A time of recurrence model for large earthquakes

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ABSTRACT: We have developed a new type of model for the recurrence time between earthquakes greater than some (large) reference magnitude e.g. M 7. The model is the sum of two parts, representing 'aftershocks' and 'background' earthquakes. The model allows ready computation of quantities of interest in seismic hazard. In particular it allows a calculation of the distribution of time of occurrence to the next (large) earthquake in a region given any elapsed time since the last. The model is characterised by four parameters, of which two are critical: the relative weights of the two parts and a time constant for the 'background'. It appears that 30 - 45% of M ≥ 7 earthquakes can be regarded as aftershocks, the balance being 'new' earthquakes. The model fits quite well the time between earthquakes of M ≥ 7 in New Zealand since 1840. However, comparison with similar global models suggests that some M ≥ 7 aftershocks are missing from the NZ catalogue. Conditional probability curves show that, in New Zealand, the probability of an M 7 earthquake following an M 7 earthquake is several times greater than predicted by current Poisson models for intervals up to about three years following the first earthquake.

1 INTRODUCTION - CURRENT APPROACHES TO MODELLING EARTHQUAKE RECURRENTNESS

Seismic hazard models have application in structural design, insurance and emergency management. One of their components is a model for the occurrence of earthquakes. Typically this includes recurrence of earthquakes on known active faults and a 'background' seismicity model to account for earthquakes too small to produce surface faulting and for larger earthquakes that may occur at locations other than known active faults (e.g. Stirling et al., 2002). Such seismicity models invariably assume that 'background' or 'new' earthquakes occur randomly in time. Empirical studies show that indeed when aftershocks are removed from earthquake catalogues the remaining events are well modelled by a random, Poisson process (Gardner and Knopoff, 1974, Smith, 1994). Why this should be so is an unresolved fundamental question about earthquake genesis that is well outside the scope of this study.

For a region where the seismicity is spatially homogeneous, a Poisson process is described by a single parameter, the mean rate of occurrence of earthquakes. It is assumed that all earthquakes above a nominated level (completeness level), e.g. M 5, have been recorded. The distribution of magnitudes above this level is modelled by the Gutenberg-Richter distribution (Kagan, 1999; Christophersen, 2000), or a modified version of it to allow for the presence of active faults (Stirling et al. 2002). The mean value is determined from the catalogue of earthquakes for the region when aftershocks have been removed. The removal of aftershocks is termed declustering (we use the term aftershock to mean any earthquake that occurs as a consequence of a prior earthquake). If the seismicity is inhomogeneous, it may be modelled with kernel functions (Stock and Smith, 2002a,b).

A problem with this approach to modelling seismicity arises from the removal of aftershocks. A number of algorithms for this exist (e.g. Gardner and Knopoff, 1974; Davis and Frohlich, 1991). However, it is now known that aftershocks, in a generalised sense, may occur years after the initiating mainshock (Kagan and Jackson, 1999; Parsons, 2002) and so a significant fraction of the catalogue time may be
represented by 'aftershock time' during which no new, independent earthquakes are assumed to occur. However, if new earthquakes are truly random, then they may occur at any time, including during the period of the aftershocks. The result of this difficulty is that estimates of the mean rate of new earthquakes from declustered catalogues tend to underestimate the true rate of occurrence of new events (e.g. Tormann et al., 2004).

An alternative approach that avoids this difficulty is to simultaneously model the aftershocks and the new earthquakes. A common method for this is the Epidemic-Type Aftershock Sequence (ETAS) model (Ogata, 1988, 1998; Console et al., 2003) in which every catalogue earthquake is considered to be a potential generator of aftershocks (hence 'epidemic').

