

On the flexural ductility of very lightly reinforced concrete sections

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ABSTRACT: If a concrete member is very lightly reinforced the ultimate moment capacity may be less than the bending moment required to crack the member. In these circumstances only one crack (or possibly a construction joint) may open at the highly stressed part of the member, and strains in the reinforcing will be concentrated at that location. If the steel yielding is concentrated at one location rather than distributed over a plastic hinge length, as in a normally reinforced member, the strains are much higher and, with low-cycle fatigue effects, could lead to fracture of the reinforcing steel.

In this paper a method is proposed for estimating the maximum displacements that members of this type can sustain without steel fracture, and the process is illustrated by applying it to a very lightly reinforced concrete spillway pier.

The maximum rotation at the cracked section is controlled by the maximum yield strain that the reinforcement can sustain without fracture, and how far the yield strains can penetrate into the concrete above and below the crack (i.e. yield penetration). The maximum yield strain that the bars can sustain depends on the low-cycle fatigue capacity of the bars. The yield penetration above and below the crack depends on the bond-slip relationship of the reinforcing bars.

The method of assessing the displacement capacity is based on the investigations by others into the low cycle, large strain fatigue performance of reinforcing steel, and into the bond slip and yield penetration of reinforcement at crack locations.

It is believed that the method provides a useful tool for assessing the ductility of very lightly reinforced concrete sections.

1 INTRODUCTION

Design standards such as the New Zealand Concrete standard specify design criteria that aim to provide reinforced concrete members with adequate ductility to withstand the forces and deformations imposed by earthquakes. However, it is recognised that members that do not comply with these design standards may still have sufficient ductility to withstand significant earthquake loads, and methodologies have been developed to assess the earthquake performance of such members (e.g. Priestley, 1995).

An example of a “non-complying” member that is encountered, especially in buildings designed to earlier codes, is a member that is so lightly reinforced that its ultimate moment capacity is less than the bending moment required to crack the member. In these circumstances only one crack (or possibly a construction joint) may open at the highly stressed part of the member, and strains in the reinforcing will be concentrated at that location. These types of members are more common in existing structures designed to less demanding seismic requirements.

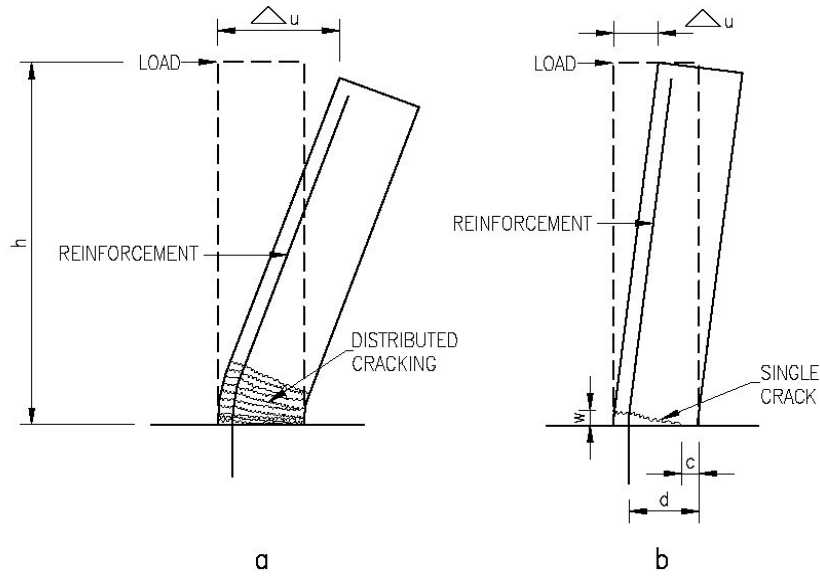


Figure 1: Post elastic deformation of (a) a normally reinforced cantilever column and (b) a very lightly reinforced cantilever column

If the steel yielding is concentrated at one crack location, as illustrated in Figure 1(b), rather than distributed over a plastic hinge length, as in a “normally” reinforced member (Figure 1(a)), the strain in the reinforcement for a given displacement is much higher and could lead to fracture of the reinforcing steel. The ultimate displacement capacity (Δ_u) and hence the ductility of the very lightly reinforced column in Figure 1(b) will therefore be less than that of the “normally” reinforced column.

Methodologies have been published for assessing the ultimate flexural displacement capacity of “normally” reinforced concrete beams, columns and walls (e.g. NZSEE 2002). The authors are not aware that methods for estimating the ultimate flexural displacement capacity of very lightly reinforced sections have previously been published, or that cyclic testing of such structures has been undertaken. This paper outlines the development of an assessment procedure and illustrates it by applying it to a lightly reinforced spillway pier.

