Decision tools for earthquake risk management, including Net Present Value and Expected Utility

W.D. Smith

GNS Science, Lower Hutt, New Zealand

G.A Vignaux

Victoria University of Wellington, New Zealand

2006 NZSEE Conference

ABSTRACT: The results of earthquake risk assessments should be presented in ways that will help facilitate risk management decisions. So the measures of risk that are chosen need to be those that will assist decision-makers. Annualised Loss may not be the best basis on which risk management decisions can be made. The Conditional Expected Value of the loss, defined for a suitable set of probability ranges, is a promising measure of the risk because it is similar to a scenario loss and can be readily comprehended by decision-makers. Utility Theory provides a further measure by taking account of individuals' perceptions of the severity of losses. It can be combined with the concept of Net Present Value to give an overall measure of the risk in terms of the value judgements of the individual decision-maker. The reduction in risk that would result from proposed mitigation works can be readily assessed, so that the decision-maker who is faced with the costs of mitigation is in a position to assess the benefits.

1 INTRODUCTION

Earthquake risk analysis combines all the detail of established procedures of earthquake hazard assessment with engineering assessments of the vulnerability of assets. It is now a fairly routine procedure to set up source models for point, line and area seismic sources, and to combine these with attenuation models to produce assessments of hazard that are specific to given locations. Source models take into account earthquake mechanisms and recurrence intervals for active faults, and strong motion attenuation functions incorporate site conditions. But while hazard assessment combines source and attenuation modelling, risk assessment goes one step further, to estimate likely losses to structures by modelling their vulnerability. This results in probabilistic estimates of losses, for specific portfolios of assets.

A risk assessment is only done if there is a risk management decision to be made. The purpose of the risk assessment is to produce data that will provide a quantitative basis, as far as is possible, on which that decision can rest. It is incumbent on the risk analyst to be in dialogue with the decision-maker, in order to understand the decisions to be made and the constraints under which the decision-making process must operate, and therefore to present the results of the analysis in the most useful way (National Research Council, 1996).

In the following, several risk measures are presented and discussed. No one measure is adequate for risk management purposes, but together they form a set of measures that can provide a quantitative basis for decision-making.

2 THE EP CURVE

Risk of earthquake-induced damage to an asset such as a building can be expressed as an EP (Exceedance Probability) curve, plotted in Figure 1 in two different formats. The EP curve is widely used in insurance applications. It shows the probability that any given level of loss will be equalled or exceeded, and contains all the information about nearby faults that can affect the site, likely magnitudes and recurrence intervals on these faults, strong motion attenuation, site effects and the vulnerability of the building. The curve also takes account of the aleatory variability in the earthquake process, by representing parameters as distributions. Epistemic uncertainty can be handled by preparing a suite of curves to represent the ranges of the parameters in question, and determining percentiles of confidence.

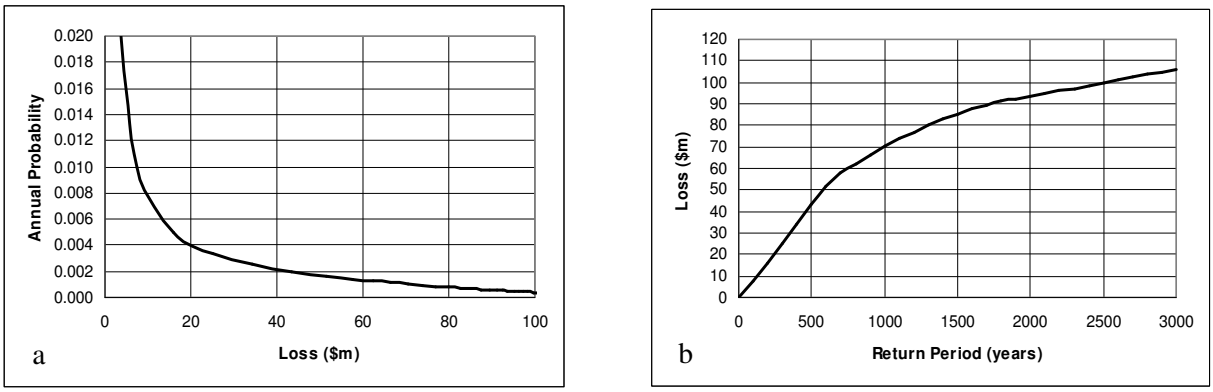

Figure 1. EP curve in the form of (a) probability of exceedance as a function of loss, and (b) event loss as a function of return period, for a portfolio of assets in central New Zealand.