In these models (e.g. Ogata 1988) the rate of occurrence of earthquakes per unit area at any location, \( \lambda(t) \), is written as

\[
\lambda(t) = \lambda_a(t) + \lambda_c(t)
\]

where \( \lambda_a(t) \) is the contribution from aftershocks and other triggered earthquakes, and \( \lambda_c(t) \) is contribution from 'new' independent events. This is the approach we take here. These models have become very sophisticated, multi-parameter models (e.g. Console et al., 2003). The models are fitted to the data by the method of maximum likelihood (e.g. Ogata, 1988; Severini, 2000). These many-parameter models create another problem: a large number of data are required to give a good fit to the model (i.e. one where the parameter uncertainties are small). Since the catalogue must be complete, and completeness magnitudes increase going backwards in time, a trade-off has to be made between the duration of the catalogue being analysed and the completeness level and consequent quantity of data.

2 A NEW APPROACH

As an initial step, we have developed a very much simpler model that has just two critical parameters. Accordingly it may be usefully fitted to catalogues very much smaller than usually required for ETAS models, such as high completeness magnitude, historical catalogues like the New Zealand catalogue of \( M \geq 7 \) earthquakes since 1840.

This model was developed using a global catalogue of earthquakes of \( M \geq 6.5 \) during the time 1985-2000. Details may be found in Christophersen and Smith (2004) – hereafter C&S. The model is a probability density function for the time \( t \) to the next earthquake, termed the inter-event time:

\[
f(t) = w_1 f_0 (1 - \exp(-t/t_1)) \exp(-t/t_1) / t + w_2 (1 / t_0) \exp(-t/t_0) \]

The parameters of the model are \( w_1, t_0, t_1 \) and \( t_0 \). \( f_0 \) is an analytically calculable normalising constant so that \( w_1 + w_2 = 1 \). The model is insensitive to \( t_0 \) which may be set to a small value e.g. 0.01 or 0.001 days, and it is relatively insensitive to \( t_1 \) which may be set equal to \( t_0 \). Discussion of these sensitivities is given in C&S. Thus the model has two effective parameters: \( t_0 \) which is the time constant for the occurrence of 'new' earthquakes, and \( w_1 \) which is the fraction of all earthquakes that are aftershocks.

The model applies to all earthquakes above a specified magnitude (e.g. \( M \geq 6.5 \) or \( M \geq 7 \)) within a circular region whose size depends on the magnitude \( M \) of largest earthquake in the region. The circle radius \( d \) is given by:

\[
d = 4 \sqrt{ \left( 10^{(M-3.39) / 10} \right)}
\]

(see C&S and Christophersen 2000). Thus for e.g. \( M = 8 \) the radius is 455 km. We term the seismicity in such a region a supercluster. An earthquake is constrained to belong to only one supercluster. Superclusters constitute a simple way of limiting the area influenced by the occurrence of any earthquake. It has the acknowledged weakness however that all earthquakes in a supercluster are consid-
erred to have the same extent of spatial influence, whereas this should evidently scale with their magnitude. Development of a spatial model with magnitude scaling is ongoing work.

Figure 1 shows the fit of the model to the dataset, which contains 341 earthquakes. The comparison is between the probability distribution function \( F(t) \) derived from \( f(t) \):

\[
F(t) = \int_{0}^{t} f(\Delta) \, d\Delta
\]

and the empirical distribution of the data. A best-fitting Poisson (random) model is also shown for comparison. The Poisson model fits badly during the time when aftershocks are frequent, but the three curves converge at times greater than about 300 days.

### 3. FEATURES OF THE MODEL

Before we consider application to the New Zealand dataset, we briefly describe some features of the model and its results, some of which are quite novel. See C&S for details.

- The form of the \( w_1 \) part of the model (equation 2) is rather different from that usually adopted (e.g. Console et al. 2003). Our modifications have been made because previously proposed models are non-physical and were intractable analytically (see Sornette and Knopoff 1997).

- Before fitting of the model to the data by the method of maximum likelihood, a correction was applied for the finite duration of the catalogue. The model presumes inter-event times could be of any duration, but the catalogue is limited to 16 years.

- Specific parameter values and 95% confidence intervals are \( w_1 = 0.45 \pm 0.1 \) and \( t_0 = 870 \pm 200 \) days. \( t_s \) and \( t_1 \) were set equal to 0.001 day and \( t_0 \) respectively. This means that for this dataset approximately 45\% of earthquakes of \( M \geq 6.5 \) in the superclusters were aftershocks.

