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Modelling of In-Structure Damping: A Review of the State-of-the-art

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ABSTRACT: System parameters in the mathematical model of a vibrating structure include mass, damping and stiffness; out of which mass and stiffness could defined as a function of the system geometry, whereas damping involves more an observed phenomenon. A consolidated review of the existing current damping models is presented. Earlier studies illustrate considerable issues associated with the use of the conventional Rayleigh model of damping. In this paper conceptual evaluations of the existing formulations are presented and the issues associated with the use or misuses of the models are documented. Despite the inherent complex uncertainties involved, scope for a more in-depth research is identified. A study on the dynamic response of a cantilever beam to a constant force is presented to illustrate the effect of using viscous and non-viscous damping models. Different damping models exhibit different peak responses highlighting the fact that viscous damping might not represent the realistic state of the structure during a dynamic excitation.

1 INTRODUCTION

By nature the vibration response is always associated with the phenomenon of damping; it could be described as nature's gift without which a structure undergoing a disturbance would tend to remain in that state, interchanging energy periodically between kinetic and potential forms, resulting in the whole system remaining in a state of chaos. Damping, in simplistic terms, could be defined as the process by which a certain portion of the energy in a vibrating system is irreversibly lost causing a decaying trend in the system response.

Modelling of a natural phenomenon normally involves representing the process in the form of a mathematical model, with certain system specific parameters controlling the evolution of the process. Mass, damping and stiffness are the system parameters in the mathematical model of a vibrating structure; in which mass and stiffness can be derived as a function of the system geometry and material characteristics, whereas damping involves representing an observed phenomenon. Despite having a large literature on the subject, the underlying physics is only known in a phenomenological ad-hoc manner, making damping an overall mystery in the general dynamic analysis of structures. A major reason of this could be the fact that there is no single universally accepted model for damping (Woodhouse 1998). The ambiguity involved in the modelling of damping is mainly due to the intricacies involved in understanding the *state variables* controlling the damping forces (Adhikari 2000).

Conceptually the choice of a mathematical model depicting damping should also be a function of the type of analysis; i.e. elastic or inelastic. Common practice is to use the classical viscous damping model originated by Rayleigh, through his famous 'Rayleigh dissipation function', in which instantaneous velocities are only considered as the relevant state variables and on employing Taylor's expansion results in a model which captures the damping through the formation of a 'dissipation matrix' analogous to mass and stiffness matrices (Adhikari 2000). The most common practice existing in the industry is to use this model with a preconceived damping ratio, irrespective of the type of analysis involved.

The main objective of this paper is to present the currently available models of damping and the associated theories in a consolidated manner; wherever possible the effect of different models on the response of the system is highlighted. A brief conceptual review on issues arising due to the use of

Rayleigh damping in inelastic seismic analysis is presented (Numerical simulation reviews on the most general form of linear damping is recorded). These simulations interestingly highlight the fact that there may be differences in the peak responses obtained by different damping model leading to a certain level of uncertainty, which could prove detrimental emphasising the significance of the use of the correct damping model even in elastic response analysis.

2 BACKGROUND

2.1 Types of Damping

In this section a brief theoretical overview of the types of damping phenomena are presented. The availability of numerous mechanisms capable of energy dissipation has resulted in a host of adjectives illustrating the phenomena of damping; (e.g. Columb, sliding, friction, structural, viscous, hysteretic etc.) But if the focus is zeroed down on in-structure damping, then it could be broadly described to be falling into three main categories (Desilva, 2007) :

- *Material Damping/ Internal Damping*

  This arises mainly due to energy dissipations caused by micro-structural interactions resulting in defects such as grain boundaries; local thermal effects due to temperature gradients; dislocations in the grain lamina etc. There is a large variety of mathematical models representing this energy dissipation phenomenon due to the availability of a diversity of materials. With all the uncertainties involved, still two general types of mathematical models could be identified: visco-elastic and hysteretic model. For small strains, the most general model representing material damping in the linear scenario is the visco-elastic model, considering damping through the viscoelastic correspondence principle (Woodhouse 1998). The visco-elastic model is frequency dependant, unlike the hysteretic model.

