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Problems with Seismic Design Based on Elastic Stiffness

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ABSTRACT Compared to its small population, New Zealand has had a disproportionate influence internationally on seismic design philosophy. It will be shown that many of the contributions of different New Zealanders to understanding seismic performance have been at least related to problems perceived in the consequences of designing structures based on elastic stiffness estimates.

1 INTRODUCTION: NEW ZEALAND CONTRIBUTIONS TO SEISMIC DESIGN

Seismic design philosophy is in a constant state of change. Elastic design from the 1920's gave way to strength design in the 1960's; lateral force vectors corresponding to approximately 0.1g uniform with height (buildings) or length (bridges) gave way to period-dependent base shear, and modal shape (approximately!) distribution of the base shear. This was followed by multi-mode assessment of base shear and lateral force distribution, and reduction of elastic force levels in recognition of the salutary influence of ductility. Capacity design to ensure a suitable heirarchy of strength in a structure was used to modify the results of elastic analyses. In the 1990's and 2000's, performance based seismic design became fashionable, and with it, the realization that strength and performance were not closely related, while displacement and performance could be.

Together with developments and changes to design philosophy, developments in improving structural response were taking place. The importance of careful detailing of plastic hinges was recognized, based on extensive experimental research, and methods of damage avoidance, such as seismic isolation were developed.

New Zealand has had a significant role to play in the development of both seismic design philosophy and detailing. In this introductory section a few of the more significant aspects, internationally, of New Zealand's role are briefly summarized. A more detailed summary of New Zealand's internal development of seismic design is available in Megget (2006).

1.1 Capacity Design

The concept of capacity design, where a desireable mechanism of inelastic response under seismic attack is ensured by providing a strength heirarchy (weak-beam/strong column; shear strength > flexural strength) was initially proposed by John Hollings in 1968 (Hollings, 1968) related to frame buildings. The concept was developed further by Park and Paulay (1976) and researchers at Canterbury University and the (then) Ministry of Works into the basic capacity design equation:

$$\phi_S S_R \geq \phi^o \omega S_E \qquad (1)$$

In Eq.(1) the strength S_E determined from the design lateral forces is modified by an overstrength factor ϕ^o recognizing the possibility of overstrength materials and strain hardening in the potential plastic hinges, and by a dynamic amplifcation factor ω acounting for higher mode effects. The left side of Eq.(1) represents the dependable required strength of the action being capacity protected.

Capacity design has had a significant influence on seismic design, not just in New Zealand, but in many seismically active regions, and could be considered to be the most important contribution of New Zealand to seismic design. Factors for flexural overstrength and dynamic amplification developed in New Zealand for frames and structural walls (e.g., Fig.1) more than 30 years ago have remained unchallenged until recently (e.g. Fig.2), when it became apparent that ductility demand should be incorporated in the basic capacity design equation (Priestley et al 2007).