2 PARAMETERS CONTROLLING MAXIMUM DISPLACEMENT

In Figure 1, the ultimate displacement $\Delta_u = \Delta_y + \Delta_p$, where Δ_y is the elastic displacement and Δ_p is the displacement due to plastic or post-yield strains in the structure. In the case of a lightly reinforced column, (Figure 1(b)) the plastic component of Δ_u is given by Equation (1).

$$\Delta_p = w h / (d - c) \quad (1)$$

Where w = crack width, c = neutral axis depth, h is the height of the column and d = distance between the compression face and the most remote tension reinforcement.

If w is known, then Δ_p and Δ_u can readily be calculated.

The maximum crack width is controlled by the maximum yield strain that the reinforcement can sustain without fracture and how far the yield strains can penetrate into the concrete above and below the crack (i.e. “yield penetration”).

The maximum yield strain that the bars can sustain depends on the low-cycle fatigue capacity of the bars. The yield penetration above and below the crack depends on the bond-slip relationship of the reinforcing bars.

These parameters will now be examined in more detail.

3 MAXIMUM LOW CYCLE FATIGUE STRAIN

Mander (1994) experimentally evaluated the low-cycle fatigue behaviour of ASTM A615 grade 40 ordinary deformed-steel reinforcing bars with a specified minimum yield strength of 275 MPa, and ASTM A722 high-strength prestressing thread bars with a specified ultimate strength of 1,083 MPa under axial-strain-controlled reversed cyclic tests with strain amplitudes ranging from yield to 6%. The cyclic loads cause cumulative damage and finally fracture. The behaviour of the specimens tested by Mander conformed well to the commonly used strain-life models for low-cycle fatigue. One of these models, the Koh-Stephens model (Koh & Stephens 1991) has been adopted for the proposed assessment procedure in this study (see Equation (2)).

$$\Delta\varepsilon = 0.159(2N_f)^{-0.448} \quad (2)$$

Where $\Delta\varepsilon$ = the total strain range; N_f = cycles to failure for a fixed strain range of $\Delta\varepsilon$.

For a member responding to an earthquake, the strain range between reversals in the response will vary. It is proposed that the damage accumulated is independent of the order in which the cycles of strain occur and that damage can be measured in terms of a scalar damage indicator D where:

$$D = \sum_i \frac{n_i}{N_f(\Delta\varepsilon_i)} \quad (3)$$

Where D = damage indicator (= 1.0 at failure); n_i = number of strain cycles with strain range $\Delta\varepsilon_i$ and $N_f(\Delta\varepsilon_i)$ = number of cycles to failure at strain range $\Delta\varepsilon_i$. Equation 2 is similar to the well known ‘‘Palmgren-Minor’’ sum that is used to quantify fatigue damage from stress cycles. Others (e.g. Mander, 1994) have suggested that the cumulative effect of cycles of varying strain range should be combined by considering the total plastic strain energy absorbed by the reinforcement. However this approach does not appear to have been verified with test data and the Palmgren-Minor sum procedure is just as reasonable and easier to apply in practice.

4 BOND-SLIP AND YIELD PENETRATION

Engstrom (1992) has studied the effect of reinforcement yielding on the bond-slip relationship of deformed reinforcing bars. Based on this work he derived the following expression for the maximum crack opening expected at first yield of a reinforcing bar in a lightly reinforced section:

$$w_y = 0.576 \left[\frac{d_b f_y^2}{\tau_{\max} E} \right]^{0.714} + \frac{f_y}{E} 4d_b \quad (4)$$

He also derived an equation for the additional crack width opening after yielding as:

$$w_p = \varepsilon_u \left[\frac{f_y \left[\frac{f_{su}}{f_y} - 1 \right]}{0.28\tau_{\max}} \right] \frac{d_b}{4} \quad (5)$$

where: d_b = the bar diameter; f_y = yield strength of the bars; f_{su} = ultimate strength of bars; E = elastic modulus of bars; ε_u = maximum strain that the steel can withstand without fracture; τ_{\max} = peak bond stress on the bond-slip curve prior to yielding.

