted through the test data as shown in Figure 3. The form of the hyperbolic relationship is:

\[
M = \frac{\phi}{a} + \frac{b\phi}{K_{\phi} \cdot M_{\phi}}
\]

(1)

where: \(M\) and \(\phi\) are the foundation moment and rotation respectively.
\( K_i \) and \( M_{ult} \) are the initial rotational stiffness and moment capacity of the foundation, and \( a \) and \( b \) are numerical values used to refine the fit of the curve to the data (quite a reasonable fit is obtained if \( a \) and \( b \) are set to 1.0).

In Figure 3 the data presented in Figure 2 for the initial three (stiffest) pull-backs were used to define the foundation hyperbolic moment-rotation curve. The reason that the stiffest responses were used for the curve fit is because this will minimise any SFSI effect. (The observed degradation of the subsequent foundation moment-rotation curves is, presumably, because of the irreversible deformation of the ground beneath the edges of the foundations.) Also shown in this figure is the estimated moment capacity of the foundation for the applied vertical load. The graph shows clearly that the hyperbolic curve fits the recorded data very well and that the moment capacity controls the curve for the extrapolation beyond the recorded data.

An important parameter in the development of the hyperbolic curve fit is the initial rotational stiffness of the foundation which can be obtained using the formulae of Gazetas (1991). Since this is the stiffness at very small load the small strain elastic modulus of the soil might seem to be the appropriate value to use. However, then the first part of the moment-rotation curve is far too stiff, so an “operational” stiffness, about one third to a quarter of the small strain value (EuroCode 8, CEN 2003), was used in calculating the hyperbolic curve in Figure 3.

### 3 FINITE ELEMENT MOMENT-ROTATION CURVES AND HYSTERETIC DAMPING

Figure 4 shows the moment-rotation curve obtained with the Abaqus (Simulia 2010) three dimensional nonlinear finite element software. The rigid foundation is resting on the surface of saturated clay with the properties obtained from the site investigation at the Auckland site. The soil behaviour was nonlinear and the interface condition between the foundation and the underlying soil was able to accommodate detachment and reattachment of portions of the interface. Comparison of Figures 3 and 4 makes it clear that the field data is modelled accurately by the hyperbolic moment-rotation curve and also by the sophisticated finite element software.

An important additional parameter is the damping provided by the soil beneath the foundation. Elastic radiation damping can be estimated using the methods of Gazetas (1991); this is velocity dependent. In addition there is hysteretic material damping from the cyclic loading of the soil near the foundation; hysteretic damping in soil is frequency independent. Having concluded that the nonlinear three dimensional finite element analysis is capable of modelling the push-over moment-rotation behaviour of shallow foundations with uplift, the next step is to apply this to cyclic loading to estimate shallow foundation hysteretic damping. Figure 5 has such damping information for a square foundation on a dense sand layer (friction angle 40 degrees) calculated with the Plaxis 3D software.

**Figure 4** Abaqus finite element modelling (circular markers) of the shallow foundation pull-back data compared with the hyperbolic curve fit from Figure 3.
4 BUILDING-FOUNDATION SUBSTITUTE STRUCTURE MODEL

The shape of the shallow foundation moment-rotation curve is not unlike the moment rotation curve for a reinforced concrete element. Figure 6 reproduces a diagram from the paper by Shibata and Sozen (1976). The procedure followed in this paper is to make use of the ideas proposed by Shibata and Sozen, but to assume that the structural elements are elastic whereas the shallow foundation is where the nonlinearity in the structure-foundation system is located. Figure 7 is taken from Priestley et al (2007) and it shows how the approach of Shibata and Sozen can be used to reduce a multi storey frame structure to an equivalent single degree of freedom model; they refer to this as a substitute structure. Priestley et al explain how the parameters for the substitute structure, \( h_s \), \( m_e \), and \( K_e \), can be evaluated.

One additional aspect needs to be included in the substitute structure model, the compliance of the soil beneath and adjacent to the foundation. This is done by representing the structure-foundation system as a SDOF structure supported on a rotational spring and a horizontal spring. Equation 2 gives the natural frequency of the substitute structure-foundation system.

Figure 5 Hysteretic damping relationship for a square shallow foundation on dense sand.