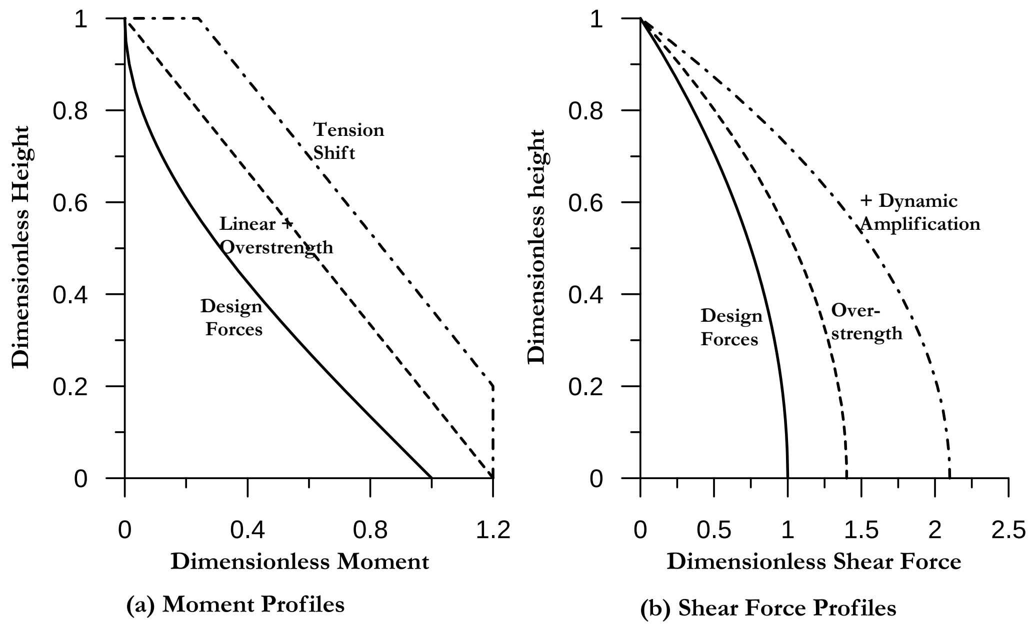

Fig. 1 Early Recommendations for Capacity Design of Structural Walls (Blakely et al 1975)

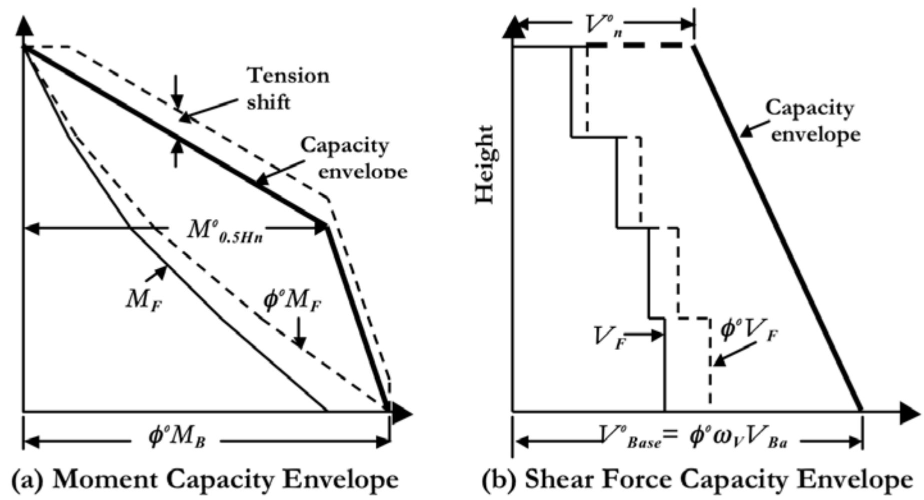

Fig.2 Recent Modifications to Capacity Design of Structural Walls (Priestley et al 2007)

In Fig.2 the midheight moment and shear force at base and top of the wall are functions of the design ductility demand.

1.2 **Coupled Walls**

Tom Paulay (Paulay 1972) developed the concept of replacing conventional flexural flexural and shear reinforcement in coupling beams of coupled walls with diagonal reinforcement as a solution to the problem of extremely high drifts that must be expected in these beams, purely based on geometrical considerations (Fig.3a).

From the geometry of Fig.1a it is obvious that the coupling beam drift is amplified above the wall drift at the same level by the expression

