Proceedings of the Ninth Pacific Conference on Earthquake Engineering
Building an Earthquake-Resilient Society
14-16 April, 2011, Auckland, New Zealand

Damage Assessment of Seismically-Excited Buildings through Incomplete Measurements

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ABSTRACT: A real structure possesses large number of degrees of freedom. It is impossible and impractical to have measurements at all degrees of freedom. This study presents a damage assessment technique for seismically-excited buildings from few floor response measurements. The system realization using information matrix (SRIM) identification technique was firstly employed to estimate the modal properties such as frequencies and damping ratios of the instrumented building. However, the complete mode shapes can not be obtained due to incomplete measurements. A mode shape recovery technique was developed to reconstruct the complete first mode shape of the building system. An optimization process was performed to minimize a prescribed objective function representing the difference between the measured and estimated outputs at the instrumented locations. A story damage index (SDI) computed through the recovered first mode shape was applied to express the degree of story damage. Floor measurements with noise contamination of a five-story shear building under earthquake excitation were conducted for numerical verification. Moreover, a three-story benchmark building was considered to examine the accuracy and applicability of the proposed damage assessment technique via experimental data. It is shown that the proposed method obtained results in fairly good agreement with those of full measurements and is of value in practical application.

1 INTRODUCTION

In recent years, with the advanced systems of data acquisition and signal processing, there has been an increasing interest in the development of structural health monitoring (SHM) in methodologies that are capable of detecting and quantifying structural damage in areas of a structure, at which some non-destructive tests (NDT) for detail damage evaluation are implemented. Calculating the change of modal frequency to detect damage is wildly used in structural health monitoring systems because damage is always accompanied by a reduction of stiffness as well as modal frequency. Damage in different locations and components actually leads to different frequency changes in various modes. Nevertheless, it remains difficult to determine the damage location just by observing the changes of modal frequencies.

Among the modal parameters of a structure system, the mode shape is obviously the only location related parameter. Therefore, many researchers have attempted to establish mode-shape-based indices, such as modal curvature index (Pandey et al. 1991; Farrar and Jauregui 1998), index MAC (Modal Assurance Criterion) (Allemang and Brown 1982) and index COMAC (Coordinate Modal Assurance Criterion) (Iieven and Ewins 1988), to identify damage degree and location of a structure. All above indices have simple expressions and have been applied in identifying the location of damage. However, it has been shown that they have low sensitivity to damage in some cases (Ndambi et al. 2002; Brasiliano et al. 2004). Considering both changes of modal frequencies and mode shapes to detect the occurrence and location of structural damage is more reliable than just relying on either one of them. The modal flexibility damage index (MFDI) (Pandey and Biswas 1994) is the most well-known one. The principle of this method is based on the comparison of flexibility matrices obtained from two sets of mode shapes. Moreover, this method involves the normalization of mode shape since the mode shape values are not fixed. The advantage of the mode-shape-based technique is containing spatially related information, thus damage location is available. While this technique requires a large number of measurements at different locations to accurately characterize mode shapes.

Many researchers developed other damage indices for various types of structures. Brasiliano et al. (2004) evaluated the residual error method in the movement equation to verify its efficiency when applied to continuous beams and frame structures. Kim and Chun (2004) derived an index to apply to buildings. Kim et al. (2003) employed frequency-based and mode-shape-based damage detection methods for locating and quantifying damage in pre-stressed concrete beams. Bernal (2002) proposed a technique to localize damage in structures using damage locating vectors (DVLs) that have the property of inducing stress field whose magnitude is zero in the damaged elements. The DLVs are associated with sensor coordinates and are computed systematically as the null space of the change in measured flexibility. Wang et al. (2007) developed a story damage index (SDI) which is expressed as a simple formula based on modal frequency and mode shape extracted from real earthquake records. For all the aforementioned methods, the incomplete mode shape is the primary problem because limited number of sensors are available for a real instrumented structure.

In structural engineering, it is not practical to have measurements at all degrees of freedom. In most applications, an incomplete set of time history response records are available and this impairs the applicability of the SHM methods. In the case of an incomplete set of measurements, only limited information about the system parameters can be retrieved (Lus et al. 2003). In addition, dealing with a limited set of input-output measurements raises issues related to the uniqueness of the identified solution (Franco et al. 2006).

