Analytical Model for Shear Strengthening of RC Beam-Column Joints Using Composite Materials

U. Akguzel, S. Pampanin

University of Canterbury, Christchurch, New Zealand.

ABSTRACT: An analytical model, based on principal tensile stresses vs. joint shear deformation rules, for the evaluation of reinforced concrete (RC) joints strengthened with fibre reinforced polymer (FRP) composite materials is presented. For this purpose, existing model available in literature is simplified and step-by-step iterative procedure is given which separately evaluates the shear contributions provided by the composite material and by the confined concrete. To achieve the target performance of the retrofit strategy, step-by-step design procedure is adopted to generate a M-N interaction diagram, or performance domains, for the beam-column-joint subassembly. Finally, results of parametric analyses are shown to highlight the effect of different retrofitting schemes and axial load levels on the effectiveness of strengthening along with the brief discussion on deformation based retrofit design. The procedure is shown to yield simple and efficient design calculations and can easily be implemented by practicing engineers.

1 INTRODUCTION

During the last two decades several strengthening methodologies have been proposed to enhance the shear resistance of RC beam-column joints using fibre reinforced composite (FRP) materials. In spite of the few experimental studies, relatively less work has been dedicated on the analytical modelling/analysis and design of FRP-strengthened joints. Gergely et al. (2000) presented a design example for an interior joint of bridge bent tested by Pantelides et al. (1999). By using steel stirrups analogy FRP contribution to the shear capacity of the joint was analyzed and composite layout was specified. Ghobarah and Said (2001) and El-Amoury and Ghobarah (2002) also proposed a design methodology based on providing fibre reinforcement to replace the missing joint shear reinforcement to upgrade the shear capacity of existing RC beam-column joints, respectively. Antonopoulos and Triantafillou (2002) proposed an extended version of Pantazopoulos and Bonacci’s model (1992) to account for the effect of externally bonded FRP on the shear capacity of the panel zone region.

In this contribution, the theoretical aspects and alternative simplified method for the analysis of RC joints with focus on the strength assessment of panel zone region prior to and after the retrofit intervention are covered in this paper. Step-by-step procedures with numerical example for the construction of M-N performance domain for each structural element are explained in detail. In order to highlight the effect of axial load level on the shear strength of retrofitted joint with different retrofitting schemes outcomes of parametric analysis is shown.

2 PROPOSED MODIFIED MODEL

Extensive research on the behaviour of beam-column joints have demonstrated that the panel zone (joint region) can be represented as two-dimensional panels reinforced in orthogonal directions under the effect of in-plane shear and normal stresses as shown in Figure 1 (Pantazopoulos, 1992).

As-built joint after deformation, the overall geometry can be described by the average angle of shear distortion, $\gamma$, the principal strains, $\varepsilon_1$ and $\varepsilon_2$, which are related to those in the longitudinal and transverse directions, $\varepsilon_l$ and $\varepsilon_t$, and the average compressive stresses, $\sigma_c$ and $\sigma_l$, respectively. Moreover, assuming that (1) the maximum principal stress in the concrete, $\sigma_c$, is always less than the tensile
capacity, for the sake of simplicity which can be taken as zero; and (2) the directions of principal strains and stresses coincide (this is nearly correct if the reinforcement has not yielded), following relationships are given in Figure 1 by utilizing Mohr’s circle.

Figure 1: Exterior RC joint strengthened with FRP: (a) design dimensions; (b) horizontal force equilibrium; (c) vertical force equilibrium (modified after Antonopoulos et al.); (d) kinematics of as-built joint and Mohr circle

development of plane strain and stress.

The strengthened joint can be idealized as a three-dimensional element with dimensions of \( h_c \) (overall depth of column in the direction of the horizontal shear to be considered), \( b_w \) (width of beam), and \( h_b \) (height of beam). Schematic illustration and average stresses of RC joint strengthened with FRP are shown in Figure 1. The notations shown in the figure are: distance from column centre to contraflexure point, \( l_b \), half clear span length, \( l_b' \), height of beam, \( h_b \), height of column, \( l_c \), effective depth, \( d \). Additionally, for the force equilibrium requirements horizontal column shear force, \( V_c \), and beam shear force \( V_b \) and compressive axial forces of the column and beam (if any) are represented as \( N_c \) and \( N_b \), respectively. The joint shear stresses are assumed to be uniformly distributed over the boundaries of the joint which are presented by direct member action as well as by reinforcement bond.