- *Boundary damping/Structural Damping*

  This is mainly due to the relative motion of components with a mechanical structure. From a modelling point of view this is more difficult to simulate as the mathematical models become more abstract in nature due its system dependance. The Columb friction model is a representative mathematical model for boundary damping. One of the common types of boundary damping is friction in steel joints, element slippage etc (Chopra 1995). In a fully built up system the boundary damping is found to be at least an order of magnitude higher than the material damping (Woodhouse 1998).

- *Fluid viscous Damping*

  This arises from energy dissipation associated with drag forces and is mainly concerned with motion of mechanical system in fluids. This form of damping mainly happens in for e.g. vehicle suspensions with “shock absorbers” which really epitomise the classical viscous dashpots.

2.2 COMMENTS ON THE CURRENT REPRESENTATION OF DAMPING IN GENERAL DYNAMIC ANALYSIS

As has been already stated, the current practice of incorporating damping in dynamic analysis is through Rayleigh’s “dissipation matrix” which depends on the instantaneous velocity as the state variable controlling the damping force. Now the most interesting question to be asked is whether this model of “dissipation matrix” is capable of capturing all the above described types of damping phenomenon? To answer this question, a conceptual review on the mathematical side of Rayleigh damping is presented.

In strict mathematical terms, Rayleigh's matrix is actually representative of those systems which is mainly driven by fluid damping due to its inherent dependence on the instantaneous velocity. In nature, there are no clear evidences to suggest why other effects like material damping and boundary damping would have to depend only on instantaneous velocity (Adhikari 2000). The use of a classical viscous model tends to represent the effect in a macro scale. The interesting fact is that while inertia and stiffness force is represented at an element level; the damping force is represented at a system level. This leads to a conflict in the scales and as a result, the present dynamic model equipped with classical viscous damping tend to represent the damping phenomena in a phenomenological manner. The other interesting fact is that the current justification of the use of the classical viscous damping is under the assumption of "small damping". In a later section in the paper, this assumption is also questioned by numerically emphasising the fact that, even if the damping is small it could have critical effects if not represented properly.

3 MODELS OF DAMPING

For mathematical convenience the models of damping could be broadly classified as

- Damping in continuous system
- Damping in Discrete systems

3.1 Damping in Continuous Systems

Four mathematical models have been found in literature representing damping in continuous system (Banks &Inman 1991). These damping models are described with Euler-Bernoulli beam continuum represented by

$$u_{tt}(x,t) + L_1 u_t(x,t) + L_2 u(x,t) + \frac{\partial^2}{\partial x^2}\left[\frac{EI(x)}{\rho} u_{xx}(x,t)\right] = f(x,t) \quad (1)$$

The term $L_1 u_t(x,t)$ is determined by an external damping mechanism where as the $L_2 u(x,t)$ is determined by an internal damping mechanism. The other terms follow the standard interpretations, with '*f(x,t)*' representing a time and space variant force, 'u' representing deformations and '*E*', '*I*' and ρ representing the material and geometric properties .

- *Viscous Air Damping/ Viscous External damping*

  This is often represented by

$$L_1 = \gamma I_0 \quad (2)$$

  Where γ is the viscous damping constant of proportionality and I_0 is the identity operator.

- *Kelvin-Voigt Damping*

  This damping is of the form

$$L_2 = c_d I \frac{\partial^5}{\partial x^4 \partial t} \quad (3)$$

and

$$L_1 = c_d I \frac{\partial^4}{\partial x^4} \quad (4)$$

- *Time Hysteresis Damping*

  In time hysteresis model, inclusion of hysteretic damping is achieved by

$$L_1 = \int_{-\infty}^{t} g(\tau) u_{xx}(x, t+\tau) d\tau \quad (5)$$

  where $g(\tau)$ is a dissipation kernel function.

- *Spatial Hysteresis Damping*

  The another type is the spatial hysteresis model in which,

$$L_1 = \frac{\partial}{\partial x}\left[\int_0^t h(x,\gamma)[u_{xx}(x,t) - u(\gamma,t)]d\gamma\right] \quad (6)$$

  where $h(x,\gamma)$ is a dissipation kernel function.

  As the damping in continuum elements goes beyond the scope of this paper no further discussions on these models are presented; interested readers should refer to Banks and Inman (1991).

