Hysteresis Behaviour of Reinforced Concrete Non-Ductile Beam-Column Joints

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ABSTRACT: This paper demonstrates an analytical approach for predicting the hysteresis behaviour of nonseismically detailed reinforced concrete beam-column joints. Considering the versatility and mathematical tractability of Bouc-Wen-Baber-Noori model for single-degree-of-freedom system, this model has been adopted with suitable modification for the research. The model, in its analytical form of a set of differential equations, can capture the true characteristics of non-ductile reinforced concrete beam-column joints, like stiffness and strength degradation, pinching, softening and hardening. Livermore Solver for Ordinary Differential Equations of double precision version has been used for solving the differential equations, involved in the model. As hysteresis behaviour of non-ductile RC beam-column joints is not only dependent on the individual magnitude of the parameters, included in the differential equations, but also on their interaction, Genetic Algorithm has been selected as a system identification tool to carry out systematic parameter estimation with reasonable accuracy. The analytical responses have been compared with the experimental results of six internal and six external non-ductile reinforced concrete beam-column joints from literature. Good correlation between the analytical and experimental results proves the effectiveness of the model and accuracy of the solver and algorithm.

1 INTRODUCTION

In low or non-seismic regions, reinforced concrete buildings are normally designed based on gravity loading only, contrary to the modern seismic design specifications. These buildings have little or no transverse reinforcement in beam-column joint regions. Therefore it is of great concern that the performance of these structures may not be adequate to sustain earthquake-induced loads. Lack of realistic models of the non-ductile structural components can be proved detrimental to the seismic reliability assessment of reinforced concrete structures. Analytical model of beam-column joints under seismic loading requires a force displacement relationship capable to yield the true behaviour of the structural components at all displacement levels. This is a very stringent requirement considering the numerous parameters contributing to strength and stiffness degradation and pinching. Pinched loops are typical in non-ductile reinforced concrete beam-column joints under cyclic loading due to the presence of high shear forces, opening and closing of cracks, rebar slippage at beam-column interfaces. In order to explore the performance of non-ductile reinforced concrete beam-column joints, several experimental investigations were conducted by various researchers (Hakuto et al., Liu et al. etc). As it is not always feasible to test every aspects, an utmost effort has been undertaken at Nanyang Technological University, Singapore to obtain better understanding on the hysteresis behaviour of non-ductile reinforced concrete beam-column joints and to model the hysteresis loops analytically.

2 HYSTERESIS MODEL

2.1 Basis

The objective of this study is to develop a hysteresis model which is generic, computationally efficient and mathematically tractable such that it is applicable to the random input functions. The basis for the model is the hysteresis model introduced by Bouc, Wen, Baber and Noori (BWBN).

2.2 Model development

The hysteresis behaviour of each beam-column joint has been considered as a single-degree-of-
freedom system consisting of a mass connected in parallel to a viscous damper, a linear spring and a non-linear hysteretic spring. The schematic model of such a single-degree of freedom hysteretic system is shown in Figure 1.

Figure 1: Schematic Model of a single-degree of freedom Hysteretic System

The equation of motion of such system is:

$$m\ddot{u} + c\dot{u} + F_r[u(t), z(t), t] = F(t)$$

(1)

where \(u\) is the relative displacement of the mass \(m\) with respect to the ground motion and the dot (\(\cdot\)) signifies the differential with respect to time; \(c\) is the linear viscous damping coefficient; \(F_r[u(t), z(t), t]\) is non damping restoring force consisting of the linear restoring force \(ak\dot{u}\) and the hysteretic restoring force \((1 - \alpha)kz\); \(\alpha\) is the stiffness ratio or rigidity ratio; \(z\) is hysteretic restoring force; \(F(t)\) is the time dependant forcing function.

Dividing both sides of (1) by \(m\), the following expression is obtained:

$$\ddot{u} + 2\xi_0\omega_0\dot{u} + \alpha\omega_0^2u + (1 - \alpha)\omega_0^2z = f(t)$$

(2)

where \(\xi_0\) is linear damping ratio \((c/2)\sqrt{k_0m}\); \(\omega_0\) is the pre-yield natural frequency of the system \(\sqrt{k_0/m}\); \(f(t)\) is the mass-normalized forcing function.