Under the equilibrium conditions, for older joints the average compressive transverse and longitudinal stresses, \( \sigma_t \) and \( \sigma_l \), can be written as

\[
\sigma_t = -\rho_p E_f \varepsilon_t
\]

\[
\sigma_l = -\rho_s E_s \varepsilon_l - \frac{N_c}{h_b b_e}
\]

where \( E_f \) = elastic modulus of fibre; \( \varepsilon_t \) = average strain in the FRP in the transverse or long. direction; \( \rho_p \) = FRP reinforcement ratio in the transverse or longitudinal direction (Eq. 10, 11). Plane stress and strain components can be written in terms of \( \varepsilon_t \) and \( \tan \theta \) using Mohr circle as given in Figure 1 with the constitutive relations \( f_t = E_f \varepsilon_t, f_l = E_s \varepsilon_l \) and \( f_p = E_p \varepsilon_p \) where \( E_f \) and \( E_s \) are the elastic modulus of fibre and steel, respectively. Following the same way, shear stress contribution due to FRP \( \nu_f \) can be written as

\[
\nu_f = \frac{\rho_p E_f \varepsilon_p}{\tan \theta}
\]

The procedure described above leads to a quadratic polynomial of \( \tan \theta \):

\[
\left( \frac{1}{E_c} + \frac{1}{E_f} \right) \tan^4 \theta + \left( \frac{\rho_p E_f}{h_b b_e \rho_s E_s} \right) \tan^3 \theta - \left( \frac{1}{\rho_p E_f} + \frac{1}{\rho_s E_s} \right) = 0
\]
2.1 Iterative procedure to evaluate the joint shear strength due to composite material

The shear capacity of the strengthened RC joint can be determined by a simple iterative procedure. The input consists of (1) geometric data: $h_b$, $b_n$, $h_c$, $b_c$; (2) material properties: compressive strength of concrete, $f_c$; elastic modulus of concrete, $E_c$; tensile strength of concrete, $f_t$; FRP thickness per layer, $t_f$; design rupture strain, $\varepsilon_{fr}$; (3) FRP application details: depth of FRP sheet on beam surface, $d_{f,b}$, depth of FRP sheet on column surface $d_{f,c}$, number of sheet on beam $n_{f,b}$, number of sheet on column $n_{f,c}$, number of beam sides covered with FRP, $n_{s,f,b}$, number of column sides covered with FRP, $n_{s,f,c}$, FRP reinforcement ratio in the transverse or long. direction can be found as follows:

$$\rho_{p} = (n_{f,b} n_{s,f,b} t_f d_{f,b})/(h_b b_w)$$

$$\rho_{n} = (n_{f,c} n_{s,f,c} t_f d_{f,c})/(h_c b_c)$$

As an initial step, the transverse strain, $\varepsilon_t$, is incremented. For each value of $\varepsilon_t$ is solved for $\tan \theta$ so that the shear stress due to FRP, $\nu_{f,c}$ can be calculated by Eq. (8). Next, normal stress in FRP, $f_{f,c}$, along direction $t$ (at midheight of joint) is determined to do the failure check at the end of each step by comparing strains to yield values. At the end of each iteration steps two conditions should be checked:

1) Failure of FRP: As an ultimate limit state, debonding of the composite material is treated here according to the fractural mechanics-based model of Holzenkampfer (1994) which is given by:

$$f_{f,c,b} = f_{f,c}^{max} = c_1 \sqrt{(E_f f_{ct})/(t_f n_{f,b})} \quad \text{for} \quad l_{b,t} \geq l_{b,max}$$

where empirical coefficients $c_1$ and $c_2$ are taken as 0.64 and 2 as suggested by Neubauer et al. (1997) and Holzenkampfer (1994), respectively. FRP development length $l_{b,t}$ along the direction $t$ (in mm) is

$$l_{b,max} = \sqrt{(E_f t_f) / (c_2 f_{ct})}$$

2) Diagonal compression failure of joint panel concrete: In the absence of stirrups, strength of the diagonal compression strut will govern the failure mechanism of the joint. At any point in iteration compression failure check can be done as indicated in the literature (Priestley et al., 1996)

$$p_t \leq 0.3 f_c$$

where $p_t$ is diagonal compression stress and $f_c$ is compressive strength of concrete.