$$\theta_{CB} = \theta_W \left(\frac{L_W + L_{CB}}{L_{CB}} \right) \tag{2}$$

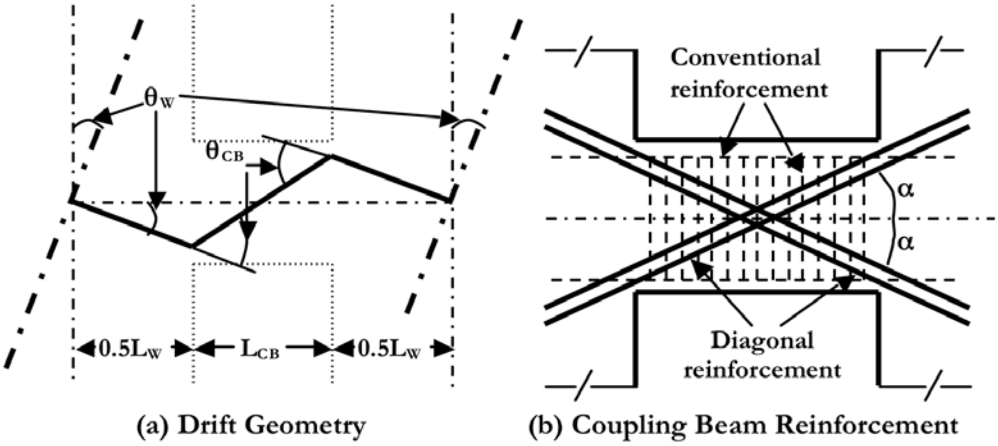

Fig.3 Deformations of Coupling Beams

A consequence of the squat geometry of typical coupling beams is that is that they tend to yield at extremely low drifts – long before the wall-base plastic hinges form. Paulay recognized early on that this resulted in vastly different ductility demands on the walls hinges and the coupling beam hinges. Difficulties resolving this conflict with the design requirement of a specified global ductility demand for the structural type are discussed later in this paper.

1.3 **Estimation of Ductility Capacity**

A fundamental basis of performance-based seismic design is the determination of curvature ductility capacity of critical plastic hinges, and the corresponding global structual displacement ductility capacity. Work carried out at the University of Canterbury by researchers under the guidance of Park and others from the 1970's to the 1990's (e.g. Priestley and Park 1984) related to the performance of bridge structures appears to be the earliest work in quantifying displacement capacity and relating it to demand. Plastic hinge length was based on extensive experimental results, and limit-state curvatures were related to the amount, distribution and detailing of transverse reinforcement. Information from the experimental program has found its way into a number of overseas design approaches, as well as into New Zealand design practice.

1.4 **Seismic Isolation and Added Damping**

Although the concept of seismic isolation is not new (Pliny describes a form of base isolation developed by the Greeks for the Temple of Artemis, build in the 5th C.BC, and various methods of seismic isolation have been advanced since the Messina earthquake of 1908), the development of practical seismic isolation results from collaboration between the University of Berkeley (Jim Kelley) and the (then) Physics and Engineering Laboratory of the (then) DSIR in Lower Hutt (Ivan Skinner, Bill Robinson and others). Of the many different types of seismic isolation and added damping proposed by Ivan Skinner's group, the lead/rubber bearing developed by Bill Robinson, which combines isolation from the period shift associated with support of the structural mass on elastomeric bearings, with energy dissipation (added damping) from the excellent hysteretic characteristics of lead cores confined by the rubber bearing. The results of the New Zealand research and development of practical isolation systems has been widely implemented overseas in both bridge and building structures.

1.5 **Realistic Seismic Analysis**

The computer code *Ruaumoko* developed at the University of Canterbury over the past 40 years by Athol Carr and a multitude of graduate students (the original platform was developed as the PhD dissertyation of Richard Sharpe) is now one of the most powerful nonlinear time-history codes available anyway. It has a very large hysteretic element library, and has gradually developed from the original 2D version to have full 3D capability, and the ability to perform adaptive pushover analyses, graphical post-processing and generation of artificial accelerograms. The code is internationally respected, and used by researchers and consultants in many countries.

1.6 **Torsional Response of Buildings**

In the mid-1990's Tom Paulay realized that distributing additional resistance to walls to cope with torsional response should not be based on initial stiffness analysis, since member stiffness was unknown at the start of the design process, and the stiffness eccentricity had little relevance once the structure entered the inelastic phase of response (Paulay 2001). He determined that strength eccentricity that was more relevant, and thus the designer could contol this by ignoring the results of elastic analysis, and distributing strength in such a way that the eccentricity of the centre of strength from the mass centroid was minimized (see Fig.4, where C_R and C_V are the centres of stiffness and strength respectively).