Figure 6 The basis of Shibata and Sozen’s substitute structure concept.
Figure 7 Implementation of the substitute structure as presented by Priestley, Calvi and Kowalsky.

\[ \omega_{\text{substitute}}^2 = \frac{\omega_s^2}{1 + \frac{K_h}{K_n} + \frac{K_h h_e^2}{K_e}} = \frac{K_e}{m_e} \]  

where: \( \omega_{\text{substitute}} \) and \( \omega_s \) are the natural frequencies of the substitute structure-foundation system and the fixed base structure respectively,  
\( K_h \) and \( K_e \) are the stiffnesses of the fixed base substitute structure and substitute structure-foundation system respectively,  
\( K_h \) and \( K_e \) are the horizontal and rotational stiffness of the foundation respectively,  
and \( h_e \) is the height of the substitute structure lumped mass (\( m_e \)).

5 EXAMPLE CALCULATIONS

Calculations were done to illustrate the design process outlined above. A 10 storey building with a 24 m square foundation resting on dense sand was considered. The sand had a friction angle of 40 degrees and a small strain shear modulus of 66 MPa. The spectrum for the CHHC record for the Christchurch earthquake of February 22, 2011 was used as the input. The steps in the calculation process are:

(i) Obtain soil profile details and properties and select a design earthquake spectrum.
(ii) Reduce the soil small strain shear modulus to an operational value (EC8 part 5, Table 4.1, CEN 2003).
(iii) Assign dimensions to the trial shallow foundation and determine the rotational and lateral stiffness (use the Gazetas (1991) expressions).
(iv) Evaluate the vertical load on the foundation and obtain the moment capacity of the foundation under that load.
(v) Use equation (1) to develop the moment-rotation relationship for the foundation
(vi) Evaluate \( h_e \), \( m_e \) and \( K_e \) for the substitute structure (Figure 7).
(vii) Use equation (2) to evaluate the natural period of the substitute structure-foundation system using the elastic stiffnesses for the foundation.
(viii) For the first iteration use the period from (vii) to get the spectral acceleration and evaluate the foundation actions and rotation.
(ix) Repeat step (viii) at different system periods so that a set of responses is obtained. For each iteration, assign the damping value (Figure 5).
(x) Plot the results from (ix) onto the foundation moment-rotation curve and hence determine the actions and period where the demand crosses the foundation curve.

Figure 8 gives the results of the above series of calculations for the 24 m square foundation. Figure 8a gives the foundation properties. Figure 8b gives the details of the “design” solution. Figure 8c has the response spectrum for the design earthquake motion with the various trial solutions marked. Figure 8d
Figure 8 Results of the example calculation. (a) properties of the structure-foundation system, (b) details of the response where the earthquake demand matches the foundation behaviour, (c) ground motion response spectrum showing the various trials, (d) moment-rotation data for the various trials in relation to the foundation moment-rotation curve.

shows the moment and rotation values obtained for the various iterations (step (ix) above). From here we see that for trial number four the demand matches a point on the moment-rotation curve. Note that the “bumpy” nature of the response spectrum in Figure 8c is reflected in the responses plotted in Figure 8d.

6 DISCUSSION

The process outlined above is attractive because of its simplicity and “hands-on” calculation. However, other approaches are possible. Commercial structural analysis software suites provide an option to represent the foundation as a bed of springs, and some have springs that can detach and reattach. An example, using detachable springs, is given in a companion paper by Storie and Pender (2013). Spring-bed models, with uniformly distributed springs of equal stiffness, do not give the correct rotational stiffness for a shallow foundation if the spring stiffnesses are assigned to match correctly the vertical stiffness of the foundation. To overcome this problem the FEMA 356 document (Federal Emergency Management Agency 2000) recommends placing stiffer springs at the edges of the foundation whilst keeping the overall foundation vertical stiffness the same. Figure 9 shows the hyperbolic moment-rotation curve developed herein in relation to push-over curves calculated with the SAP2000 software (CSI 2011) for the two spring-bed foundation models. This shows that the uniform spring-bed model matches very closely the hyperbolic relationship obtained from equation 2. On the other hand, the FEMA 356 spring-bed over-predicts the foundation moment for rotations less that about 0.03 radians. This mismatch occurs at rotations which are important for the evaluation of SFSI effects on shallow foundation design.

Yet another approach has been presented by Toh and Pender (2010).
CONCLUSIONS

In this paper we have outlined briefly an approach to the design of shallow foundations that accounts for loss, and subsequent re-establishment, of contact at the edges (rocking) of the foundation during earthquake cyclic loading. The method is amenable to “hand” calculations and complements spring-bed modelling using structural analysis software. In this paper the method has been presented within a force-based design framework. It has also been developed within a displacement based design framework, Algie (2011).

REFERENCES