3 DESIGN OF THE FRP RETROFIT SYSTEM

Before the appropriate retrofit solution adopted assessment of the deficient beam-column joints should be utilized by taking into account the expected damage mechanisms along with the internal hierarchy of strength in the system. Recently in the literature a simple procedure has been proposed to evaluate the expected sequence of events through the comparison of internal hierarchy of strengths within a beam-column-joint system (Calvi et al., 2002; Pampanin et al., 2007b) through the construction of capacity and demand curves within a M-N (moment-axial load) performance domain. Appropriate rehabilitation techniques can be applied by rearranging the sequence of events to bring them up to capacity design provisions.

Design recommendations are based on limit-states design principles. In assessing the nominal strength of a member, the possible failure modes and subsequent strains and stresses in each material should be assessed. Beam, column and joint capacities are referred to a given limit states (e.g. for joints: cracking, equivalent ‘yielding’ or extensive damage and collapse) and evaluated in terms of equivalent moment in the column, $M_{e,b}$ at that stage, based on equilibrium considerations within the beam-column joint system. Capacity design principles (Paulay and Priestley, 1992) should be employed for the seismic retrofit which assumes that the structure develops its full capacity and members are capable of resisting the associated required shear strengths.
3.1.1 Evaluation of beam and column section capacities

The behaviour of a member subjected to bending or combined bending and axial load can be best studied by performing conventional moment-curvature analyses. M-N capacity curves for beams and columns corresponding to a given limit states can be derived and plotted in a performance-domain to define the sequence of events. The confinement effects of the FRP on the section can be taken into account following procedures available in the literature (e.g. Spoelstra and Monti 1999).

3.1.2 Evaluation of joint strength prior and after retrofit

Firstly, as-built joint will be analyzed to obtain M-N interaction curve by using step-by-step procedure. In order to achieve this, the equivalent column moment can be expressed in terms of column shear force \( V_{jh} \) using simple statics for an exterior beam-column joint under the external and internal actions along with the notations given in Figure 1.

\[
M_{col} = \frac{V_{jh}}{l_c} - \frac{V_{jh}}{l_{b, jd}} \left( \frac{l_c - h_b}{2} \right)
\]  

(15)

where \( jd \) is the lever arm in the beam that can be assumed as 0.9d.

It should be noted that the nominal horizontal shear stress \( v_{jh} \) at the mid-depth of the joint core is \( V_{jh}/(b_v h_t) \). From Mohr’s circle \( v_{jh} \) and the nominal principal tensile stress in the joint \( p_t \) are found as

\[
p_t = f_t/2 + \sqrt{(f_t/2)^2 + v_{jh}^2}
\]  

(16)

\[
v_{jh} = \sqrt{p_t^2 + f_t^2}
\]  

(17)

where \( f_t \) is nominal compressive stress on the column at the mid-depth of the joint core is given by

\[
f_t = N_v/(h_t b_v)
\]  

(18)

where compressive stress, \( N_v \), is taken as negative. Note that, based on experimental evidences (Pampanin et al., 2002; Hertanto, 2006) for tee-joints with plain round bars and end hooks the first crack in the joint core can be assumed to occur at a principal tensile stress level \( p_t = 0.19 \sqrt{f_t} \) in MPa.

It is noteworthy that, especially for old-type joints setting limits for the principal stress values in the joint (both compressive and tensile) is more consistent with the underlying mechanics of the problem. Because the ultimate failure mode is governed by the crushing of compression strut instead of stirrup yielding as seen in properly designed joints after the drastic degradation in the strength due to the first crack in the joint panel. By this way, influence of axial loads on the joint capacity through the columns can be monitored clearly either in as-built or retrofitted condition.

