A macro-element for pile head response to cyclic lateral loading

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ABSTRACT: The standard method of modelling the lateral load response of a pile head is to use computer software packages. Many of the conventional software packages model the interaction of the pile shaft and the soil using either the linear or nonlinear Winkler springs approach. An alternative way of analysing the elastic behaviour is to use a simple expression for the pile head stiffness. This approach has been extended in this paper to represent the nonlinear pile head lateral response. Numerical simulations showed that this approach is capable of achieving the same monotonic response prediction as the Winkler spring software. Thus, the response of the whole pile shaft can be represented by a simple set of equations focusing on the pile head. This is referred to as a macro-element. This paper presents an analytical technique of the elastic continuum approach used for the cyclic response of the laterally loaded pile problem. This approach will then be compared with the cyclic Winkler calculations and field data obtained from previously published results.

1 INTRODUCTION

Piles are frequently used to resist lateral forces that result from loading of supported structures. The response of pile foundations to lateral forces depends on the interaction between the pile and the soil surrounding. This lateral force at the pile head can be the governing design constraint for a single pile and pile groups supporting different types of structures. The lateral force may be a result of earthquakes, machine vibrations, wave actions, wind, blasts, or impacts and may be transient or cyclic in nature.

Researchers have employed various analytical approaches for the analysis of single piles subjected to cyclic lateral loadings. The approaches can be categorised by the treatment of the soil medium in the analysis.

- The Winkler spring model or modulus of subgrade reaction approach represents the soil supporting the pile by an array of uncoupled springs. These springs can be taken to be linear elastic or nonlinear and the shapes of the load-deformation relationships are described by the p-y curves. Finite-element or finite-difference techniques are then be used to determine the response of the pile and spring system to the applied loadings (Matlock et al. 1978, Novak et al. 1978, and Reese and van Impe, 2001).
- A theoretical homogeneous continuum approach in which the soil surrounding the pile is modelled as a homogeneous elastic continuum (Poulos 1971a, b).
- A finite element or finite difference approach in which the pile and the soil surrounding it are discretised using finite-element or finite difference techniques and desirably using nonlinear representations of soil stress-strain relationships. Examples of this are the studies by Blaney et al. (1976) and Kuhlemeyer (1979).

The analysis of the lateral pile head stiffness is frequently carried out by using computer software based on the Winkler spring idealisation of the localised interaction between the pile shaft and the soil such as LPILE (Reese and van Impe, 2001) and RUAUMOKO (Carr, 2005). However, the aim of this paper is to demonstrate an analytical method as an alternative to such software for estimating the
deformation at the pile head due to the cyclic lateral loadings.

2 SOIL-PILE INTERACTION MODEL

2.1 Elastic Continuum Model (ECM)

The elastic continuum model was developed to account for the continuity of the soil mass. The assumption of the soil as an elastic continuum was proposed by Mindlin (1936). Due to the similarity between the beam problem and the pile problem, these solutions were later adapted by Poulos (1971a, b) to investigate the response of a laterally loaded pile. The elastic continuum model is based on the Young’s modulus of the soil, \( E_s \). This value can be determined either by: (a) field tests, (b) laboratory testing from relatively undisturbed samples and (c) back calculation from pile load tests. Table 1 presents several published values showing the correlations between undrained Young’s modulus and undrained shear strength \( (E_s/s_u) \) from the literature. Poulos and Davies (1980) stated that the best way to determine the \( E_s \) is to back calculate from lateral pile load tests.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Normalised Secant Modulus</th>
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</thead>
<tbody>
<tr>
<td>Skempton (1981)</td>
<td>50 to 200</td>
</tr>
<tr>
<td>Poulos and Davies (1980)</td>
<td>15 to 95</td>
</tr>
<tr>
<td>Sullivan (1980)</td>
<td>100 to 250</td>
</tr>
<tr>
<td>Simon (1976)</td>
<td>40 to 3000</td>
</tr>
<tr>
<td>Bjerrum (1972)</td>
<td>500 to 1500</td>
</tr>
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</table>