Engstrom found that if the reinforcement in tension has a large amount of cover ($>6d_b$) the concrete will shear off between the ribs so that the bar can pull out of the concrete through a hole with a diameter equal to the bar diameter plus twice the rib depth. However, if the reinforcement yields before the bond stresses reach the pullout value the yielding ribs cannot sustain the bond forces and

there is a premature loss of bond capacity. Other researchers (Harajli et al 2002, Homayoun et al 1996) have shown that when the cover concrete splits (or there is splitting between adjacent bars) the bond-slip relationship deteriorates in a similar way to that caused by yielding. Splitting failures become more likely when the clear cover is less than 5 to 6 times the bar diameter, d_b , or the clear spacing between the bars is less than 10 to $12d_b$. Engstrom (1998) also carried out testing on yielding bars that only had clear cover of $1.0d_b$. These tests indicate that the effects of yielding and splitting are to some extent additive and that the bond-slip relationship is more variable when splitting of the cover concrete occurs.

The testing indicates that the maximum bond stress, τ_{max} , is a function of the compressive strength of the concrete. For normal strength concrete (strength, $f_c' = 20$ to 30 MPa) the upper limit of the maximum bond stress given by Engstrom was $\tau_{max} = 2.5(f_c')^{1/2}$. For high strength concrete (strength, $f_c' = 100$ to 115 MPa) the upper limit of the maximum bond stress, τ_{max} , was $= 0.45 f_c'$ when no cover concrete splitting occurred. However, for tests with bar clear cover of only $1.0d_b$ when splitting failures are expected, the upper limit of the maximum bond stress, τ_{max} , was reduced to $2.5(f_c')^{1/2}$ as for normal strength concrete. These are upper limit values for the maximum bond stress. The tests indicated that the maximum bond stress of the bond-slip curve lies between these upper limit values and a lower limit equal to half the upper limit.

The extent of yield penetration either side of the crack is strongly dependant on the ratio of the ultimate strength of the reinforcing bars to their yield strength rather than the actual value of the bars ultimate strength. It can be seen from equation (3) that if this ratio is 1.0 there will be no yield penetration and no additional crack opening due to bar yielding. The maximum depth of yield penetration will occur when the bars reach their ultimate strength, f_{su} . However, in the proposed assessment procedure the effective length of yielding bar implied by equation (5) is assumed to remain constant after first bar yielding.

Equations (4) and (5) were derived from monotonic tests where the bars were loaded in a single cycle to failure. Under cyclic loading into the strain-hardening zone the steel tends to lose its yield point and “soften”. This increases the effective ratio of ultimate to yield strength (f_{su}/f_y) and the cyclic loading would be expected to reduce the bond forces along the bar. Both these factors would increase the yield penetration along the bar and increase the crack opening capacity available.

5 APPLICATION OF THE METHODOLOGY

The methodology reported here was developed as part of a study into the earthquake performance of dam spillway piers that were constructed in the late 1960s.

The piers are 2440mm thick and are reinforced with 32mm diameter deformed-steel bars at 229mm centres in each face. Cover to the reinforcement is 125mm. The construction specification required the steel to have a minimum f_y of 230 MPa. However, testing has indicated that this steel probably had a lower 5% characteristic strength of about 280 MPa. Opus Central Laboratories have recently tested some 32 mm bars taken from the Newmarket Viaduct that was built circa 1963-64. These bars had a ratio of ultimate to yield strength (f_{su}/f_y) of 1.56. As part of this investigation Pacific Steel advised that this ratio is typically 1.50 for post 1950 D32 bars produced by Pacific Steel. A ratio of 1.5 was adopted for this assessment.

The mean concrete compressive strength is estimated from compression tests on core samples to be 60 MPa and allows for the increased concrete strength with aging. As splitting type bond failure is expected for the spillway pier bars the maximum bond stress, τ_{max} , in equations (4) and (5) was assumed to be $2.5(f_c')^{1/2}$ (19.5 MPa) rather than the higher value of $0.45 f_c'$ (27 MPa) expected when no splitting of the cover concrete occurs. However, the additive effect of concrete cover splitting and bar yielding on the overall bond-slip relationship (other than maximum bond stress value) was conservatively ignored.

The bending moments required to (1) crack the pier and (2) yield the reinforcement are estimated to be 4600 kN-m and 2100 kN-m respectively. All post-yield deformation will therefore be concentrated at

a single crack (or construction joint) location near to the base of the wall, because the cracked strength is insufficient to spread the cracking and hence the reinforcement yielding up the wall. While some spalling of concrete is expected due to high localized concrete compression strains when the crack is opened to its full extent, this is not expected to reduce the restraint provided to the bars by the concrete sufficiently to allow buckling of the bars. The shear stress in the pier at flexural capacity is very low, so it will not fail in shear.