The hysteretic restoring force is a function of hysteretic displacement \(z\) and the relationship between \(z\) and \(u\) according to the standard BWBN model is shown in the following expression:

$$\dot{z} = h(z) \left\{ \frac{\dot{u} - v(\beta|\dot{u}|z|^{n-1}z + \gamma|\dot{u}|z^n)}{\eta} \right\}$$

(3)

where \(\beta\), \(\gamma\) and \(n\) are hysteretic shape parameters; \(A\) determines tangent stiffness; \(v\) and \(\eta\) are strength and stiffness degradation parameters respectively; \(h(z)\) is pinching function.

For a non-pinching and non-degrading system, it is considered that the hysteresis is defined by a continuous function and the hysteretic stiffness is always zero at the local maximum or minimum. It is the point on the load-slip curve where velocity changes its sign. So, at an infinitesimal small distance \(dz\) away from \(z_{max}\), where the velocity is close to but not equal to zero and \(\dot{z}_{max} \approx \dot{z}\),

$$\dot{z}_{max} \approx 0 = A\dot{u} - (\beta|\dot{u}|z|^{n-1}z + \gamma|\dot{u}|z^n) \quad \text{or} \quad z_{max} = \pm \left(\frac{A}{\beta + \gamma}\right)^{\frac{1}{n}}$$

(4)

Although inclusion of \(A\) grants increased versatility of the model, this parameter is somewhat redundant as both hysteretic stiffness and hysteretic force, a function of hysteretic displacement, can be varied by the rigidity ratio \(\alpha\) and the hysteresis shape parameters \(\beta\), \(\gamma\) and \(n\). Thus, \(A\) has been set to unity to remove the redundancy.

2.3. Stiffness Ratio or Rigidity Ratio

Stiffness ratio or Rigidity ratio \(\alpha\) is the ratio of the final asymptote tangent stiffness to the initial stiffness. The magnitude of \(\alpha\) varies from 0 to 1. For linear system, \(\alpha\) is 1 and for complete nonlinear system, it is zero. In the original BWBN model, this parameter was considered as of constant magnitude. However, from experimental results of non-ductile beam-column joints under cyclic loading, it has been observed that the strength does not continue increasing with incremental displacement, as shown
in Figure 2. As a result, the stiffness of the structural components decreases. Therefore, stiffness ratio or rigidity ratio is expressed as a function of $D_{\text{max}}$:

$$\alpha = \alpha_0 e^{(-0.1 \cdot D_{\text{max}})}$$  \hspace{1cm} (5)

When $u > 0$, $D_{\text{max}}$ is the maximum positive displacement whereas when $u < 0$, $D_{\text{max}}$ is the absolute value of the maximum negative displacement.

The pinching stiffness (minimum tangent stiffness of the curve where unloading finishes and reloading begins) is around $\alpha_0 \omega_0^2$ and the stiffness decreases with development of maximum displacement. This proves that the model using varying $\alpha$ is more accurate.

2.4. Hysteresis Shape Parameters

Three hysteresis parameters $\beta$, $\gamma$ and $n$ and their interactions determine the basic hysteresis shape. Absolute values of $\beta$ and $\gamma$ inversely influence hysteretic stiffness and strength, as well as the smoothness of the hysteresis loops. For $n = 1$, the relationships between $\beta$ and $\gamma$ and their effects on hysteresis are described below and shown in Figure 3.

- $\beta + \gamma > 0$
- $\gamma - \beta < 0$
- $\beta + \gamma > 0$
- $\gamma - \beta = 0$
- Strong Softening on loading and unloading, narrow loop
- Weak Softening on loading, mostly linear unloading
- Strong Hardening
- Weak Hardening
- $0 > \beta + \gamma$
- $\beta + \gamma > \gamma - \beta$

For increasing values of $n$, the loading path of a softening hysteresis approaches the ideal elastic-plastic function while the unloading path approaches a straight line. In contrast, for increasing values of $n$, transition from linear to hardening sharpens with narrowing of hysteresis loops.