A simple set of equations applied at the pile head are needed for the calculation. The first set of equation given below are to estimate the elastic displacement, \( u_{gl} \), and rotation, \( \theta_{gl} \), of the pile shaft at the ground-line. More details and further examples can be found in Pender (1993). The pile is embedded in a homogenous soil layer and both pile and soil are assumed to behave elastically in this model.

\[
\begin{align*}
    u_{gl} &= f_{UH} H + f_{UM} M \\
    \theta_{gl} &= f_{UH} H + f_{UM} M
\end{align*}
\]

where: \( H \) is the applied pile head horizontal load, \( M \) is the applied pile head moment, and \( f_{UH}, f_{UM}, f_{UH}, f_{UM} \) are the flexibility coefficients of the pile shaft.

Frequently the actions are applied at some distance above the ground-line and the pile shaft displacements are required at that position for design. By assuming the pile to be a cantilever from the ground-line to a free head connection to the attached structures, the lateral displacement \( u_{pile} \) and rotation \( \theta_{pile} \) of the pile at the pile-structure connection can be determined using the following equations:

\[
\begin{align*}
    u_{pile} &= u_{gl} + \theta_{gl} L + \frac{HL^3}{3EI} + \frac{ML^2}{2EI} \\
    \theta_{pile} &= \theta_{gl} + \frac{HL^2}{2EI} + \frac{ML}{EI}
\end{align*}
\]

where: \( H \) and \( M \) are the horizontal force and moment applied at the pile-structure connection, \( L \) is the length of the pile shaft projection above the ground-line, and \( EI \) is the flexural property of the pile.

Next, the flexibility coefficients for the pile shaft at the ground-line are required. These are available for various types of soil profiles having a constant modulus, a modulus increasingly linearly with depth from the ground surface, and a modulus increasing from zero at the ground surface as the square root of the depth. The constant modulus case is applicable to an over-consolidated sedimentary soil or one derived from in-situ weathering from rock (residual soil). The linear distribution with depth
models normally consolidated clay or the degraded modulus distribution with depth in cohesionless soil profile after cyclic loading. The square root modulus distribution represents cohesionless soils at small strains. For these materials, the small strain shear modulus is known to be a function of the square root of the effective stress and the square root distribution simply reflects the increase in effective stress with depth (Pender 2007). The definition diagrams for these three cases are given in Figure 1. However, only the case of pile embedded in homogeneous soil, an acceptable assumption for heavily over-consolidated clays, is presented in this paper. The relevant equations for this profile model are given by Davies and Budhu, (1986).

Figure 1 The three soil profile models. (D – pile shaft diameter, \(E_sD\) – soil Young’s modulus at a depth of one pile diameter.)

The ratio of the Young’s modulus of the pile and the soil is the basic parameter in the flexibility coefficient equations. This equation is expressed in terms of a stiffness ratio given below:

\[
K = \frac{E_p}{E_s} \quad \text{for constant soil modulus}
\]  

where: \(E_p\) is the Young’s modulus for the pile material, and \(E_s\) is the Young’s modulus of the soil.

The flexibility coefficients are given by:

\[
f_{slr} = \frac{1.3K^{-0.18}}{E_QD}; \quad f_{slr} = \frac{2.2K^{-0.45}}{E_sD^2}; \quad \text{and} \quad f_{slr} = \frac{9.2K^{-0.73}}{E_sD^3}
\]

where: \(D\) is pile diameter.

The above flexibility coefficients assume that the piles are “long”, that is the lower parts of the pile shafts experience no lateral deflection or rotation from the ground-line actions. It is known that the length of pile shaft over which lateral deformations occur is several pile shaft diameters; this is defined as the “active length” given by:

\[
L_a = 0.50DK^{-0.36}
\]

The position and value of the maximum pile shaft moment are given by:

\[
L_{max} = 0.40L_a \\
M_{max} = I_{slr}DH \\
I_{slr} = aK^b
\]

where the coefficients \(a\) and \(b\) can be determined with sufficient accuracy from the equations given below:
The above equations are subjected to condition that \( 0 < f < 6 \). For larger values of \( f \), the maximum pile shaft moment is equal to the pile head moment. It should be emphasised that the equations are only valid for estimating the lateral deflections and rotations of the pile shaft at the ground-line when the soil and the pile behave elastically and for piles which are longer than their effective lengths.