- The model appears to produce consistent results for different specified minimum magnitudes. However, as the minimum magnitude is increased the number of data diminishes and so the parameter uncertainty increases. Nonetheless, the calculated value of \( w_1 \) for the global catalogue of \( M \geq 7 \) earthquakes 1977-2000 is 0.3 (\( \pm 0.11 \)), suggesting that the fraction of earthquakes with magnitudes \( \geq M \) that are aftershocks may diminish as \( M \) increases.
• The functional form of the model is such that the expected number (mean) of the distribution and the distribution function $F(t)$ (equation 3) are analytically calculable. For the fitted data-set, the observed mean time between earthquakes was 515 days; the value calculated from the model was 507 days.

• Also, and importantly, the model can be used to calculate the probability distribution of times to the next earthquake given a time that has elapsed since the last one. This stands in distinction to the Poisson (random) model which has no memory: the Poisson distribution does not change with time since the last event. The conditional distribution function for an elapsed time $t_L$ is:

$$F(t|t_L) = \frac{F(t + t_L) - F(t_L)}{1 - F(t_L)}$$  \hspace{1cm} (5)

where $F(t)$ is as given by equation 4. Note that for $t_L = 0$ this reverts to being $F(t)$.

Table 1: Earthquakes of $M \geq 7$ in central New Zealand (Gisborne to Arthur's Pass) since 1840 (after Smith, 1994).

<table>
<thead>
<tr>
<th>No.</th>
<th>Name/Region</th>
<th>Year</th>
<th>Month</th>
<th>Day</th>
<th>Time interval (days)</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Distance from 1855 (km)</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Wanganui</td>
<td>1843</td>
<td>7</td>
<td>8</td>
<td>-39.9</td>
<td>175.0</td>
<td>166.7</td>
<td>7.5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Marlborough</td>
<td>1848</td>
<td>10</td>
<td>15</td>
<td>1923.3</td>
<td>-41.5</td>
<td>173.8</td>
<td>100.5</td>
<td>7.1</td>
</tr>
<tr>
<td>3</td>
<td>Wairarapa</td>
<td>1855</td>
<td>1</td>
<td>23</td>
<td>2294.8</td>
<td>-41.4</td>
<td>175.0</td>
<td>0.0</td>
<td>8.2</td>
</tr>
<tr>
<td>4</td>
<td>Hawkes Bay</td>
<td>1863</td>
<td>2</td>
<td>22</td>
<td>2951.0</td>
<td>-40.0</td>
<td>176.5</td>
<td>200.4</td>
<td>7.5</td>
</tr>
<tr>
<td>5</td>
<td>Cape Farewell</td>
<td>1868</td>
<td>10</td>
<td>18</td>
<td>2062.3</td>
<td>-40.0</td>
<td>173.0</td>
<td>229.3</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>N. Canterbury/ Glenn Wye</td>
<td>1888</td>
<td>8</td>
<td>31</td>
<td>7258.0</td>
<td>-42.6</td>
<td>172.3</td>
<td>259.7</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>Nelson</td>
<td>1897</td>
<td>12</td>
<td>7</td>
<td>3383.3</td>
<td>-40.0</td>
<td>175.0</td>
<td>155.5</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>Arthur's Pass</td>
<td>1929</td>
<td>3</td>
<td>9</td>
<td>11420.0</td>
<td>-42.8</td>
<td>171.9</td>
<td>296.5</td>
<td>7.1</td>
</tr>
<tr>
<td>9</td>
<td>Buller</td>
<td>1929</td>
<td>6</td>
<td>16</td>
<td>97.0</td>
<td>-41.7</td>
<td>172.2</td>
<td>235.2</td>
<td>7.8</td>
</tr>
<tr>
<td>10</td>
<td>Hawke's Bay</td>
<td>1931</td>
<td>2</td>
<td>2</td>
<td>596.5</td>
<td>-39.3</td>
<td>177.0</td>
<td>288.3</td>
<td>7.8</td>
</tr>
<tr>
<td>11</td>
<td>Hawke's Bay (aftershock)</td>
<td>1931</td>
<td>2</td>
<td>13</td>
<td>11.0</td>
<td>-39.5</td>
<td>177.5</td>
<td>298.7</td>
<td>7.3</td>
</tr>
<tr>
<td>12</td>
<td>Pahiatua</td>
<td>1934</td>
<td>3</td>
<td>5</td>
<td>1117.8</td>
<td>-40.6</td>
<td>176.3</td>
<td>143.6</td>
<td>7.6</td>
</tr>
<tr>
<td>13</td>
<td>Wairarapa I</td>
<td>1942</td>
<td>6</td>
<td>24</td>
<td>3031.0</td>
<td>-40.9</td>
<td>175.9</td>
<td>93.6</td>
<td>7.2</td>
</tr>
<tr>
<td>14</td>
<td>Wairarapa II</td>
<td>1942</td>
<td>8</td>
<td>1</td>
<td>37.0</td>
<td>-41.0</td>
<td>175.8</td>
<td>80.3</td>
<td>7.0</td>
</tr>
<tr>
<td>15</td>
<td>Inangahua</td>
<td>1968</td>
<td>5</td>
<td>23</td>
<td>9428.5</td>
<td>-41.8</td>
<td>172.0</td>
<td>249.2</td>
<td>7.4</td>
</tr>
</tbody>
</table>
4. APPLICATION TO NEW ZEALAND