2.5. Hysteretic Energy

The hysteretic energy absorption is used in the model to approximate the system degradation. The
energy absorbed by the hysteretic element is the continuous integral of the hysteretic force, $f_h$ over the total displacement $u$, expressed as

$$
\varepsilon(t) = \int_{u(0)}^{u(t)} f_h \, du = (1 - \alpha) \omega^2 \int_{u(0)}^{u(t)} z(u, t) \, du \cdot \frac{dt}{dt} = (1 - \alpha) \omega^2 \int_0^T z(u, t) \cdot \dot{u}(t) \, dt
$$

(6)

2.6. Degradation

Strength and Stiffness degradation parameters, $\nu$ and $\eta$ respectively, are functions of the total dissipated hysteretic energy as shown in the following expressions:

$$
\nu(\varepsilon) = 1 + \delta_\nu \varepsilon \quad \text{and} \quad \eta(\varepsilon) = 1 + \delta_\eta \varepsilon
$$

(7)

where $\delta_\nu$ and $\delta_\eta$ are constants specified for rates of strength and stiffness degradation respectively at different displacement levels. When $\delta_\nu$ and $\delta_\eta$ values are zero, the structure does not degrade its strength and stiffness.

2.7. Pinching function

The expression of pinching function $h(z)$ used in analytical model is as follows:

$$
h(z) = 1 - \zeta_s \omega_0^2 \omega_0^2 z \cdot \frac{1}{\zeta_2^2}
$$

(8)

where $0 \leq \zeta_1 < 1$ determines the severity of pinching or magnitude of initial drop in slope $dz/du$; $\zeta_2$ causes the pinching region to spread and $q$ is a constant that sets the pinching level as a fraction of $z_u$. Both $\zeta_1$ and $\zeta_2$ vary with the total dissipated energy by hysteretic element, $\varepsilon$ as follows:

$$
\zeta_1(\varepsilon) = \zeta_s \left[1 - e^{-p_\varepsilon}\right] \quad \text{and} \quad \zeta_2(\varepsilon) = \left(\psi + \delta_\psi\right)(\lambda + \zeta_1)
$$

(9)

where $p$ is a constant that contributes to the rate of initial drop in slope; $\zeta_s$ is measure of total slip; $\psi$ is a parameter that controls the amount of pinching; $\delta_\psi$ is a constant for desired rate of pinching spread and $\lambda$ is a parameter that controls the rate of change of $\zeta_2$ with change of $\zeta_1$.

3 HYSTERESIS MODEL SOLVING

The complete hysteresis model can be represented in its analytical form by the following equations as

$$
\dot{u} + 2\zeta_0 \omega_0 \dot{u} + \alpha \omega_0^2 u + (1 - \alpha) \omega_0^2 z = f(t)
$$

(10)

$$
\dot{z} = \left\{1 - \zeta_s(1 - e^{-p_\varepsilon}) - (\zeta s \left[1 - e^{-p_\varepsilon}\right])^2/(\gamma + \zeta_1(1 - e^{-p_\varepsilon}))\right\}^{1/\gamma}
$$

$$
\times \left\{\frac{\dot{u} - (1 + \delta_\nu \varepsilon)\dot{\varepsilon}}{1 + \delta_\eta \varepsilon}\right\}^{1/\gamma}
$$

(11)

$$
\varepsilon(t) = (1 - \alpha) \omega_0^2 \int_0^T z(u, t) \cdot \dot{u}(t) \, dt
$$

(12)

Here all the notations carry their usual significances.

In the equations (10), (11) and (12), all the derivatives appear in the first power and variables vary with time at a highly different rate. Hence the hysteresis model constitutes a stiff set of ordinary differential equations (ODE), which can be solved numerically using Gear’s backward differential formulae. Though several numerical methods to solve stiff sets of stiff ODEs are available, the most widely used solver is the Livermore Solver for the Ordinary Differential Equations (LSODE). LSODE solves a wide range of ODEs including stiff systems for which it uses the Gear Method and it is capable of internally computing the Jacobian matrix. Moreover, the input functions do not need to be continuous, even discrete data points can be read from an external file. LSODE requires the user to convert the system of ODEs into first order form to obtain an array of first order ODEs.

$$
\frac{dy}{dt} = f(t, y)
$$

(13)

where $y$ is a vector containing the set of ODEs and $f$ is a vector-valued function of $t$ and $y$. 