All things considered, general formulation for as-built joint equivalent column moment \( M_{col} \) can be written as follows

\[
M_{col} = \left( p_t^2 + p_t \cdot \frac{N_v}{h_t b_v} \right) b_v h_t \left( \frac{l_c - h_b}{2} \right) \left( \frac{l_c}{l_{b, jd} - h_t/2} / 0.9d_b - 1 \right)
\]  

(19)

A more condensed form of the above equation yields to

\[
M_{col} = \frac{v_{jh}(1000)}{\beta} [kNm]
\]  

(20)

where \( \beta \) is regarded as a geometric coefficient which is related to the dimensions of beam-column joint according to notations given in Figure 1.
\[ \beta = \frac{2l_h - 1.8d_{lb}}{0.9d_{lb}l_t - h_b} \]

where \( A_e \) is the effective area in the joint. Assessment of the capacity curve of a deficient exterior beam-column joint steps are as follows:

**Step 1:** Determine effective area \( A_e = h_c b_w \)

**Step 2:** For exterior joints limit state in terms of \( p_t \), corresponding to first cracking \( (p_t = 0.19 \sqrt{f_c}) \)

**Step 3:** Calculate the geometric coefficient \( \beta \)

**Step 4:** Calculate as-built joint capacity in terms of \( M_{col} \) for different axial load levels

As a next step, to construct the capacity curve of the strengthened joint, calculation steps are given below. If debonding is not prevented, debonding stress \( (f_{f,deb}) \) consequently debonding strain \( (\varepsilon_{f,deb}) \) will be taken as a limit value for the capacity.

**Step 1:** Calculate FRP reinforcement ratio and debonding strain, \( \varepsilon_{f,deb} \)

**Step 2:** Determine \( \tan \theta \) by solving quadratic polynomial of \( \tan^2 \theta \) (Eq. 9: \( a \tan^4 \theta + b \tan^2 \theta - c = 0 \))

**Step 3:** Calculate average joint shear stress due to fibre \( \nu_f \)

**Step 4:** Calculate retrofitted joint capacity in terms of equivalent column moment

The total capacity of joint can be expressed in terms of principal tensile stress due to the as-built solution \( p_{ut} \) and the fibre contribution \( p_{ft} \) (i.e. \( p_{ut} = p_{ut} + p_{ft} \)). At this stage, supplied base shear \( V_{bs} \) in the joint can be evaluated using total principal tensile stress \( p_{ut} \) as a function of applied axial load and according to specified joint shear deformation limit state. Conceptual deformation based retrofit design using FRP materials for shear strength enhancement of deficient beam column joints is given in Figure 2.

![Conceptual deformation based design procedure](image)

**Figure 2:** Conceptual deformation based design procedure

### 3.2 Design example

Step-by-step capacity assessment of an exterior 2/3 scale as-built and retrofitted beam-column joint tested at the University of Canterbury will be carried out as an example (Akguzel, 2007). Geometry, reinforcing details and material properties are given in Figure 3.
Retrofitting scheme consists of 1 layer of application on the beam and 2 layer on the column. The GFRP materials used for this rehabilitation have nominal thickness of 0.36 mm, elastic modulus of 76,000 N/mm² and ultimate strain of 1.8%. Starting with the as-built joint calculation steps are given as follows:

**Step 1:** Determine effective area: \( A_e = h_b b_w = (0.23)(0.23) = 0.0529 \text{ m}^2 \)

**Step 2:** Limiting principal tension stress \( p_t = 0.19 \sqrt{f_c} = 0.19 \sqrt{18} = 0.806 \text{ MPa} \)

**Step 3:** The geometric coefficient \( \beta \) is calculated as:

\[
\beta = \frac{2l_f l_c - 1.8d_{lb}}{0.9d_{lb} A_e (l_c - h_c)} = \frac{2(1.409)(2) - 1.8(0.305)(1.524)}{0.9(0.305)(1.524)(0.0529)(2 - 0.33)} = 129.86
\]

**Step 4:** As an example for 50 kN of axial load, the equivalent joint moment \( M_{j,UP,50} \) would be:

\[
f_c = N_e / (h_b b_w) = 50 / ((0.23)(0.23)) = 0.945 \text{ MPa}
\]

\[
M_{j,UP,50} = \left( \sqrt{P_i^2 + P_t f_c} \right)(1000) / \beta = \left( \sqrt{(0.806)^2 + (0.806)(0.945)} \right)(1000) / 129.86 = 9.15 \text{ kNm}
\]

For the assessment of strengthened joint, if debonding is not prevented, debonding stress \( (f_{j,deb}) \) consequently debonding strain \( (\varepsilon_{j,deb}) \) will be taken as a limit value for the capacity. As an example, axial load value is taken 50kN as taken in as-built joint performance analysis.