The vulnerability can be modelled by formulations such as in HAZUS (FEMA, 1997), which uses ground motion spectra to predict a deformation state and hence cost of damage. Another approach is currently in use in New Zealand (Smith 2003a) where there is a considerable amount of insurance-derived data in the form of damage ratios related to MM intensities. In either case, combining the vulnerability modelling with the hazard information produces an EP curve. If it is the risk at a single site that is to be determined, the method of Cao et al (1999) may be used to integrate through the frequency-magnitude source functions and the attenuation relation, and apply the vulnerability to establish the EP curve. Wesson et al (2004) have used this technique to obtain the EP curve for damage to housing in Northridge, California. For a geographically distributed portfolio of assets, however, it is necessary to work at the event level, e.g. using a Monte Carlo procedure to set up a synthetic earthquake catalogue and expose each asset to each event, while building up statistics of losses (Smith 2003a).

3 PROBABLE MAXIMUM LOSS

This measure is used widely in insurance. Although in name it seems to refer to some sort of estimate of the maximum loss that can occur, in practice it is often taken to mean the loss that corresponds to some specified mean return period, such as 250 or 500 years. But there is no consistent definition. It is clear from Figure 1b that the loss for these assets can exceed $100 million, but the annual probability that this will occur is very low indeed. So PML is an ill-defined measure, possibly useful for insurance but not so useful for other forms of risk management.

4 ANNUALISED LOSS

This widely used measure is defined as the expected value of the probability density function for the loss. The EP curve can be expressed (Figure 1a) as *P(x)*, i.e. with probability of exceedance as a function of loss. The expected value can be shown to be given by

$$E[x] = \int_0^\infty P(x)dx \tag{1}$$

Equivalently,

$$E[x] = \int_0^1 x(P)dP \tag{2}$$

E[x] is thus the area under the EP curve in Figure 1a. In Equation 2 the cumulative probability function *P(x)* has been replaced by its inverse *x(P).*

But while the annualised loss (*AL*) is used widely in insurance, it is a very limited measure and not always applicable in other areas of risk management. A substantial study of earthquake risk (FEMA, 2001) has estimated annualised loss from earthquake throughout the USA. The expected losses are aggregated on a state by state basis, and expressed as both dollar losses and loss ratios, the latter referring to the loss as a fraction of replacement value. However that study pointed out that parameters other than the annualised loss may be valuable, in particular the annual probability of exceeding a significant threshold of loss (i.e. the EP curve), and that annualised risks may appear small and give the wrong impression of risk due a single event. Even as a tool for ranking mitigation options, the annualised loss is very limited. Kaplan & Garrick (1981) noted that "A single number is not a big enough concept to communicate the idea of risk."

5 CONDITIONAL EXPECTED VALUE OF THE LOSS

Haimes (1998) suggests that other useful statistics are the conditional expected values. He partitions the probability axis and calculates the expected value of the loss, given that it lies within a specified probability range. So for the range P_1 to P_2, where $P_2>P_1$, the conditional expected value is

$$E[x \mid P_1 < P < P_2] = \frac{\int_{x_2}^{x_1} xp(x)dx}{\int_{x_2}^{x_1} p(x)dx} \tag{3}$$

where x_1 is the damage level that corresponds to probability P_1 and x_2 to P_2. The probability density function *p(x)* may not be tractable if *P(x)* is represented only by a few points and not by a continuous curve, but Equation 3 reduces to

$$E[x \mid P_1 < P < P_2] = \frac{\int_{P_1}^{P_2} x(P)dP}{P_2 - P_1} \tag{4}$$

with the inverse function *x(P)* as in Equation 2. The conditional expected values can thus be determined directly from the cumulative probability distribution. Note that Equation 2 is a special case of Equation 4, with P_1=0, P_2=1.

Smith (2004) suggests three probability ranges: 0.032 to 0.32, 0.0032 to 0.032, 0.00032 to 0.0032, referring to these as the 10-year event, 100-year event and the 1000-year event respectively. So they are short-term, medium-term and long-term measures. For assets such as nuclear power plants a 10,000-year event may be necessary. Note that these are not just points on the EP curve; the 100-year event, for instance, is an integrated representation of all losses with annual probabilities between 0.0032 and 0.032.

The attractiveness of the conditional expected value is that it resembles a scenario loss, and as such can be readily understood and used by decision-makers. Its use addresses the FEMA (2001) point that the annualised loss may not give a true picture of the severity of large events.