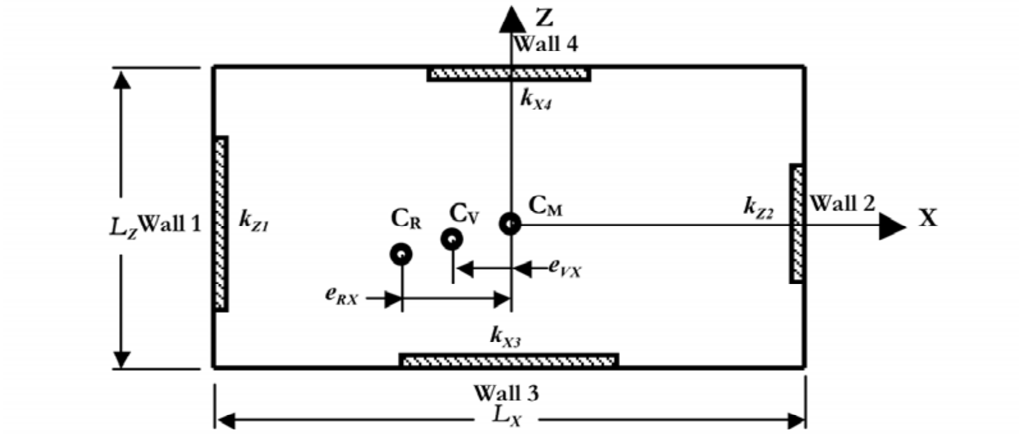

Fig.4 Plan View of Building with Definition of Terms for Torsional Analysis

Subsequent work by Beyer and Priestley (reported in Priestley et al 2007) carried this concept further within a displacement-based design environment. They showed that reducing the strength eccentricity also reduced the stiffness eccentricity and that modifying the rotational stiffness by using effective stiffness at the design response, rather than the elastic stiffness, resolved the design problems.

$$J_{R,\mu} = \sum_{1}^{n} \frac{k_{zi}}{\mu_{sys}} (x_i - e_{RX})^2 + \sum_{1}^{n} k_{Xi} (z_i - e_{RZ})^2 \qquad (3)$$

This is expressed in Eq.(3), where the elastic stiffness of the walls parallel to the direction of excitation (Z) is reduced by the expected system displacement ductility.

1.7 **Further Influences**

The influence of New Zealand initiative on international understanding of seismic design described above is by no means complete, but space limitations put limits on what can adequately be discussed. In addition to structural aspects as noted, the work at Auckland University on soil/structure interaction directed by Mick Pender, on masonry research (directed by Jason Ingham), and on inelastic beam elongation (Richard Fenwick and Les Megget) have received international recognition. Similarly, the work at the University of Canterbury on liquefaction by Davis, Berrill and others has had international significance.

2 PROBLEMS WITH ELASTIC STIFFNESS

The contributions identified in Sections 1.1 to 1.6 can all be interpreted as attempts to modify the results obtained from elastic analysis of seismic response to obtain performance which will improve behaviour. This has lead to a greater examination of problems associated with initial-stiffness based seismic design. These can be categorized as 1) assumptions related to quantifying stiffness and 2) distributing seismic forces through a structure based on initial stiffness. These are examined briefly before suggesting alternatives.

2.1 Quantifying elastic stiffness

Elastic stiffness is used to distribute seismic forces between different elements of the seismic resisting system, and, in many modern codes (such as the New Zealand Loadings code – NZS1170.5) to determine the elastic periods, and hence the seismic base shear. An exception to this is the practice – still common in many codes - to use arbitrary height-dependent equations for building period.

A major problem is that elastic stiffness depends not just on section dimensions and material properties, but critically on section strength, despite all codified approachs assuming that stiffness is independent of strength. An example is shown in Fig.5, which shows the influence of axial load ratio and reinforcement ratio (both of which influence section strength) on the effective stifness ratio of circular bridge columns. It will be seen that the effective stiffness can vary between 12 and 90% of the gross-section stiffness.