Based on the discussions mentioned above, studies on damage assessment technique for a structure with incomplete measurements are quite important. This paper is organized as follows. First, system dynamic and characteristic equations are derived. The story stiffness is expressed in terms of the floor mass, modal frequency and mode shape of a particular mode. Because of the complete mode shape can not be obtained through incomplete measurements, an optimal mode shape recovery technique is thus proposed to reconstruct the complete first mode shape of the system. The SDI (Wang et al. 2007) based on the complete first mode shape is then employed to express the degree of story damage. A five-story shear building under earthquake excitations was conducted for numerical verifications considering measurement noise. Moreover, shaking table test of a three-story benchmark building with different damage scenarios was conducted to examine the accuracy and applicability of the proposed damage assessment technique. It is shown that the proposed method obtained results in fairly good agreement with those from full measurements and is favourable for real implementation.

2 THEORETICAL DERIVATION

2.1 Dynamic and characteristic equations

Considering an N story shear building with mass m_l at the lth floor and with stiffness k_l and damping c_l at the lth story, the dynamic equations of motion due to ground acceleration $\ddot{u}_g(t)$ can be expressed as

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=-\mathbf{M}\mathbf{r}\ddot{u}_g(t) \tag{1}$$

where **M**, **C** and **K** are the $N\times N$ mass, damping and stiffness matrices. $\mathbf{x}(t)$ represents the $N\times 1$ vector of the floor displacements relative to the ground at a time t; and **r** indicates the $N\times 1$ influence vector. With other words, neglecting the damping and assuming that ω_j and $\boldsymbol{\varphi}_j$ are the jth modal frequency and the $N\times 1$ non-zero eigenvector of the system, the characteristic equation can be written as

$$(\mathbf{K}-\omega_j^2\mathbf{M})\boldsymbol{\varphi}_j=\mathbf{0} \tag{2}$$

2.2 Story stiffness expressed by the modal parameters

By solving Equation (2) from the last to the first rows, a general expression for the lth story stiffness can be obtained as

$$k_l = \omega_j^2 \sum_{n=l}^{N} \frac{m_n \phi_{nj}}{\Delta\phi_{lj}}\ ;\ j = 1, 2, \ldots, \text{or } N \tag{3}$$

where

$$\Delta\phi_{lj} = \begin{cases} \phi_{lj} - \phi_{(l-1)j} & \text{for } l = 2 \sim N \\ \phi_{lj} & \text{for } l = 1 \end{cases} \tag{4}$$

To interpret Equation (3) from a physical point of view, one can imagine a building vibrating in its *j*th mode. In this situation, the vibration frequency of the building is just the *j*th modal frequency, ω_j, and the displacement at the *l*th floor is the *l*th row of the *j*th mode shape, ϕ_{lj}. Therefore, the *l*th floor acceleration equals to $\omega_j^2 \phi_{lj}$. Also, the inertial force of the *l*th floor will be its mass multiplied by the acceleration, $m_l \omega_j^2 \phi_{lj}$. Since the resultant force applying to the *l*th story is the summation of the inertial force above the story, $\sum_{n=l}^{N} \omega_j^2 m_n \phi_{nj}$, and the *l*th story drift is equal to the relative displacement between the *l*th and the (*l*-1)th floors, $\phi_{lj} - \phi_{(l-1)j}$, then it is evident that the stiffness of the *l*th story can be obtained by dividing the resultant force by the story drift and Equation (3) can be solved easily.

3 DAMAGE ASSESSMENT THROUGH INCOMPLETE MEASUREMENTS

This section introduces the procedure of damage assessment through incomplete measurements. In the first step, the SRIM system identification technique is applied to estimate the dynamic properties of the building. If the sensors are installed in appropriate locations, modal frequencies and damping ratios can be obtained by the SRIM technique. However, mode shapes can not be obtained completely due to incomplete measurements because the mode shape is a location related parameter. The SRIM technique can only give the sensing location mode shape values. Therefore, an optimal mode shape recovery technique is proposed to recover the complete first mode shape of the building system. Finally, damage assessment is conducted by using a story damage index. Figure 1 shows the flowchart of the procedures introduced above.