6 NET PRESENT VALUE OF ALL FUTURE LOSSES

The overall severity of the risk can be measured by discounting future losses at an appropriate rate of interest. The principle here is that a future loss is not as serious as an imminent loss, even if they are of the same monetary value. If the annual discount rate is r, the Net Present Value of a loss L, n years into the future is $L/(1+r)^n$. So a measure of the total risk is obtained by taking the annualised loss AL and accumulating it into the future, discounting for each year. This is a geometric series, whose sum to n terms is

$$NPV(AL) = AL\frac{1+r}{r}\left(1-\frac{1}{(1+r)^n}\right) \tag{5}$$

and the sum to infinity is

$$NPV(AL) = AL\frac{1+r}{r} \tag{6}$$

The choice of the discounting rate is a matter that must be addressed, but a value of 3% is not unreasonable for long-range decisions. At this value the finite sum is very close to the infinite limit after 150 years. The NPV could serve as a risk measure for a decision-maker contemplating mitigation expenditure. Comparison of the cost of the mitigation with the reduction in the NPV provides a measure of the benefit. Hopkins and Stuart (2003) have used this approach in estimating the benefit of strengthening earthquake-risk buildings.

If used on its own, however, its weakness is precisely that addressed by Haimes (1998), who pointed out that the expected value (i.e. annualised loss) is not an adequate measure of the risk. The reason is that the distribution of annual losses is so broad and so skewed that no central measure represents it adequately (Smith, 2003b).

7 UTILITY THEORY

Keeney (1980) has addressed the issue of subjective value judgements in decision-making. "Objective, value-free analysis is undesirable because it simply avoids the problem. What is needed is a logical, systematic analysis that makes the necessary professional and value judgements explicit. The resulting analysis should be responsive to the client's needs and justifiable to the public and the regulatory authorities."

Utility Theory (e.g. Friedman & Savage, 1948) is well established in economics. It examines the subjective value, or utility, that decision-makers assign to gains or losses. Its central tenet is that utility is not generally a linear function of the loss, because most people are risk-averse, and rarely risk-seeking or risk-neutral. The relevance for earthquake risk assessment is that risk management decisions ought to reflect the attitudes that the people who are affected by them have toward the risks and possible losses. Porter et al (2004) have introduced ideas of utility into decision analysis for retail investments in seismic regions.

Raiffa (1968) demonstrates the utility principle by asking which of the following two options is preferred: (A) a $50 gift, or (B) a lottery ticket which yields either zero or $100, with equal probability. Despite the fact that the expected value of Option B is clearly $50, most people select Option A. $100 is better than $50, but it is apparently not twice as good. If we then ask the question "At what value of gift in (A) would the decision-maker be ambivalent about the choice, with (B) unchanged?" the spread of individual choices will be represented by a distribution, but its mean is usually about $35. So the *certainty equivalent* of Option B is about $35. Furthermore, the certainty equivalent tends to scale linearly with the amount of money at stake (Kahneman & Tversky, 1982).

For possible gains x ranging from x_{min} to x_{max}, define the utility function $u(x)$ such that $u(x_{min})=0$ and $u(x_{max})=1$. If at $x = p$ the decision-maker is ambivalent about the choice between options A and B above, then $u(p) = 0.5$. In the above example, $p = 35$ and the behaviour is risk-averse. Function $u(x)$ is a curve that can be defined by the above 3 points, as in Figure 2a where the abscissa is a scaled measure of the gain, in the range [0,1].

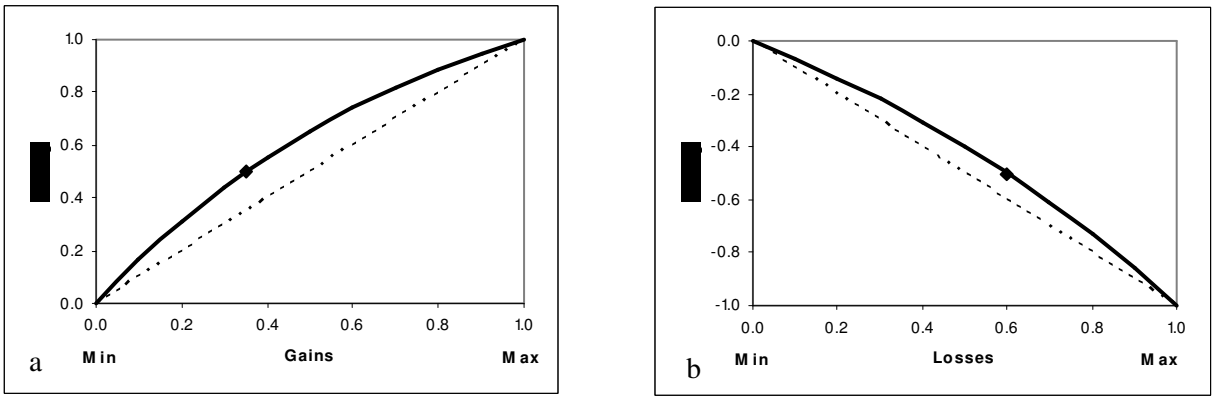

Figure 2. Utility function $u(x)$ for gains (a) and losses (b), compared with the risk-neutral utility (broken line). The risk-averse certainty equivalents of 0.35 for gains and 0.6 for losses are also marked.