**Step 1:** Calculate FRP reinforcement ratio in each direction and debonding strain, \( \varepsilon_{j,deb} \).

\[
f_{j,deb} = f_{j,\text{max}} = c_i \sqrt{(E_{j,f} / f_c)} / (t_i n_{j,b}) = (0.64) \sqrt{(76000)(2.1) / (0.36)(1)} = 426 \text{ kN}
\]

\[
\varepsilon_{j,deb} = (426)(76,000) = 0.56 \%
\]

\[
\rho_p = (n_{j,b} n_{j,c} / h_b b_w) = [(1)(2)(0.36)(300)] / [(230)(230)] = 0.00284
\]

\[
\rho_p = (n_{j,c} / h_b) = [(2)(0.36)(200)] / [(330)(230)] = 0.00544
\]

**Step 2:** Determine \( \tan \theta \) by solving quadratic polynomial of \( \tan^2 \theta \) (Eq. 9): \( a \tan^4 \theta + b \tan^2 \theta - c = 0 \),

\[
a = \left( \frac{1}{E_{j,c}} + \frac{1}{\rho_p E_{j,c}} \right) = \left( \frac{1}{19.940} + \frac{1}{(0.00284)(76000)} \right) = 0.00468
\]
\[
b = \frac{N_{f,d}}{b f_{c} E_{c} e_{f,dab}} = \left(\frac{50,000}{(230)(0.00284)(0.00544)(76000)^{2}(0.0056)}\right) = 0.00189
\]
\[
c = (-1)\left(\frac{V_{f} + 0.13}{E_{f} + 0.13}\right) = \left(\frac{19.940 + 0.0000044}{76000}\right) = -0.002467
\]
\[
\Delta = \frac{-b + \sqrt{b^{2} - 4ac}}{2a} = \frac{-0.00189 + \sqrt{(0.000189)^{2} - 4(0.000468)(-0.002467)}}{2(0.000468)} = 0.552 \Rightarrow \tan \theta = \sqrt{0.552} = 0.743
\]

**Step 3:** Calculate average joint shear stress due to fibre, \(v_{f}\)

\[
v_{f} = \frac{\rho_{b} E_{c} e_{f,dab}}{\tan \theta} = \frac{(0.000284)(76000)(0.0056)}{0.743} = 1.63\text{MPa}
\]

**Step 4:** Calculate retrofitted joint capacity in terms of equivalent column moment,

\[
M_{j,R12,50} = v_{f} (1000) / \beta = (1.63)(1000)/129.86 = 12.6\text{ kNm}
\]

Total shear capacity of the strengthened joint can be found by adding the retrofitted joint capacity to the as-built joint capacity: \(M_{j,R12,50} = 9.15 + 12.6 = 21.75\text{ kNm}\)

### 3.3 Validation of the proposed approach

In order to validate the proposed analytical procedure test results of ten shear strengthened exterior beam-column joint specimens from two different experimental campaign are compared with analytical predictions. Detailed information about the test procedures and results can be found in the references (Nassi, 2002; Vecchietti, 2001; Pampanin et al., 2007b; Antonopoulos and Triantafillou, 2002).

**Table 1. Comparisons of analytical model with test results**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>(p_{f,1}/\sqrt{f_{c}})</th>
<th>(p_{f,1}/\sqrt{f_{c}})</th>
<th>(\text{Anal}/\text{Exp.}_{\text{Pavia}})</th>
<th>(\text{Anal}/\text{Exp.}_{\text{Antonopoulos}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>F11</td>
<td>0.86</td>
<td>0.83</td>
<td>0.96</td>
<td>0.97</td>
</tr>
<tr>
<td>F22</td>
<td>0.94</td>
<td>0.99</td>
<td>1.05</td>
<td>1.23</td>
</tr>
<tr>
<td>F21</td>
<td>0.96</td>
<td>0.91</td>
<td>0.95</td>
<td>1.05</td>
</tr>
<tr>
<td>F12</td>
<td>0.79</td>
<td>0.88</td>
<td>1.11</td>
<td>1.23</td>
</tr>
<tr>
<td>F22A</td>
<td>0.94</td>
<td>0.97</td>
<td>1.03</td>
<td>---------</td>
</tr>
<tr>
<td>F22W</td>
<td>1.02</td>
<td>0.98</td>
<td>0.96</td>
<td>1.12</td>
</tr>
<tr>
<td>F22in1</td>
<td>0.88</td>
<td>1.01</td>
<td>1.15</td>
<td>---------</td>
</tr>
<tr>
<td>S33</td>
<td>0.65</td>
<td>0.77</td>
<td>1.19</td>
<td>---------</td>
</tr>
<tr>
<td>S33L2</td>
<td>0.84</td>
<td>0.77</td>
<td>0.92</td>
<td>---------</td>
</tr>
<tr>
<td>S63</td>
<td>0.78</td>
<td>0.88</td>
<td>1.13</td>
<td>---------</td>
</tr>
</tbody>
</table>