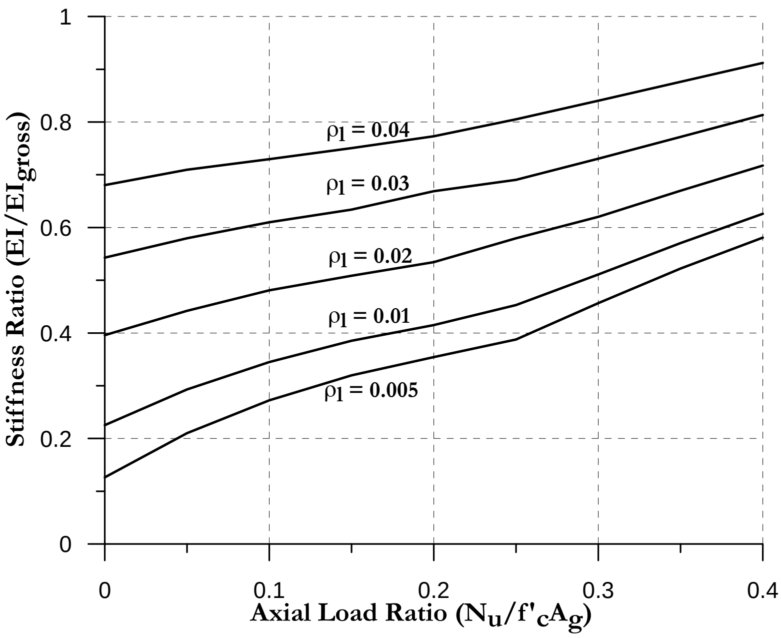

Fig.5 Effective Stiffness Ratio for Circular Bridge Columns (Priestley et al, 2007)

Extensive analyses, reported in Priestley et al 2007, of different concrete section shapes, including rectangular and circular columns, walls, and rectangular and flanged beams indicate that the yield curvature is effectively independent of strength, for a given section shape, as shown in Fig.6b, and hence the common assumption of stiffness being independent of strength (Fig.6a) is completely invalid. Stiffness is thus proportional to strength, and the current practice of distributing strength in proportion to an initial estimate of stiffness is meaningless.

It is a common misconception that the argument developed above applies only to concrete and rein-forced masonry sections and not to strictural steel. However, moment capacity of steel I-Beam or U-C sections is varied by changing the flange thickness for a given section depth. It is easy to show that the consequence of varying flange thickness is to change section flexural strength and stiffness in almost direct proportion.

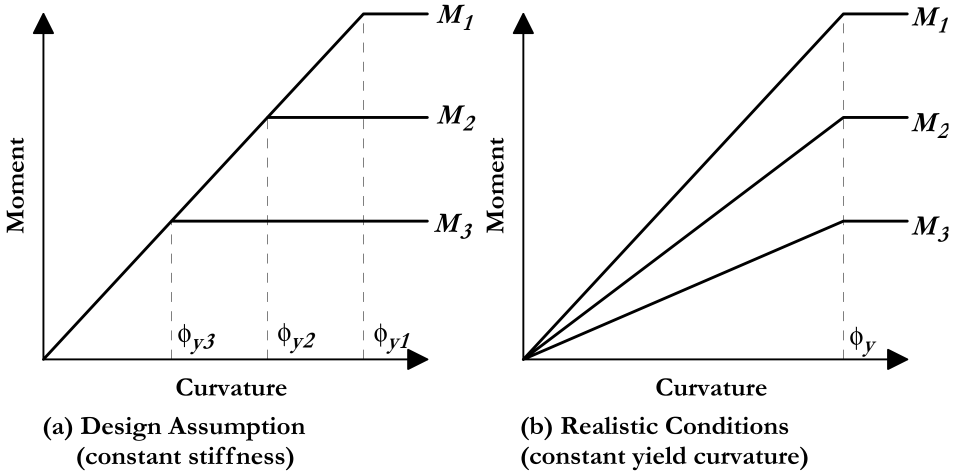

Fig.6 Influence of strength on moment-curvature characteristics

2.2 **Distributing strength in proportion to elastic stiffness**

As well as errors in calculation of elastic periods, and hence in determination of design base shear force, the inability to determine elastic stiffness at the start of the design process affects the distribution of seismic resistance between diffeent structural elements. Current design for ductile response is based on the assumption, exemplified in Fig.6a, that when strength is distributed in proportion to elastic stiffness, all structural elements can be forces to yield simultaneously. The reality, shown in Fig.6b, is that this is impossible.