However, it is well known that the behaviour of laterally loaded piles is frequently nonlinear even under relatively small load levels. Davies and Budhu (1986) investigated the effects of non-linear lateral interaction and local failure between the embedded pile shaft and the soil by calculating a modification factor applied to the elastic prediction of the pile behaviour. The following equations give the pile head displacement, rotation and maximum moment for the free head pile with nonlinear soil behaviour:

\[
\begin{align*}
    u_y &= I_u u_E \\
    \theta_y &= I_\theta u_E \\
    M_{My} &= I_{My} M_{ME}
\end{align*}
\]

where:
- \( I_u, I_\theta, I_{My} \) are the yield influence factors,
- \( u_E \) is the elastic pile head displacement at the ground-line from equations (1),
- \( \theta_E \) is the elastic pile head rotation at the ground-line from equations (2), and
- \( M_{ME} \) is the maximum elastic pile shaft moment from equation (8).

For constant Young’s modulus with depth, the modification equations are:

\[
\begin{align*}
    I_u &= 1 + \frac{h - 2.9k^{0.2}}{10.5k^{0.45}}; \quad I_\theta = 1 + \frac{h - 2.9k^{0.2}}{12.5k^{0.33}}; \quad \text{and} \quad I_{My} = 1 + \frac{h - 2.9k^{0.2}}{20k^{0.29}}
\end{align*}
\]

where:
- \( h = H/s_u D^2 \)
- \( k = K/1000 \)
- \( s_u = \) undrained shear strength of the soil

These equations are applied to predict the observed lateral response for a number of published experimental studies on single laterally loaded piles in over-consolidated clays as discussed below.

### 2.2 Case Studies

**Austin**

Reese et al. (1975) conducted a series of field tests on laterally loaded piles embedded in stiff fissured clay with plasticity index ranging from 30\% to 70\%) at a site near Austin, Texas. The test piles, with outside diameters of 625 mm and 170 mm, were installed in a large pit from which the topsoil had been removed. The bending stiffnesses \( E_p I_p \), for both piles are 138 MNm\(^2\) and 3 MNm\(^2\), respectively. The ratio \( E_p/s_u \) was assumed to be equal to 500. The experimental results are compared with the calculations performed for predicting load-displacement curve based on Davies and Budhu equations (9) and (10) and the back-calculated \( p-y \) curve. Figure 2 shows that the results obtained for the load-displacement relation as predicted by the model are very encouraging and are even better than \( p-y \) predictions for the small diameter pile.

**Bagnolet**

Kerisel (1965), quoted by Van Impe and Reese (2001), reported the results of the static lateral load tests on a closed-ended bulkhead caisson. The pile has an equivalent diameter of circular section of 0.43 m and the bending stiffness \( E_p I_p \) was given as 255 MNm\(^2\). The yield strength of steel was 248
MPa and the bending moment of the pile just reaching yielding ($M_y$) is at 204 kNm. The tests were performed at Bagnolet, Paris in a fairly uniform deposit of medium stiff clay. The water table was below the pile tips, but the degree of saturation was over 90% and it was assumed that undrained shear strength can be employed in the analyses. Figure 3 shows the comparisons of the results obtained from the test and the analyses performed for predicting load-displacement curve based on Winkler’s approach and the present approach given by equations (9) and (10). As may be seen, good to excellent agreement was found between the experimental results and computed values of the pile head displacement for the range of static loads that were applied.

Figure 2 Load-displacement data obtained from field tests (after Reese et al. 1975).

Figure 3 The experimental and computed displacement values for Bagnolet pile tests.