3.2 Damping in Discrete Systems

In normal situations the dynamic analysis is always carried out with discrete systems. Broadly the damping in discrete systems could be classified as

3.1.1 Viscous Damping

Viscous damping is mainly achieved by the incorporation of Rayleigh's Dissipation function given as

$$F = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\dot{q}_i\dot{q}_j = \frac{1}{2}\dot{\boldsymbol{q}}_{\boldsymbol{j}}\boldsymbol{C}\dot{\boldsymbol{q}}' \quad (7)$$

where '**C'** represents a non-negative definite symmetric matrix. Rayleigh further showed that one way of forming the **'C'** matrix is by a linear combination of the Mass and Stiffness, which is given as,

$$\boldsymbol{C} = \alpha\boldsymbol{M} + \beta\boldsymbol{K} \quad (8)$$

where α and β are calculated as function of frequency using a preconceived damping ratio.

This model is commonly used to model damping in MDOF systems (Multi-Degree of Freedom Systems) and its popularity is mainly due to the fact that it uses the already computed mass and stiffness matrices and demands only the calculation of the constants α and β (Carr 2007). The main advantage of this theory is that the proportionality of the modes is preserved; thereby facilitating the classical modal analysis to be performed more or less similar to the un-damped vibration. But there is no explicit justification for the preservation of this proportionality phenomenon; and in reality the test results indicate complex nature of the eigen modes. This imply the non-proportionality aspect of the mode shapes and indicates the presence of non-classical damping (Adhikari 2000).

The main issue with this model is its strong dependence on the frequency of the structure as the constants are evaluated as a function of the frequency. There have been a large number of of studies looking into this frequency dependence observed in practice, and interestingly the majority of these studies emphasise that material level damping is a strong function of x^n and a weak function of w, where 'x' refers to displacement and w refers to angular frequency (Adhikari 2000, Bandstra 1983, Baburaj etal.1994). Nevertheless, to date this model is still the most popular damping model employed in dynamic analysis.

The other variations of the Rayleigh damping could be obtained by putting either of the constants zero; α=0 results in stiffness proportional damping and β=0 results in mass proportional damping. There are other viscous models available in the literature like Caughey damping and Wilson-Penzien damping, which are all again derived from the Rayleigh dissipation matrix and hence exhibit strong frequency dependence. Caughey damping is normally used to specify damping ratios in more than two modes and is called in literature as the general classical damping matrix (Chopra 1995). Due to its convoluted expression involving frequencies, Wilson and Penzein derived an easier way to formulate the matrix; but both these models are expensive from a computational point of view as both of them derives a fully populated ***'C'*** matrix (Carr 2007). The next section briefly reviews the use of this model in seismic analysis.

3.1.1.1 Effect of the use of Rayleigh Damping in Seismic Analysis

The interesting point in using the Rayleigh damping especially in the case of inelastic seismic analysis is to decide whether to use the initial stiffness damping or tangent stiffness damping. If initial stiffness is used in the damping matrix formation, unrealistic damping forces are obtained in joints undergoing sudden change in stiffness (Brenal 1994, Hall 2006). This unrealistic force results in the underestimation of displacements and overestimation of internal member forces (Carr 2007, Zareian et al. 2010). Several modelling approaches to overcome the limitations of Rayleigh damping based on initial stiffness have been proposed. Interested readers should refer to Brenal (1994), Leger et al. (1992), Hall(2005), Charney (2008) and Zareian et al (2010). These studies actually hold up the fact that a caution should be enforced while using Rayleigh's viscous damping in seismic analysis. Val et al. (2005) highlighted the fact that even in elastic analysis of structure subject to earthquake ground motions, a use of viscous damping model may not accurately capture the peak responses.

3.1.2 Nonviscous Damping

The quest to realistically model the damping phenomena was mainly triggered by the above observed discrepancies with Rayleigh model. These led to the development of non-viscous models. Models in which the damping force is a function of past history of motion via convolution integrals over a suitable kernel function constitutes non-viscous damping. They are called non-viscous because the force depends on state variables other than just the instantaneous velocity (Adhikari et al 2003). The most generic form of linear damping given in the form of modified dissipation function is as follows (Woodhouse 1998)

$$F = \frac{1}{2}\dot{q}' \int_0^t g(t-\tau)\,\dot{q}(\tau)d\tau \tag{9}$$

where $g(t)$ represents the kernel function. The Rayleigh's dissipation function can be derived as a special case of this modified dissipation function when $g(t-\tau) = C\delta(t-\tau)$ where $\delta(t)$ is the Dirac's delta function. This could also be looked as a time hysteresis model applied to discrete systems. The generality of this model is evident in the aspect that the kernel function $g(t)$ could adopt any causal model where the energy functional is non-negative (Adhikari et al. 2003).