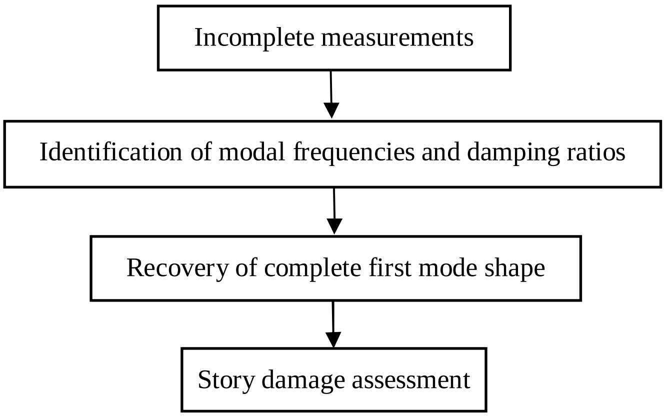


Figure 1 Flowchart of the proposed identification procedures

3.1 Optimal mode shape recovery technique

An optimization process is proposed to determine the complete first mode shape of the system. From Equation (3), with the prior knowledge of the floor masses, the story stiffness can be determined from the first modal frequency ω_1 and the first mode shape $\boldsymbol{\varphi}_1$ as follows:

$$\hat{k}_l = \omega_1^2 \sum_{n=l}^{N} \frac{m_n \hat{\phi}_{n1}}{\Delta \hat{\phi}_{l1}} \tag{5}$$

In Equation (5), the symbol "ˆ" denotes "estimated". In addition, ω_1 can be obtained by the SRIM technique. However, $\hat{\boldsymbol{\varphi}}_1$ is composed of two parts:

$$\hat{\boldsymbol{\varphi}}_1 = \begin{bmatrix} \boldsymbol{\varphi}_{kn,1} \\ \hat{\boldsymbol{\varphi}}_{un,1} \end{bmatrix} \tag{6}$$

where $\boldsymbol{\varphi}_{kn,1}$ and $\hat{\boldsymbol{\varphi}}_{un,1}$ are the known (identified) and unknown parts (to be estimated) of the first mode shape, respectively. After k_l is determined by Equation (5), the stiffness matrix **K** can be formed.

In order to determine the unknown part of the first mode shape, an optimization process that minimizes the objective function $J(\hat{\boldsymbol{\varphi}}_{un,1})$ representing the root-mean-square (rms) error between the measured and estimated outputs at the instrumented locations is performed. The objective function is expressed in a mathematical form as

$$J(\hat{\boldsymbol{\varphi}}_{un,1}) = \sqrt{\sum_{t=t_i}^{t_f} \left[\hat{\mathbf{y}}(\hat{\boldsymbol{\varphi}}_{un,1}, t) - \mathbf{y}(t)\right]^T \left[\hat{\mathbf{y}}(\hat{\boldsymbol{\varphi}}_{un,1}, t) - \mathbf{y}(t)\right]} \tag{7}$$

where $\hat{\mathbf{y}}(\hat{\boldsymbol{\varphi}}_{un,1}, t)$ and $\mathbf{y}(t)$ represent the estimated and the measured outputs at the time step t, varying from an initial time t_i to a final time t_f. For estimating $\hat{\mathbf{y}}(\hat{\boldsymbol{\varphi}}_{un,1}, t)$, stiffness matrix $\hat{\mathbf{K}}$ is formed by story stiffness $\hat{k}_l$ in Equation (5), the Rayliegh damping is assumed and can be computed as

$$\hat{\mathbf{C}} = a_0 \mathbf{M} + a_1 \hat{\mathbf{K}} \ , \quad \text{where} \quad \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = 2 \begin{bmatrix} \frac{1}{\omega_i} & \omega_i \\ \frac{1}{\omega_j} & \omega_j \end{bmatrix} \begin{Bmatrix} \xi_i \\ \xi_j \end{Bmatrix} \tag{8}$$