3 PREDICTION OF SINGLE PILES RESPONSE IN AUCKLAND RESIDUAL CLAYS

Calculations of the response of a single laterally loaded pile embedded in Auckland residual clay were done using Mathcad 11 (PTC 2007). The nonlinear response at the top of the pile shaft subjected to cyclic lateral loading was evaluated. The piles were assumed to be vertical with a constant circular cross-section and embedded in a homogeneous soil layer with a constant Young’s modulus with depth. The effect of different projections of the pile shaft above the ground-line was evaluated and for each
In the following analyses, steel has a yield strength of 350 MPa and consequently the yield moment of the pile, $M_y$, was computed to be 226 kNm.

Table 2 lists the properties of the pile and soil used for predicting the pile head response. In the following analyses, steel has a yield strength of 350 MPa and consequently the yield moment of the pile, $M_y$, was computed to be 226 kNm.

<table>
<thead>
<tr>
<th>Steel pile pipe</th>
<th>Auckland Residual Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer diameter (OD) : 273.1 mm</td>
<td>Undrained shear strength, $(s_u)$ : 100 kPa</td>
</tr>
<tr>
<td>Wall thickness (WT) : 12.7 mm</td>
<td>Soil stiffness ratio $(E_s/s_u)$ : 300</td>
</tr>
<tr>
<td>Young’s modulus $(E_p)$ : 200 GPa</td>
<td>Poisson ratio $(\nu)$ : 0.5</td>
</tr>
</tbody>
</table>

Figure 4 Computed values of load-displacement and load-rotation for Case 1

Figure 5 Computed values of load-displacement and load-rotation for Case 2

Figure 6 Computed values of load-displacement and load-rotation for Case 3
Equations (9) and (10) were developed by Budhu and Davies for monotonic lateral loading of piles. From the calculation it was found that the direct application of these equations to cyclic loading did not produce useful results as the stiffness when the direction of loading was reversed decreased too far. By making a small change after the manner of Masing’s rule to equation (10) this was remedied, at least for the calculations presented in this paper. The term \( (h - 2.9k^{0.2}) \) appears in the three terms in equation (10), we found that by halving this expression reasonable results were obtained as shown in Figures (4) to (6).

For the series of simulations, cyclic lateral loads were applied at different pile head projections. In Case 1, loads were applied at a pile shaft projection distance of one pile diameter, followed by Case 2 and Case 3, with projections of 2 and 4 pile shaft diameters, respectively. The successive loads were widely separated in magnitude so that the cycling at the previous load was assumed to have no effect on the first cycle at the next load. The computed results of the pile head load-displacements and pile head load-rotation obtained from the analyses are shown in Figure 4 – 6. Comparing the results for the three cases, it can be summarised that as the ground-line moment increases, due to the increased lever arm, the maximum bending moment goes up for the same final value of the pile head loads applied (refer Figure 7). This result suggests that, if a lateral load is applied to a pile at a great distance above the ground line, the behaviour of the pile will depend primarily on the pile stiffness and the soil characteristic have no effects. Blaney and O’Neill (1986) stated that by looking at results of other tests in the literature, it is important to apply the load as close to the ground surface as possible without touching the soil so that the response of the pile-soil system can be measured accurately, rather than being dominated by the flexibility of the above ground part of the pile shaft.

![Computed maximum bending moment for the three cases](image)

**4 CONCLUSIONS**

The paper demonstrated the application of a simple set of pile stiffness equations to analyse both linear and non-linear pile-soil interaction. For lateral pile-soil interaction, it was very important to be able to take into account the non-linear load-displacement response of the pile. The Davies and Budhu equations provided a convenient way to model the pile-soil behaviour. The good agreement with the results of full-scale tests suggested that the elastic continuum model is an elegant approach to the laterally loaded pile problem. Further, the objectives of this paper to provide a simple tool that can be used in place of pre-packaged software and to an independent way of checking that the software is working correctly were achieved.
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REFERENCES