4
Subsequently, it can be written that

\[
\begin{align*}
\begin{bmatrix}
y_1(t) \\
y_2(t) \\
y_3(t) \\
y_4(t)
\end{bmatrix} &= 
\begin{bmatrix}
u(t) \\
\dot{u}(t) \\
z(t) \\
\varepsilon(t)
\end{bmatrix}
\end{align*}
\] (14)

The hysteresis model equations (10), (11) and (12) can be rewritten based on Equation (14) as follows:

\[
\begin{align*}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= -2\xi_0\omega_0 y_2 - \alpha \omega_0^2 y_1 - (1 - \alpha) \omega_0^2 y_3 + f(t) \\
\dot{y}_3 &= \left\{1 - \xi_0 \left(1 - e^{-\psi y_3}\right) e^{-(y_3,sgn(y_2) - q(1/(1+\delta y_3)\beta + \gamma))^{1/n}}/(\psi + \delta y_3)^2(\lambda + \xi_0(1-e^{-\psi y_3}))^2\right\} \\
&\times \left\{y_3 - (1+\delta y_3)(\beta y_3 + \gamma) + \gamma y_3 y_3^{n-1} + \delta y_3 y_3^n\right\}^{1+\delta y_3} \\
\dot{y}_4 &= (1 - \alpha) \omega_0^2 y_2 y_3
\end{align*}
\] (15) (16) (17) (18)

Once the displacement function and data acquisition frequency is known and the parameters are estimated using a suitable system identification technique, LSODE solver is applied to solve these equations.

4 PARAMETER ESTIMATION

The structure of Genetic Algorithm (GA), a parameter estimation tool, is characterized by four nested loops.

The innermost loop (Loop 4) is the actual GA that generates a population, checks the solver calculation (LSODE) and selects and mates pairs to crossover and mutate. Solver checking is necessary because parameters are generated at random. To prevent the GA from falsely recognizing the erroneous sums of squares as better fit, the solver computation is checked after each run. If the error flag is detected, the particular parameter set that tripped the solver is assigned the highest sums of squares of the population, which is associated with the worst fitness encouraging the GA to quickly drop the parameter set. Loop 3 executes the GA a user-specified number of times, each time with a different randomly chosen initial population. Loop 2 progressively decreases or shrinks the parameter interval. The GA is an adaptive algorithm in the sense that it is able to discover erroneous initial input ranges for the parameters. If wrong interval is specified and the optimal parameter lies outside the interval, results tend to be clustered near the one side of the interval that should be readjusted. The GA subsequently shifts the interval in the direction of the clustering and starts over, which is the task of Loop 1.

One of the great benefits of using GA is the interval selection for each parameter does not affect the end result, but can make a significant difference in CPU time needed to reach the ultimate solution. Though GA takes longer time to converge than calculus-based techniques, but a trend is recognizable relatively fast and the user can quickly gain insight into the problem at hand.

5 CALIBRATION OF ANALYTICAL MODEL WITH EXPERIMENTAL RESULTS

To check the accuracy of selection of the pinching function and the identified parameter values from Genetic Algorithm, the hysteresis model after parameter identification has been compared and calibrated with the experimental results of internal and external non-ductile RC beam-column joints obtained from literature. The experimental and analytical shear force-horizontal deflection plots of selected non-ductile reinforced concrete internal beam-column joint specimens Units O4 and O5 tested by Hakuto et al. (2000), Units NS1 and NS2 by Li et al. (2010) and Units 1 and 2 tested by Liu et al. (2002) are shown in Figure 4 and the estimated parameters are tabulated in Table 1. The experimental and analytical shear force-horizontal deflection plots of selected non-ductile reinforced concrete external beam-column joint specimens Units O6 and O7 tested by Hakuto et al. (2000), Units EJ2 and EJ3 tested by Liu et al. (2002) and Units JO1 and JO2 by Bedirhanoglu et al. (2010) are presented in Figure 5 and the estimated parameters are summarized in Table 2.
Figure 4: Experimental and Analytical Shear Force versus Horizontal Deflection Plots of Non-ductile Reinforced Concrete Internal Beam-Column Joints
Figure 5: Experimental and Analytical Shear Force versus Horizontal Deflection Plots of Non-ductile Reinforced Concrete External Beam-Column Joints

a) Specimen O6 (Hakuto, 2000)
b) Specimen O7 (Hakuto, 2000)
c) Specimen EJ2 (Liu, 2002)
d) Specimen EJ3 (Liu, 2002)
e) Specimen JO1 (Bedirhanoglu, 2010)
f) Specimen JO2 (Bedirhanoglu, 2010)
Table 1: Estimated Parameters for the Non-ductile RC Internal Beam-Column Joint Specimens