where the constant coefficients a_0 and a_1 can be determined by any two arbitrary modal frequencies and damping ratios. With the knowledge of matrices **M**, $\hat{\mathbf{K}}$ and $\hat{\mathbf{C}}$, $\hat{\mathbf{y}}(\hat{\boldsymbol{\varphi}}_{un,1}, t)$ can be estimated by using a numerical method. The proposed optimal mode shape recovery technique has the following features. (1) Only the first mode shape needs to be recovered to form the stiffness matrix **K**. (2) Due to the regularity of the first mode shape, a systematic initial guessing method can be employed. (3) $\boldsymbol{\varphi}_{kn,1}$ is obtained directly by the SRIM technique. Thus, the numbers of unknown part $\hat{\boldsymbol{\varphi}}_{un,1}$ can be reduced.

3.2 **Definition of story damage index**

Define the damage of a building as the reduction in percentage of story stiffness before and after damage. Based on Equation (3), the damage index of the *l*th story, termed SDI$_l$, can be expressed as (Wang et al. 2007)

$$\mathrm{SDI}_l = 1 - \frac{k_l^*}{k_l} = 1 - \frac{\omega_j^{*2} \sum_{i=l}^{N} \frac{m_i \phi_{ij}^*}{\Delta \phi_{lj}^*}}{\omega_j^2 \sum_{i=l}^{N} \frac{m_i \phi_{ij}}{\Delta \phi_{lj}}} \tag{9}$$

where the asterisk (*) denotes the damage state. The value of SDI$_l$ ranges from 0 (no damage) to 1.0 (collapse), providing a convenient index to indicate the location and degree of story damage. As seen in Equation (9), the value of SDI$_l$ can be easily obtained based on only the 1st modal frequency and mode shape. This is highly beneficial in practice because the 1st modal parameters can be identified accurately through system identification technique even with noise-contaminated measurements.

4 VALIDATIONS VIA NUMERICAL SIMULATION

4.1 Simulation of a five-story shear building

To verify the accuracy of the proposed damage assessment technique, a five-story shear building is considered for numerical simulation. Only the base and the first floor accelerations are measured as seen in Figure 2. The 1999 Taiwan Chi-Chi earthquake record (TCU076, PGA=200 gal) is used as the base excitation. Table 1 lists the physical and modal parameters of the five-story building. Each floor has same weight while the story stiffness ratio decreases from 1.0 to 0.8 along the height.

Table 1 Physical and modal parameters of the five-story shear building

Physical Parameters						
Floor		1 F	2 F	3 F	4 F	5 F
Floor Mass (kg)		1.80E+05	1.80E+05	1.80E+05	1.80E+05	1.80E+05
Mass Ratio		1	1	1	1	1
Story Stiffness (N/m)		8.00E+08	7.60E+08	6.84E+08	5.81E+08	4.65E+08
Stiffness Ratio		1	0.95	0.9	0.85	0.8
Modal Parameters						
Mode		1	2	3	4	5
Frequency (Hz)		2.93	8.31	13.06	16.76	19.28
Damping Ratio (%)		2.00	3.38	5.00	6.20	7.10
Mode shapes	5F	3.86	-1.34	0.82	-0.55	0.24
	4F	3.49	-0.31	-0.73	1.17	-0.75
	3F	2.83	0.88	-0.88	-0.64	1.24
	2F	1.97	1.41	0.46	-0.57	-1.42
	1F	1.00	1.00	1.00	1.00	1.00

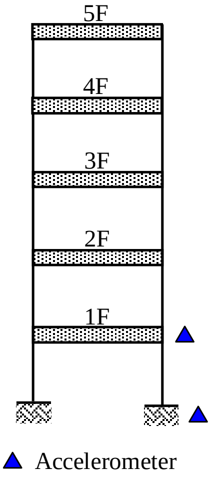
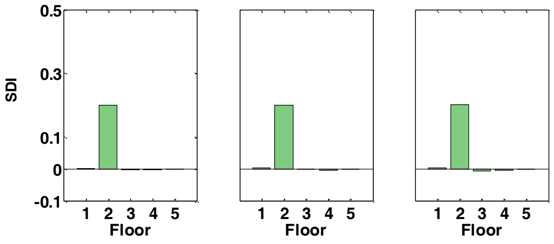