Two examples are considered to illustrate the consequences. The first is the simple bridge structure of Fig.7, being designed for longitudinal seismic response.

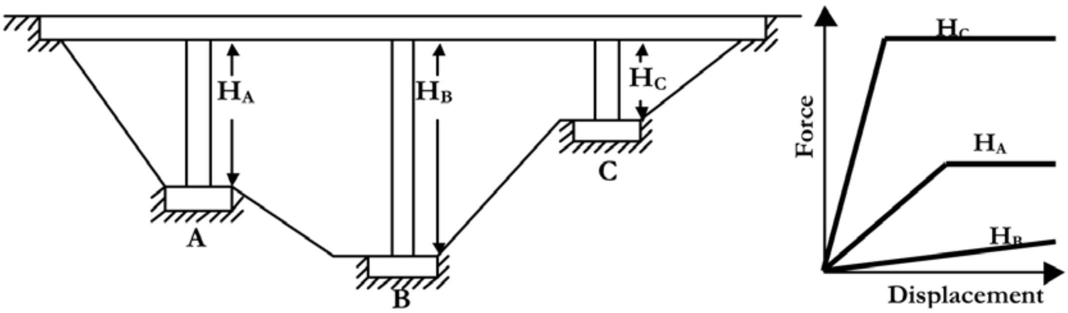

Fig.7 Bridge with Different Height Piers under Longitudinal Seismic Response

Strict application of current seismic design philosophy based on initial stiffness estimates would allocate shear between the piers in proportion to their elastic stiffness; in other words, in proportion to the inverse of height cubed, on the incorrect assumption that the piers could be made to yield simultaneously. In fact, since the yield curvature is independent of strength, the yield displacements will be proportional to height squared, provided the sections have the same external dimensions, as will normally be the case. The consequence of current philosophy will be to provide flexural strength in inverse proportion to height squared, meaning that the short pier C will be allocated a much higher reinforcement content than (e.g.) pier B. If the designer recognizes that this will increase the flexural stiffnes of the pier C section relative to that of pier B (see Fig.5 above), and reanalyzes the bridge, they will allocate a still higher fraction of the seismic resistance to pier C. This is not only illogical; it is potentially unsafe, as displacement and shear capacity of the critical pier C will be compromised as a consequence.