The values outlined above were substituted into Equations 4 and 5 to calculate the crack widths for selected strain ranges. The displacement at the top of the pier was then calculated from equation (1). These values are shown in Table 1.

The time-history of the displacement of the top of the model pier when subjected to an earthquake record that represented the dam safety evaluation earthquake is shown in Figure (2).

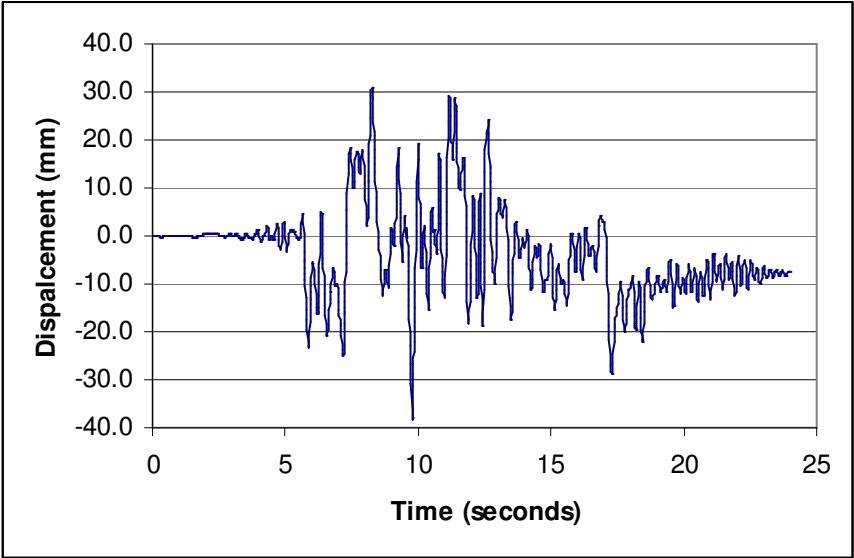


Figure 2: Displacement time-history at top of pier

The numbers of cycles, n_i in each of the strain ranges for the most severe loading condition (negative displacement) are shown in Table 1. The “rainflow” algorithm was used to evaluate n_i from Figure 2. D is the cumulative damage indicator from equation (3). Since D is less than 1.0, the reinforcement is not expected to fracture due to low cycle fatigue.

Table 1: Cumulative damage to pier subjected to earthquake time-history

Strain Range	Cycles to Failure	Total Crack Width (mm)	Wall Displ. (mm)	Strain range cycles	Cumul. Damage
$\Delta\epsilon_i$	$N_f(\Delta\epsilon_i)$	w	Δ_u	n_i	$n_i/N_f(\Delta\epsilon_i)$
	eqn. 2	eqn. 4+5			eqn. 3
0.05	6.5	10.9	52	0	0.00
0.04	11	8.7	41	1	0.09
0.03	20	6.8	32	1	0.05
0.02	50	4.7	22	12	0.24
0.01	200	2.8	13	42	0.21
				D = Σ =	0.59

The effective length of yielding bar in this example, $w_p / \epsilon_u = 205\text{mm}$, seems intuitively to be of the correct order.

It is apparent from examination of the numbers of cycles to failure (N_f) in Table 1 that strain ranges, $\Delta\varepsilon$, that are less than 1.0% can be neglected as they will not make a significant contribution to fatigue damage for the normal types of earthquake motions. It is also apparent that it should be conservative to adopt a maximum strain of about 5% to assess the flexural ductility of lightly reinforced structures, given that there are normally only a few cycles of response that are near to the peak displacement response, with other displacement cycles being much less than the peak. The displacement capacity of the structure can then be assessed on the basis of this 5% maximum strain without the need for time-history analyses. However further parametric studies are required to confirm that this is a safe limiting strain to adopt under all conditions. The number of cycles of strong response will obviously have a major influence on the fatigue damage.

6 CONCLUSIONS

A method has been developed for assessing the flexural ductility of very lightly reinforced concrete sections that are expected to have reinforcement yielding at a single crack in the plastic hinge zone. The procedure is simple and appears to be suitable for design office use. It makes use of research by others into the low-cycle fatigue performance of reinforcing bars, and the penetration of reinforcing bar yield strains into concrete adjacent to the crack.

It includes a procedure for assessing the cumulative fatigue damage from the results of earthquake time-history analyses, but it is also proposed that the maximum displacement capacity can be calculated from equivalent static or modal response methods if the maximum strain range is limited to 5%.

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