3.1.3 Frequency Independent Damping Models

As has been already stated in section 3.1.1, damping in most materials exhibit weak frequency dependence phenomenon. The concept of frequency independent damping arose when in 1927 Kimball and Lovell claimed that hysteretic damping is universal in nature. Since then there has been several studies which further strengthened their claim. One of the most popular models in this category is the linear Columb friction force model given as (Reid, 1956, Muravskii 2004)

$$F = k\left[x + \eta|x|\frac{\dot{x}}{|\dot{x}|}\right] \tag{10}$$

This model could be in general a better representative of boundary/structural damping occurring at structural joints. As material damping is negligible in comparison with boundary damping, it could well be assumed that the use of this model in the dynamic analysis would give a better representation of the overall damping phenomenon. The hysteresis loop presented by such a model for a periodic amplitude of x_0 is given in Figure 1.0

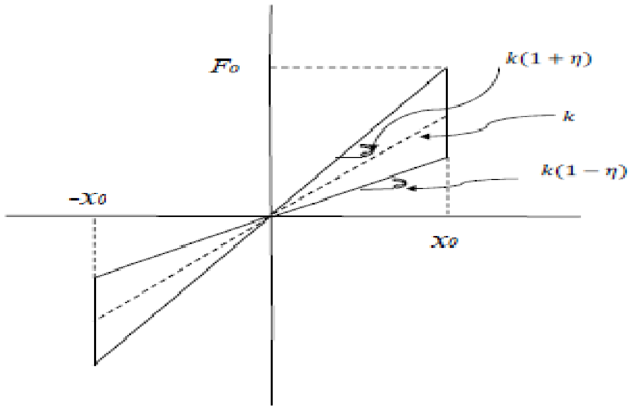

Figure 1.0 Relation between the force and displacement in the hysteretic model (adopted from Muravskii 2004).

The main issue with this model is that the force becomes discontinuous when the velocity changes the sign. Several modifications have been suggested in the literature to overcome this difficulty. One of the modified version of this model used in non-linear seismic analysis is called the quasi hysteretic model given as (Muravskii 2004)

$$F = k(x_m)\left[x + \eta(x_m)\dot{x}\frac{x_m}{\dot{x_m}}\right] \tag{11}$$

In equation 11, the mean values of displacement (x_m) and velocities($\dot{x_m}$) are used. The other way suggested to alleviate the force discontinuity is to use a different stiffness, when the velocity changes the sign. The hysteresis loop of such model would be as shown in Figure 2. For further details on this interested readers should refer to Muravskii (2004), Muravskii (1994) and Muravskii et al.(1998).

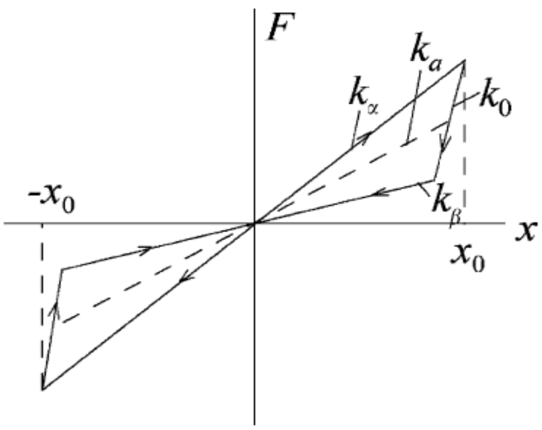

Figure 2.0 Relation between the force and displacement in the hysteretic model (adopted from Muravskii 2004).