Central New Zealand, from northern Hawke's Bay to Arthur's pass, has been seismically active with large earthquakes since European times (Table 1). We have applied our method to this dataset, although there are too few earthquakes (15) to obtain really satisfactory accuracies for the parameters of the model. The largest earthquake in this set is the M ~ 8.2 Wairarapa earthquake of 1855 (Grapes and Downes, 1997). Accordingly, the supercluster for this event would have a radius of 570 km (equation 3) centred on the 1855 event, and it would encompass the area just described, but not southwest of about 44 S, 169 E or northeast of about 37.5 S, 179.5E. The 1995 M 7.0 event off East Cape is just at the margin of the supercluster and is not included (no other event in Table 4 is in fact more than 300km away from the 1855 one).

Figure 2: Results of fitting the model equation (2) with $t_0 = 0.001$ day and $t_1 = t_0$, to the New Zealand inter-event times, Table 1. The curves are cumulative distribution functions for inter-event times: data (stepped), our model (solid) and Poisson model (dashed) on linear (a) and log(time) scales (b).

Fitting equation 2 to these 14 inter-event times by maximum likelihood yielded the parameter values $w_1 = 0.13$ and $t_0 = 3500$ days (9.6 years). Figure 2 shows the fit of the model against the data in comparison with a Poisson model. Notwithstanding large uncertainties in the parameters, the $w_1$ parameter is much smaller than that found from the global datasets, where $w_1 \sim 0.3 \sim 0.45$.

Either the lower value is (approximately) correct for New Zealand, which would imply a magnitude-frequency relationship in New Zealand rather different from the global average or, more likely, that the New Zealand catalogue of M ≥ 7 events is not complete; in particular, some short inter-event time M 7 earthquakes have been missed.

Since about 1/3 of the catalogue is pre-instrumental, this is not as implausible as it may seem. The 1848 Marlborough earthquake was part of a complex sequence in which some aftershocks were large and had a pattern of felt reports suggesting a spread of locations (Grapes et al., 2004). It is possible that at least one additional M 7 earthquake occurred in the 1848 sequence. Similarly in the aftermath of the 1855 earthquake a large (M 7) aftershock would not have gone unnoticed but it would have
been difficult to assess for size. The occurrence of one or more M 7 aftershocks in 1855 is thus plausible.

Figure 3 shows the effect of modelling a synthetic dataset in which three additional times of 1, 2 and 4 days have been added to the New Zealand M 7 dataset. The substantial effect is that \( w_1 \) changes from 0.13 to about 0.36 which is about the global average. At the same time, \( t_0 \) increases (insignificantly) from 3500 to 3800 days.