Figure 2 Five-story shear building used for numerical simulation

4.2 Effect of measurement noise

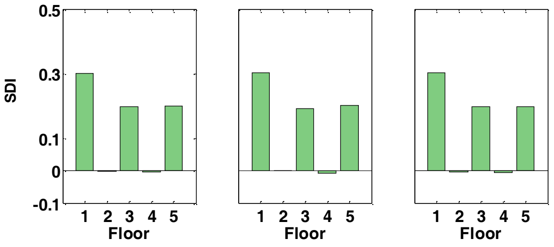
According to the proposed damage assessment procedure, using the base and first floor acceleration measurements as the input and output, respectively, the modal frequencies and damping ratios can be obtained precisely. However, mode shape values are lacking in 2F-5F. There are four unknowns for each mode. Hence, the optimal mode shape recovery technique is applied to obtain the complete first mode shape.

Two damaged cases with different noise levels (noise to signal ratio, *NSR*=5%, 10% and 20%) and damage degree in story stiffness are simulated as: case (I): single damage at 2F(0.2), case (II): multiple damage at 1F(0.3), 3F(0.2) and 5F(0.2). The number in the bracket represents the stiffness reduction ratio of the story. Figures 3-4 show the story damage index (SDI) of both cases with different *NSRs*. The values of SDI are determined by the estimated first mode shape that is evaluated by the optimal mode shape recovery technique. It is obviously seen that the SDI values agree exactly with the stiffness reduction ratios of the damaged stories.



(a) NSR(5%) (b) NSR(10%) (c) NSR(20%)

Figure 3 SDI versus NSR (case I)



(a) NSR(5%) (b) NSR(10%) (c) NSR(20%)

Figure 4 SDI versus NSR (case II)

5 VALIDATIONS VIA SHAKING TABLE TEST

5.1 Description of test frame and experiment

The experimental structure used in this study is a full-scale three-story steel frame mounted on the shaking table at the National Center for Research on Earthquake Engineering (NCREE) in Taiwan as shown in Figure 5. This structure is regarded as a benchmark building designed for the demonstration of research on structural control and health monitoring in Taiwan. It is a uniform building with total of 18 tons in weight and 9 m in height. The dimension of the rectangular floor is 3 m × 2 m in plan. The weight of each floor comes from the composite frame-plate structure and additional lead blocks atop. It is supported by four steel columns with H-shape (H150×150×7×10) section.

In order to verify the proposed damage assessment technique, two types of building systems are used to represent the "undamaged" and "damaged" structure as shown in Figure 6 and described as: (1) Undamaged: two symmetric bracings in the first story with loosed bolts at the bottom joints, (2) Damaged: bracings removed. Figure 6 also shows that only base and 1F acceleration responses are measured.

Figure 5 Photo of the shaking table test building in NCREE

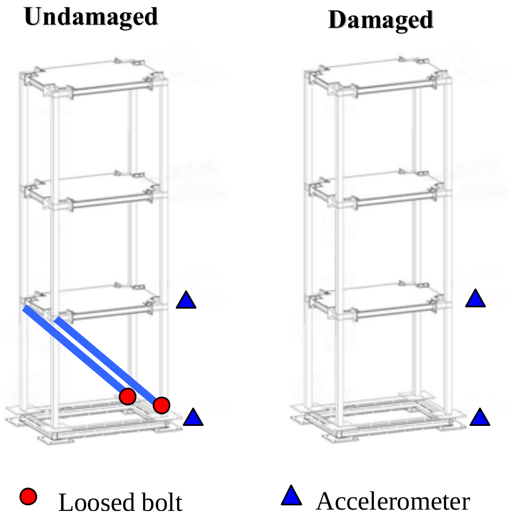


Figure 6 Simulated undamaged and damaged buildings

5.2 Comparison of incomplete measurements and full measurements

Tables 2-3 list the identified modal parameters of the undamaged and the damaged structure. The modal frequencies and damping ratios were identified by the SRIM technique and the complete first mode shape was estimated by the proposed mode shape recovery technique. For comparison, Tables 4-5 illustrate the modal parameters identified by full measurements for both undamaged and damaged cases. Because of full measurements, the SRIM technique can identify all of the modal parameters as shown in Table 4 and Table 5. It is seen from Tables 2-5 that (1) the modal frequencies and damping ratios determined from incomplete measurements are very close to those from full measurements. (2) The high MAC of the first mode shapes obtained from the proposed method and from the full measurements for undamaged case (MAC=0.9981) and damaged case (MAC=0.9984) indicates the accuracy of the proposed mode shape recovery technique.