### 3.4 Parametrical study

The effect of different GFRP layouts and axial load level on the capacity of strengthened joint is investigated through a parametric study. Firstly, effects of two different retrofit schemes on the hierarchy of strength in the joint are shown through constructing the performance domains as shown in Figure 4. Note that, retrofitting scheme is expressed as \(Rij\) where \(i\) indicates the number of sheets applied on the beam surface whereas \(j\) indicates the number of sheets on the column (e.g. \(R12\) suggest 1 layer of application on beam and 2 layer on the column). Sequence of events can also be determined due to demand imposing on the system in the same constructed domain. Recent studies (Pampanin et al., 2007a) have shown that variation of axial load due to lateral loading on a frame should be taken
into account, particularly when dealing with the assessment and retrofit of poorly detailed R.C. frames. The relationship between the lateral force $F$ and the variation of axial load $N (N = N_{\text{gravity}} \pm \alpha F)$ is function of the geometry of the building (i.e. number of bays and storeys) and can be derived by simple hand calculations or pushover analyses on the prototype frame.

Secondly, principal shear stress capacity enhancement due to retrofit for R11 scheme and overall joint strength degradation curves with the combination of contributions of GFRP and concrete under different axial loads for R11, R12 and R22 are given in Figure 5. Contribution of GFRP is quite substantial even with the minimum retrofit scheme, R11. Although the favourable effect of high axial load becomes more pronounced as more fibre placed horizontally (e.g. R12 and R22), behaviour becomes less ductile such as 20% reduction in ductility.

Figure 4: Evaluation of hierarchy of strengths: M-N performance domain

Figure 5: GFRP effect on the behaviour of strengthened joint under varying axial load (cont’d)
4 CONCLUSIONS

Simplified analytical model using kinematics and equilibrium states in the joint is presented for the analysis of RC exterior joints strengthened with GFRP materials in order to construct the performance domain. Conceptual deformation-based design approach is also shown based on the principal tensile stresses in the joint for different axial load and retrofit schemes. Finally, parametric analysis is carried out to exhibit the effectiveness of strengthening for different configurations of retrofit scheme.

Conclusions can be summarized as follows: (1) Mode of joint failure can be considered by establishing a relationship between loads applied to the joint and resulting deformations. (2) From the analysis it is shown that external bonded reinforcement has significant effect on the enhancement of the joint shear capacity even in the application of minimum retrofit strategy; (3) Shear strength of the joint either as-built or retrofitted depends on the usable compressive strength of concrete and debonding characteristics of the composite material; (4) Proposed simplified approach and step-by-step procedure can be easily adopted by designers as a powerful tool for the assessment and retrofit interventions. By this way, solutions can be adopted to bring the proper sequence of events and hierarchy of strength to meet the capacity design principles leading to a more ductile and dissipating hysteresis behaviour; (5) As a final step, deformation-based retrofit strategy can be adopted in such a way that global displacement response of the structure obtained through the assessment of demand can be easily linked to the demand level of joint shear deformations. Thus, joint retrofit design can be linked to the overall lateral drift of the structure for specified performance limit.

ACKNOWLEDGEMENTS

The financial support provided by the NZ Foundation for Research, Science and Technology (FRST) through the Research Program “Retrofit Solutions for NZ” is gratefully acknowledged. The assistance and cooperation of Mr. A. Vecchietti and Mr. R. Nassi are also acknowledged.

REFERENCES