5. CONDITIONAL INTEREVENT TIME PROBABILITIES: AN EXAMPLE

For this example we use the artificial model described above. We use this rather than the model for Table 1 data because we think the synthetic model is closer to global models and, accordingly, is more likely to represent the actual situation in New Zealand than the Table 1 model. Nonetheless, it is a synthetic model. The purpose of the example is to demonstrate the behaviour and application of the model, not to predict conditional inter-event time probabilities for New Zealand.

Figs. 16-1 - 3 show how the probability distribution for inter-event times changes with different elapsed times since the last earthquake. In each case, a time \( t_L \) has elapsed since an \( M \geq 7 \) earthquake, and the distributions give the probabilities for the times to the next \( M \geq 7 \) event. To make the comparisons easier, figure b in each case gives the ratio of model to Poisson probabilities. The elapsed times \( t_L \) have been chosen to illustrate the general behaviour. Notice that in all cases except \( t_L = 3 \), the model probabilities are little different from Poisson in absolute terms.

For an elapsed time of about 3 days since an M 7 earthquake, probabilities are initially 24 times higher than Poisson. The ratio falls steadily until it is about 1 at inter-event times of about 1000 days. Then probabilities become, and stay, less than Poisson.

\( t_L = 100 \) days approximately marks a transition. For longer elapsed times, the probability ratio is less than Poisson for all inter-event times i.e. the probability of the next earthquake occurring within any specified time is lower than for Poisson. In Fig 16-4, where \( t_L \) is about 300 days, probabilities are only 80-90% of Poisson until inter-event times of about 5000 days.

This behaviour can be summed up as follows: Following a large (M 7) earthquake, the probability of getting another large earthquake is very much higher than Poisson for short inter-event times. As time passes without an earthquake, probabilities fall, until after about 100 days without a second large event, probabilities become (slightly) less than the Poisson probabilities. This reproduces what we see in the data (Table 1): there are rather more of both short and long inter-event times than the Poisson model would predict.

6. CONCLUSIONS

1. We have demonstrated, using a New Zealand example, the potential utility of a new model formulation for the times between earthquakes in a specified region. The model allows, probabilistically, that earthquakes following an initial earthquake are a mixture of 'aftershocks' and 'new events'. The model fits well a global dataset of \( M \geq 6.5 \) earthquakes since 1985. The formulation allows the probability distribution of times to the next event to be calculated after an arbitrary time has elapsed since the initial event. Although the differences in probability from the standard Poisson model are small, they could be significant for insurance assessments and premium setting.

2. The attempt at modelling New Zealand \( M \geq 7 \) earthquakes since 1840 suggests either large New Zealand earthquakes have fewer aftershocks (in our generalised sense of 'aftershock') than earthquakes globally or that some \( M \geq 7 \) earthquakes at short time intervals after the mainshock have been missed from the catalogue.

3. The development of a useful model for New Zealand, or any region for that matter, requires a realistic model for the spatial effect of earthquakes of different magnitudes. Doubt about the possible omission of short inter-event times from the \( M \geq 7 \), 1840-present catalogue suggests
using a different dataset for the model e.g. M ≥ 6 events since 1940, for which the catalogue is thought to be substantially complete (but we note Kagan’s, 2004, general conclusions about catalogue incompleteness at short inter-event times).

Figure 16-1: Comparison of synthetic model and Poisson probability distributions for inter-event times, given an elapsed time of t_L = 3 days since the last M ≥ 7 event. a: model distribution (solid) and Poisson distribution (dashed). b: ratio of model to Poisson probabilities.

Figure 16-2: As for Fig. 16-1 except that the elapsed time t_L = 100 days.

Figure 16-3: As for Fig. 16-1 except that the elapsed time t_L = 316 days.
However, our 'supercluster' assumption is poor for a magnitude range 6 to > 8, and so proper modelling of the spatial extent of the effect of an earthquake of any magnitude is required. Since we now have a good model for the temporal behaviour, we expect that modelling the spatial effect, and then the increase in spatial effect with time, will be straightforward.

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