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$p$</th>
<th>$q$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\omega$</th>
<th>$\zeta_s$</th>
<th>$n$</th>
<th>$\psi$</th>
<th>$\delta_\psi$</th>
<th>$\delta_v$</th>
<th>$\xi_0$</th>
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<td>O4</td>
<td>1.35</td>
<td>0.2</td>
<td>0.05</td>
<td>0.011</td>
<td>-0.01</td>
<td>2.25</td>
<td>0.97</td>
<td>1.093</td>
<td>0.37</td>
<td>0.036</td>
<td>0.001</td>
<td>0.0005</td>
<td>0.0003</td>
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<tr>
<td>O5</td>
<td>1.32</td>
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<td>0.032</td>
<td>0.011</td>
<td>-0.01</td>
<td>2.15</td>
<td>0.97</td>
<td>1.085</td>
<td>0.37</td>
<td>0.036</td>
<td>0.001</td>
<td>0.0005</td>
<td>0.0003</td>
<td>0.005</td>
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<tr>
<td>U1</td>
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<td>0.2</td>
<td>0.035</td>
<td>0.011</td>
<td>-0.01</td>
<td>1.35</td>
<td>0.97</td>
<td>1.039</td>
<td>0.37</td>
<td>0.036</td>
<td>0.001</td>
<td>0.0005</td>
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<tr>
<td>U2</td>
<td>1.32</td>
<td>0.2</td>
<td>0.03</td>
<td>0.011</td>
<td>-0.01</td>
<td>1.58</td>
<td>0.97</td>
<td>1.028</td>
<td>0.37</td>
<td>0.036</td>
<td>0.001</td>
<td>0.0005</td>
<td>0.0003</td>
<td>0.005</td>
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<tr>
<td>NS1</td>
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<td>0.2</td>
<td>0.032</td>
<td>0.011</td>
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<td>0.97</td>
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<td>0.37</td>
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<td>0.0005</td>
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<td>0.005</td>
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<tr>
<td>NS2</td>
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<td>0.011</td>
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<td>0.97</td>
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<td>0.036</td>
<td>0.001</td>
<td>0.0005</td>
<td>0.0003</td>
<td>0.005</td>
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Table 2: Estimated Parameters for the Non-ductile RC External Beam-Column Joint Specimens

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<th>Parameters</th>
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<th>$q$</th>
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<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\omega$</th>
<th>$\zeta_s$</th>
<th>$n$</th>
<th>$\psi$</th>
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<td>1.082</td>
<td>0.32</td>
<td>0.036</td>
<td>0.001</td>
<td>0.0005</td>
<td>0.0003</td>
<td>0.005</td>
</tr>
<tr>
<td>EJ2</td>
<td>1.11</td>
<td>0.2</td>
<td>0.037</td>
<td>0.011</td>
<td>-0.01</td>
<td>0.95</td>
<td>0.97</td>
<td>1.055</td>
<td>0.32</td>
<td>0.036</td>
<td>0.001</td>
<td>0.0005</td>
<td>0.0003</td>
<td>0.005</td>
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<tr>
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<td>0.2</td>
<td>0.033</td>
<td>0.011</td>
<td>-0.01</td>
<td>0.98</td>
<td>0.97</td>
<td>1.035</td>
<td>0.35</td>
<td>0.036</td>
<td>0.001</td>
<td>0.0005</td>
<td>0.0003</td>
<td>0.005</td>
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<tr>
<td>JO1</td>
<td>1.18</td>
<td>0.2</td>
<td>0.04</td>
<td>0.011</td>
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<td>1.25</td>
<td>0.97</td>
<td>1.055</td>
<td>0.33</td>
<td>0.036</td>
<td>0.001</td>
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<tr>
<td>JO2</td>
<td>1.28</td>
<td>0.2</td>
<td>0.039</td>
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</table>

6 CONCLUSION

Theoretical models attempt to simulate and predict the actual mechanisms. As it is not often practical to test every aspect, people need to rely on suitable analytical models. For establishing the accuracy and applicability of the analytical model, experimental and analytical load-deflection plots for non-ductile reinforced concrete beam-column joint specimens have been compared. Good correlations between experimental and analytical hysteresis loops of the beam-column joints prove the effectiveness of modified BWBN model and accuracy of the solver and GA. Further investigation will be carried out on implementation of the hysteresis model in commercial software and establishment of relationship between model parameters and physical parameters of the beam-column joints.

7 REFERENCES