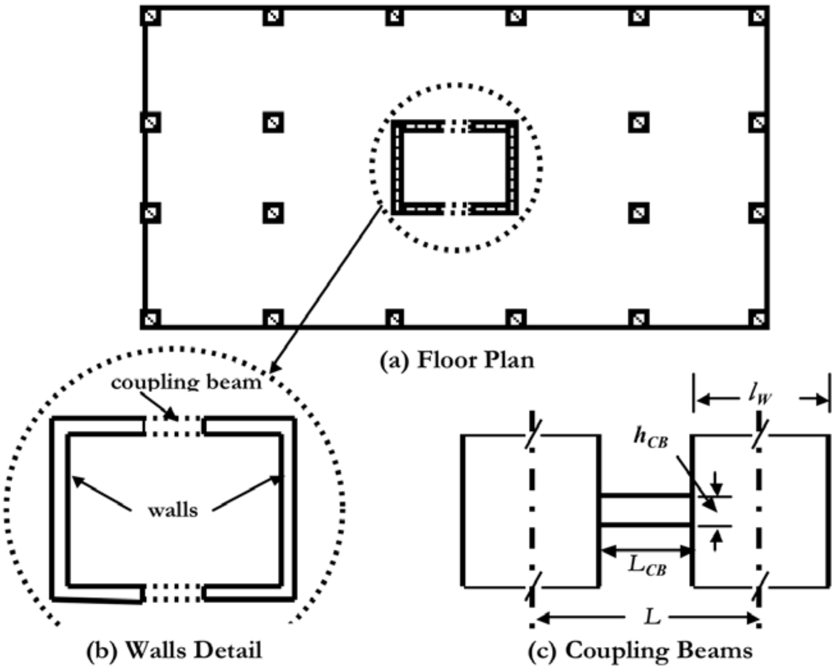

Fig. 8 Structural Layout of A Coupled Wall Building

A second example is provided by the typical coupled wall building of Fig.8 which consists of two C-section walls coupled by beams, as shown in Fig.8c. Gravity columns and flat slabs complete the structural system. Seismic resistance is provided by frame action, involving coupling beam moments and shears and axial forces in the walls, and flexural response of the walls. If elastic analysis is used to allocate base shear between these two actions, the required flexural strength of beams will vary up the height of the building, which is illogical. More importantly, the distribution of resistance between frame action and wall bending will not recognize the fact that the coupling beams will yield at a much lower structural displacement than will the walls. An example given in Priestley et al 2007 of a 12-story coupled wall building of typical proportions shows that when the walls yield, the critical coupling beam (which is at the height of the wall contraflexure point) will already have a rotational ductility demand of 8.5. Using elastic analysis to distribute strength between the two resisting mechanisms is thus completely inappropriate.

3 ALTERNATIVES TO ELASTIC STIFFNESS-BASED ANALYSIS

Direct displacement-based seismic design is an alternative approach to determining seismic base shear for a given structure, and for distributing this base shear through the structure in a fashion that recognizes inelastic, rather than elastic response. Elastic stiffness is not used in the design process at all, and hence the problems discussed above do not arise. Equilibrium considerations, and designers' judgment are used in the distribution of resistance. The procedure uses effective stiffness to design displacement demand, and equivalent viscous damping to represent the effects of ductility. Since the procedure has been presented in many previous documents it is not developed herein. Interested readers are referred to Priestley et al 2007 for full details.

Referring to Fig. 7, the logical design choice is to use the same flexural reinforcement ratio for all piers. This means that design base shear will be distributed between the piers in inverse proportion to the pier height, not to height cubed. Ductility demand of the piers will be in inverse proportion to height squared, which is considered in direct displacement-based design in determining the system equivalent viscous damping.

Referring to Fig.8, the allocation of base-shear between wall bending and coupling beam frame action is a designer's choice. This is caried out by determining the fraction of total overturning moment at the structural base to be allocated to each of the two actions in similar fashion to that described subsequently for frame analysis. Note that this greatly simplifies the design process, and allows the designer to use constant beam depths and beam reinforcement up the height of the building. Full details are available in Priestley et al 2007.

An example of how seismic resistance can be distributed to the different elements of a structure based on equilibrium considerations is now presented with reference to the frame structure of Fig. 9.