4 COMPARITIVE STUDIES ON VISCOUS vs. NON-VISCOUS DAMPING MODELS

In this section a numerical study is initiated to understand the effect of viscous and non-viscous models on the overall response of the system. A cantilever steel beam with 100mm square cross section and length 8m is subjected to a constant force of 100N. Throughout the simulation the applied force is constant so that eventually the steady state would coincide with static deflection. Simulations are performed using both classical stiffness proportional damping (α=0 in Eq. 8) and non-viscous damping (Eq. 9). A single exponential model called Biot's relaxation function is used as the Kernel function. The Biot's relaxation function is of the form

$$g(t) = \mu e^{(-\mu t)} \tag{12}$$

Where μ is a dissipation constant. A very low value of μ indicates strong non-viscous characteristics and a high value of μ indicates close to viscous characteristics (Adhikari 2000). Figure 3 illustrate the time domain representation of the displacement response. Simulations are carried out for μ=2 and μ=50.

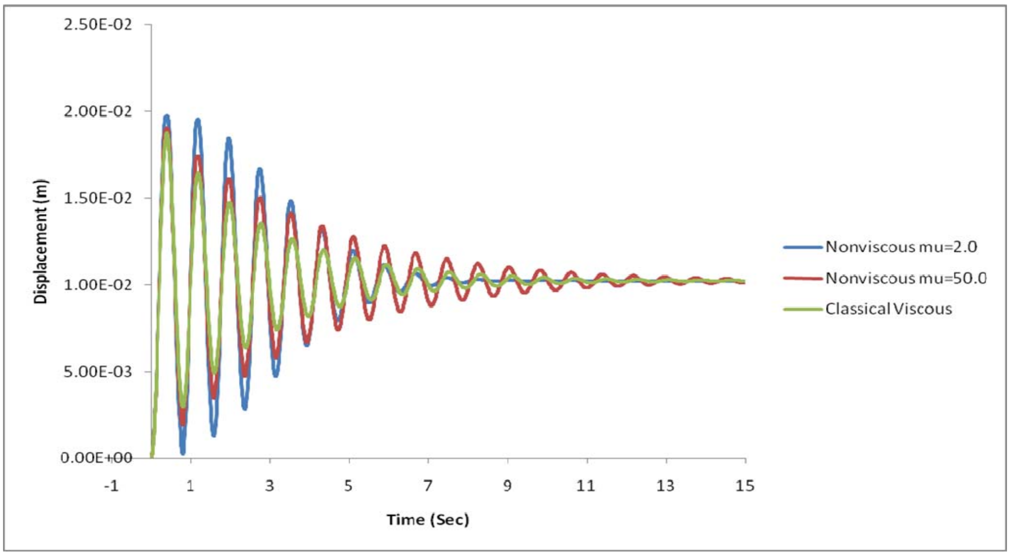

Figure 3.0 Time Domain response of displacement

It is observed that when very high non-viscous characteristics is present (i.e. μ=2) the peak response is higher compared to the classical viscous case; but the rate of decay is faster. On the other hand if a high value of μ is used the peak response is less than the one with lower μ; but it is still higher than the classical viscous damping. Now the interesting question is which of these μ values would reflect reality? At this point of time unfortunately this question remains unanswered and demands further research. Also it would be interesting to see the effect of these damping models in nonlinear analysis. In this specific simple case, on comparison with the viscous damping, there is no dramatic increase in the peak response; but this cannot be generalised and this study flags the concern that the responses calculated by viscous damping could be underestimated. There could be instances where the differences in the peak response could be higher and what is depicted as a pre-yield state by viscous damping might not be a realistic pre-yield state, because if the peak response is under estimated, the inelastic response will also be erroneously represented in the analysis. This could prove detrimental and could be quite critical in a seismic analysis scenario. Similar observations were presented by Val et al. (2005) using a elasto-slip model in which it is shown that there could be instances where the viscous damping assumption would underestimate the peak response. The other interesting fact is that, no conspicuous effect on frequency of excitation is exhibited by non-viscous damping.

5 CONCLUSIONS

A state of the art review on damping models is presented. The issues associated with the use of classical Rayleigh model of damping are highlighted in some detail. A comparative analytical study on a cantilever beam is presented to highlight the effect of damping models on the response. It is seen that different models give different responses epitomising the significance of the choice of damping models in the analysis. It is shown that non-viscous damping can produce larger peak response as compared to the viscous damping. This could mean that in a realistic situation the structure might undergo inelastic excursions; but an analytical model equipped with viscous damping might not capture this. A detailed further research is required in this particular area. Also the performance of the non-viscous models in inelastic scenario warrants further research.

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