Figure 7 shows the SDI values obtained from the proposed method (dark) and the full measurements (blank). The SDI of first story computed by the proposed method and full measurements are 0.23 and 0.19, respectively, in agreement with the true value. Only half number of sensors compared with the full measurements were used indicating the applicability of the proposed method. Slight discrepancies were found in the second and third story due to noisy measurements.

Table 2 Identified modal parameters (undamaged)

System modal parameters –undamaged				
Mode		1	2	3
Frequency (Hz)		1.13	3.36	5.17
Damping ratio (%)		2.53	1.14	0.74
Mode shape	3F*	2.78	-	-
	2F*	2.31	-	-
	1F	1.00	1.00	1.00

*recovered

Table 3 Identified modal parameters (damaged)

System modal parameters –damaged				
Mode		1	2	3
Frequency (Hz)		1.07	3.26	5.13
Damping ratio (%)		2.02	0.22	0.18
Mode shape	3F*	2.34	-	-
	2F*	2.00	-	-
	1F	1.00	1.00	1.00

*recovered

Table 4 Modal parameters identified by full measurement (undamaged)

System modal parameters –undamaged				
Mode		1	2	3
Frequency (Hz)		1.14	3.38	5.17
Damping ratio (%)		2.61	1.06	0.67
Mode shapes	3F	2.78	-0.83	0.52
	2F	2.11	0.57	-1.15
	1F	1.00	1.00	1.00

Table 5 Modal parameters identified by full measurement (damaged)

System modal parameters –damaged				
Mode		1	2	3
Frequency (Hz)		1.07	3.26	5.13
Damping ratio (%)		1.97	0.21	0.18
Mode shapes	3F	2.46	-0.84	0.54
	2F	1.93	0.50	-1.18
	1F	1.00	1.00	1.00

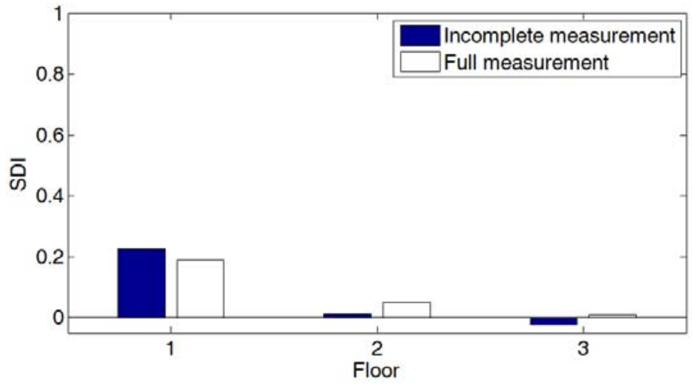


Figure 7 Comparison of SDI values from the proposed method and full measurement

6 CONCLUSIONS

This study presents a damage assessment technique for seismically-excited buildings from few floor response measurements. The SRIM identification technique was firstly employed to estimate the modal properties such as frequencies and damping ratios of the instrumented building. A mode shape recovery technique was developed to reconstruct the complete first mode shape of the building system. A story damage index (SDI) computed through the recovered first mode shape was applied to express the degree of story damage. A five-story shear building considering measurement noise was conducted for numerical simulations. Simulation results show that the SDI values agree exactly with the stiffness reduction ratios of the damaged stories. Moreover, a three-story benchmark building was considered to examine the accuracy and applicability of the proposed damage assessment technique via experimental data. It is shown that the proposed method obtained results in good agreement with those of full measurements. Only half number of sensors compared with the full measurements were used indicating the applicability of the proposed method.

ACKNOWLEDGEMENTS

This work was supported by the National Science Council of the Republic of China under Grant NSC 97-2625-M-005-004 and by the Central Weather Bureau under Grant MOTC-CWB-97-E-10. These supports are greatly appreciated.

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