There is a similar situation with losses. Ask a decision-maker if, from a planning perspective, a guaranteed loss of $5 million is preferable to the possibilities of zero or $10 million, and he is likely to opt for the $5 million certainty. His certainty equivalent might be $6 million, for instance, in which case the curve would be as in Figure 2b. Linear transformations of utility values are valid (Winston, 2004). For losses it is convenient for $u(x)$ to range from zero (best result) to -1 (worst result). The risk-averse behaviour of the decision-maker is shown by the downward concavity of the curve.

The inverse utility function u^{-1} gives the conversion from a utility measure back to the measure of x. Raiffa (1968) provides a lot more detail.

Utility Theory provides a way of modifying the annualised loss to take account of the risk perception of the decision-maker. If instead of integrating the loss, as in Equation 2, we integrate the utility of that loss, we obtain the expected utility of losses for any one year, i.e.

$$U_A = \int_0^1 u(x(P))dP \tag{7}$$

Then we can use the inverse utility function u^{-1} to obtain a modified AL, which we shall call AL_u. This is a monetary value which takes into account the extent to which the decision-maker is risk-averse.

$$AL_u = u^{-1}(U_A) \tag{8}$$

When the risk analysis is performed by a Monte Carlo procedure (e.g. Smith 2003a), as is necessary if the portfolio of assets at risk is distributed geographically, the estimation of these measures of risk is very straightforward. The analysis is done at event level, i.e. the assets are exposed to a series of earthquakes that represent the likely occurrences over a long period of exposure, such as 100,000 years. Losses are calculated for each event, and it is from these that the EP Curve is developed. The total of all the losses, divided by the length of the exposure period, is the AL. In order to obtain AL_u we apply the utility function $u(x)$ to each loss, sum these and then divide by the number of years. The inverse utility function then provides AL_u as in Equation 8.

Applying the Net Present Value concept to AL_u, using the same summation of the geometric series for the discounting, the sum to infinity becomes

$$NPV(AL_u) = AL_u \frac{1+r}{r} \tag{9}$$

Inherent in this step is the assumption that the utility of future losses is the same as for current, or imminent, losses. We believe this assumption to be reasonable. It enables us to combine Utility Theory and Net Present Value analysis to obtain a risk measure. Thus the utility-adjusted NPV is a measure of the total value of all future losses which represents (a) the decision-maker's perception (utility) of the severity of possible losses and (b) a discounted rate for losses into the future.

8 EXAMPLE

An asset owner has three buildings: two in Wellington with replacement values of $100 million and $200 million, and a third in Palmerston North (125 km away) with a value of $150 million. The EP curve is shown in Figure 1, as calculated using the seismicity model of Stirling et al (2002) and the Monte Carlo procedure of Smith (2003a). A simulation of 100,000 years was used, and typical building vulnerability properties were assumed. It is clear from Figure 1 that a loss of $40 million or more has a return period of just less than 500 years, a loss of $100 million or more has a return period of 2500 years, etc. The annualised loss for this portfolio is just $360,000, which illustrates the point made by FEMA (2001) that annualised loss may not be an indicative measure of event losses. The maximum event loss during that simulation was actually $160 million.

The conditional expected values of loss are as follows:

10-year event	$0.27m
100-year event	$4.9m
1000-year event	$55m

These may be regarded as scenario losses for events at these three return periods.

The Net Present Value of future losses, using the above annualised loss and a discounting rate of 3%, is $12.4 million when summed to infinity. The sum to 50 years is $9.6 million. So the cost-effectiveness of a mitigation programme that would prevent future losses could be assessed from these values.