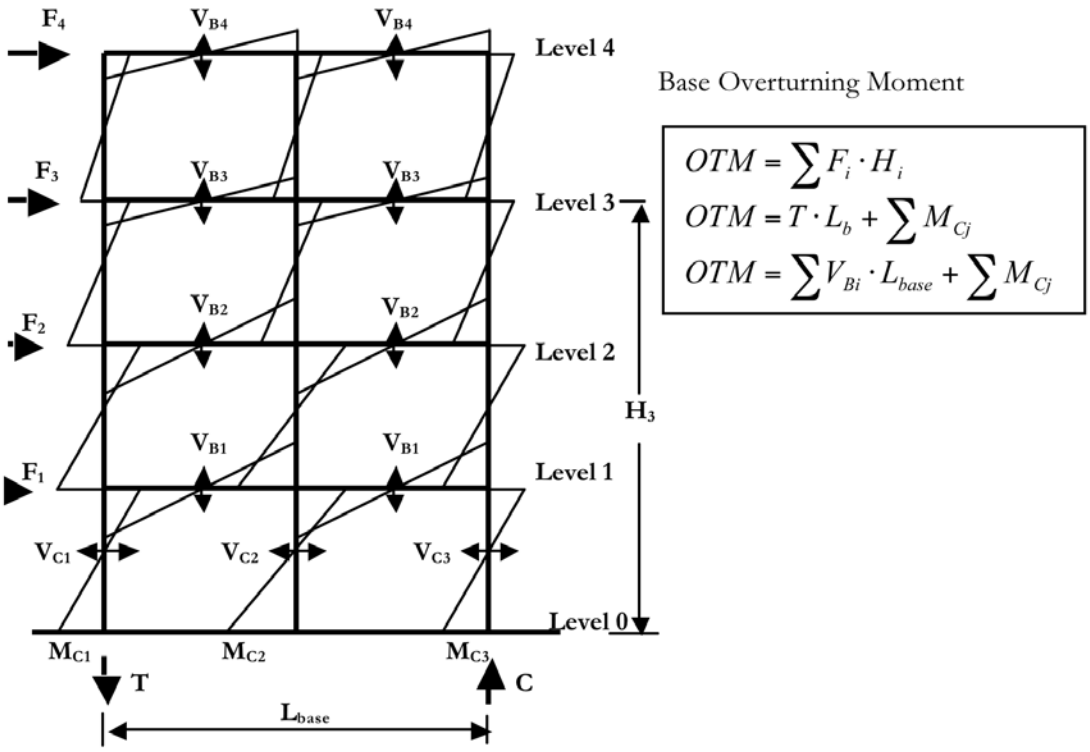

Fig.9 Equilibrium Considerations for Frame Seismic Forces

Considering the building as a vertical cantilever, the base overturning moment (OTM) from the external forces F_i must be in equilibrium with the internal forces at the building base provided by the column base moments M_C abd the column axial forces T and C. Making moments about the base of the compression column:

$$OTM = \sum_n F_i H_i = \sum_1^3 M_{Cj} + T \cdot L_{base} \tag{4}$$

Column base moments: The value of the design column base moments is a designer's choice. This may seem arbitrary, but it should be recognized that the choice of a pinned base or a fixed base is commonly accepted, but is much more arbitrary. A sensible decision is to base the column design moments on a selected height of the column contraflexure point. Choosing this as approximately 0.65x the storey height will ensure that column moments below the first floor beam soffit are sufficiently low so that a soft storey mechanism cannot occur, even when higher mode effects are considered. Since the sum of column shears in the first storey must equal the base shear, we have

$$\sum_{1}^{3} M_{Cj} = 0.65 H_1 V_{base} = 0.65 H_1 \sum_{n} F_i \qquad (5)$$

Beam seismic shears: Equilibrium requires that the seismic tension force at the base of the left column equals the sum of beam seismic shears in the left bay of the frame. Incorporating the relatinship of Eq.4:

$$\sum_{n} V_{Bi} = T = \left(\sum_{n} F_i H_i - \sum_{1}^{3} M_{Cj} \right) / L_{base} \qquad (6)$$

The distribution of the seismic tension force T between the beam seismic shears up the building is again, to some extent, a designer's choice. However, to ensure a well-conditioned frame with similar ductility demands up the building height, T should be allocated to the beams approximately in proportion to the storey shears in the level below the beam under consideration.

Beam seismic moments: Once the individual beam shears have been decided in such a way that Eq.(6) is satisfied, the lateral force-induced beam design moments at the column centrelines are given by:

$$M_{Bi,l} + M_{Bi,r} = V_{Bi} \cdot L_{Bi} \qquad (7)$$

where *l* and *r* refer to left and right ends of the beam, and L_{Bi} is the beam end. Again the designer can choose, with some freedom, how to allocate the sum of beam moments between the two ends.

Column moments and shears: Column shears need to be in equilibrium with the storey shears. Column moments can be determined either from equilibrium with beam moments at the joint centroids, or by assuming equal design moments at top and bottom of each column (except the first storey, as discussed above). The method adopted is not particularly critical since the moments corresponding to the design lateral forces will be amplified for higher mode effects by capacity-design principles.

4 CONCLUSIONS

The availability of powerful computers has led us to rely more on elastic stiffness-based analysis than is appropriate, given the uncertainty of elastic stiffness at the start of the design process, and the severe modifications that result when selected elements (e.g. beams, but not columns, in a frame) respond with ductility. Equilibrium-based analyses, and designer's judgments are more appropriate for design, and sophisticated stiffness-based analyses should be reserved for design verification, when members strengths (and hence elastic stiffness) and ductility characteristics have been decided.

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