Further expenditure could however be justified from Utility Theory. A typical utility function for x in the range [0,1] is given by

$$u(x) = a(1 - e^{bx}) \tag{10}$$

If we assume that the decision-maker has this utility function and that the certainty equivalent for losses is 0.6 of the maximum, parameters a and b are 0.784 and 0.822 respectively. The utility-adjusted NPV turns out to be $13.8 million ($10.7 million for 50 years). However if the decision-maker is more risk-averse, so that his/her certainty equivalent is 0.8 of the maximum, the utility-adjusted NPV is $23.6 million ($18.2 for 50 years). In this case parameters a and b are 0.039 and 3.281. The decision-maker should feel comfortable about this level of expenditure, because of the benefit in preventing the large losses.

In practice, a mitigation programme will not reduce all losses to zero. The analysis procedure is simply to model the risk as it would be under the mitigation proposal and find the reduction in the various measures of risk, which can then provide a basis for assessing the advantages of committing the expenditure.

9 CONCLUSIONS

No single measure of risk will be adequate for making well-informed decisions in risk management. In particular, the Annualised Loss is a poor indicator of likely losses. The Conditional Expected Values of the loss, expressed in terms comparable to scenario losses, are useful for conveying to decision-makers the likely extent of losses. The Net Present Value gives a measure of total future losses, taking into account appropriate discounting for events far into the future. Utility Theory introduces the subjective value judgements of the decision-maker, to indicate the level of expenditure with which he should be comfortable in order to mitigate the risk.

ACKNOWLEDGEMENTS

Comments and suggestions from David Rhoades and William Power have greatly improved the clarity of this paper.

REFERENCES:

Cao, T., Petersen, M.D., Cramer, C.H., Toppozada, T.R., Reichle, M.S. and Davis, J.F. 1999. The calculation of expected loss using probabilistic seismic hazard. *Bulletin of the Seismological Society of America* **89**, 867-876.

Federal Emergency Management Agency (FEMA), 1997. *HAZUS*. National Institute of Building Services Documents 5200 to 5203, Federal Emergency Management Agency, USA.

Federal Emergency Management Agency (FEMA), 2001. *HAZUS®99 Estimated Annualized Earthquake Losses for the United States*, FEMA 366, Washington, DC: US Government Printing Office, 33 pp.

Friedman, M. & L.J. Savage 1948. The utility analysis of choices involving risk. *Journal of Political Economy* LVI, No. 4, 279-304.

Haimes, Y.Y. 1998. *Risk Modeling, Assessment and Management.* John Wiley & Sons, New York.

Hopkins, David C. & George Stuart 2003. Strengthening existing New Zealand buildings for earthquake: analysis of cost benefit using annual probabilities. *Proceedings, 2003 Pacific Conference on Earthquake Engineering,* Paper No. 72.

Kahneman, D. & A. Tversky 1982. The psychology of preferences. *Scientific American* 246, 136-142.

Kaplan, S. & Garrick, J.B. 1981. On the quantitative definition of risk. *Journal of Risk Analysis* 1(1), 11-27.

Keeney, R.L. 1980. *Siting Energy Facilities*. Academic Press, 413 pp.

Keeney, R.L. & H. Raiffa 1976. *Decisions with multiple objectives: preferences and value tradeoffs.* John Wiley & Sons, 569 pp.

National Research Council, Committee on Risk Characterization 1996. *Understanding Risk: Informing Decisions in a Democratic Society,* National Academy Press: Washington, DC.

Porter, K.A.; Beck, J.L. & R. Shaikhutdinov 2004. Simplified estimation of economic seismic risk for buildings. *Earthquake Spectra* 20, 1239-1263.

Raiffa, H. 1968. *Decision Analysis*, Addison-Wesley 1970, 309 pp.

Smith, W.D. 2003a. Earthquake hazard and risk assessment in New Zealand by Monte Carlo methods. *Seismological Research Letters*, **74**, 298-304.

Smith, W.D. 2003b. Criteria for strengthening buildings: cost-benefit analysis is misleading. *Bulletin of the NZ Society for Earthquake Engineering* 36, 260-262.

Smith, W.D. 2004. The Decision Support Model for risk management: a conceptual approach. *Bulletin of the New Zealand Society for Earthquake Engineering*, 37, 149-154.

Stirling, M.W., McVerry, G.H. & K.R. Berryman 2002. A new seismic hazard model for New Zealand. *Bulletin of the Seismological Society of America* 92,1878-1903.

Wesson, R.L, Perkins, D.M., Leyendecker, E.V., Roth, R.J., and Petersen, M.D. 2004. Losses to single-family housing from ground motions in the 1994 Northridge, California, earthquake. *Earthquake Spectra* **20**, 1021-1045.

Winston, W.L. 2004. *Operations Research, Applications and Algorithms*. Duxbury Press, 4th edition.