Capacity spectra for bi-linear models and a procedure for constructing capacity design spectra

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ABSTRACT: A very useful tool for the preliminary design of structures is the elastic capacity spectrum. A pseudo-acceleration relationship has to be assumed for constructing a capacity spectrum and this assumption results in large errors for long-period structures with large damping ratios. In the present study, capacity spectra for bi-linear models are presented. Pseudo-acceleration is also assumed which results in acceptably small errors when a viscous damping ratio of 5% proportional to the tangent stiffness of a single degree freedom structure is used. For nonlinear structures with a small damping ratio and tangent stiffness, dependency of damping force could be acceptable because energy adsorption is primarily the result of structural nonlinear deformation. Capacity spectra of bi-linear models for a number of near-source records from large earthquakes and spectral ratios of two horizontal components are presented. A procedure for constructing capacity spectra of bi-linear models from an elastic 5% damped design spectra is also presented.

1 INTRODUCTION

The capacity spectrum is a very useful tool for the preliminary design of structures. For a given spectral period, design displacement and base shear coefficient (i.e., the spectral acceleration in fraction of $g$, acceleration of gravity) can be selected directly from the capacity spectrum (Freeman 1978). For a nonlinear structure, usually a substitute structure with an effective period calculated at the peak displacement, a total damping ratio (which is the sum of viscous damping ratio and the equivalent damping ratio derived from the energy dissipation due to the inelastic deformation of the structure), is used to calculate the displacement and base shear coefficient from an elastic spectrum (see Chopra 1995, and Chopra and Geol 1999).

Much research has been done in constructing inelastic spectra from 5% damped elastic design spectra (Berrill et al 1980, Newmark and Hall 1982, and Krawinkler and Nassar 1992). The commonly used method is to derive a strength reduction factor $R_\mu$, the ratio of the peak force for a single degree of freedom (SDF) structure responding elastically (i.e., $\mu = 1$) to the yielding force for the same structure that has a ductility ratio $\mu > 1$ (maximum displacement divided by yield displacement), at a given elastic period. Once $R_\mu$ is available, inelastic design spectra can be constructed. Strength reduction factors $R_\mu$ can be derived from assumptions as to the relationships of peak elastic and inelastic response, or can be derived from the analyses of nonlinear response of inelastic SDF structures. See Miranda and Bertero (1994).

These methods suffer two drawbacks. The first drawback is the assumption as to relationships between pseudo-acceleration and the displacement spectrum. Pseudo-accelerations are very similar to the total acceleration when period and damping ratios are small to moderately large (period less than 3 s and damping ratio less than 20 %), but an error of 20-30 % may arise from the pseudo-acceleration assumption for long periods and large damping ratios. See Figure 1a. The second drawback is the use of a substituted elastic structure to approximate the response of a nonlinear structure. Such an approximation may produce considerable errors in calculated structural displacements.

In the present study, we present capacity spectra for bi-linear models for strong motion records and a possible procedure of constructing inelastic capacity spectra from elastic 5% damped design spectra. In this approach, errors associated with pseudo-acceleration spectra are minimized and the need for a substituted equivalent elastic structure is eliminated. This approach is also possible for elasto-plastic nonlinear structures.
2 CAPACITY SPECTRA FOR BI-LINEAR MODELS

For SDF structure under seismic excitation, the dynamic equilibrium equation and the pseudo acceleration are described, respectively, in Equations (1a) and (1b) as

\[ m \ddot{x} + c \dot{x} + F(x) = -m \ddot{u}_g \quad (1a) \quad SA = \left(\frac{2\pi}{T_e}\right)^2 SD \quad (1b) \]

where \( m \) is the mass of the structure, \( c \) is the damping coefficient, \( F(x) \) is the force developed in the structure, \( x \) is the displacement, and \( \ddot{u}_g \) is the ground acceleration. Spectral values are the peak values of displacement, velocity or total acceleration. The assumption for a capacity spectrum is defined in Equation (1b) where \( T_e \) is elastic spectral period in seconds and \( SD \) is spectral displacement. \( SA \) equals the base shear coefficient if \( SA \) is in fraction of \( g \) (acceleration of gravity).

For a nonlinear structure with any hysteretic load-displacement loops that have no strength degradation, it is still possible to select an appropriate spectral period so that the assumption of Equation 1b is still valid, provided that the damping force is relatively small. As spectra are defined as the peak values of a time history response, an effective period \( T_{eff} \) calculated from the secant stiffness \( K_{eff} \) at the peak response, is a convenient period for developing inelastic capacity spectra,

\[ SA_\mu = \left(\frac{2\pi}{T_{eff}}\right)^2 SD \quad 2(a) \quad T_{eff} = 2\pi \sqrt{\frac{W}{K_{eff} g}} \quad 2(b) \quad T_y = 2\pi \sqrt{\frac{W}{K_y g}} \quad 2(c) \quad T_e = 2\pi \sqrt{\frac{W}{K_e g}} \quad 2(d) \]

where \( SA_\mu \) is the pseudo-acceleration for a ductility ratio \( \mu \), \( W \) is the seismic weight of the structure, \( T_y \) is the post yield period, \( K_y = \beta K_e \) is the post-yield stiffness, and \( K_e \) is the elastic stiffness.

For a nonlinear model with a post-yield stiffness much smaller than the elastic stiffness, the peak total force (i.e., the damping force plus the force due to structural deformation) does not always occur at the peak displacement, and the peak total force may occur at the peak of the damping force. Even when the damping ratio (defined from elastic stiffness) is relatively small, such as 5 %, the peak total force can still occur at the peak damping force. The damping force is large compared with the yield force of the structure, although the velocity and the damping force is still relatively small at the peak displacement. The constant damping coefficient results in very large errors in the spectral acceleration calculated from Equation (2a). A minor modification of the damping mechanism can be used here so that the degree of approximation in Equation (2a) is acceptable. If the damping coefficient is calculated by \( c = 2\xi m \omega T \), with \( \omega_T \) calculated from the tangent stiffness of the structure, an acceptable accuracy to the spectral acceleration defined in Equation (2a) is achieved for the damping ratio \( \xi = 5 \% \). Figure 1(b) shows that the errors (i.e., the differences between the total acceleration from nonlinear structural response and the pseudo-acceleration calculated from Equation (2a) as a percentage of the total acceleration) are reasonably small. For a real nonlinear structure, most energy dissipation is through hysteretic deformation, and the assumption of a tangent stiffness dependent damping coefficient is probably valid.

For the design of nonlinear structures, response spectra are calculated for a given ductility ratio, defined as the ratio of spectral displacement over the yield displacement. For a bi-linear model with a post yield stiffness ratio \( \beta \) and an elastic period \( T_e \), the effective period \( T_{eff} \), equivalent damping ratio \( \xi_\mu \), response reduction factor \( R_D \) and yield acceleration \( SA_Y \) can be calculated from Equations 3(a) to 3(d)

\[ T_{eff} = T_e \left(\frac{\mu}{1 + (\mu - 1)\beta}\right)^{\frac{1}{2}} \quad 3(a) \quad \xi_\mu = \frac{2 \beta - 1}{\mu - 1 + (\mu - 1)\beta} \quad 3(b) \quad R_D = \frac{SA_P}{SA_\mu} \quad 3(c) \quad SA_Y = \left(\frac{2\pi}{T_e}\right)^2 SD \mu \quad 3(d) \]

Note that the elastic spectral acceleration \( SA_P \) is the elastic spectral acceleration at an effective period \( T_{eff} \), and the spectral acceleration \( SA_\mu \) for a given ductility ratio \( \mu \) is also selected at the effective period, instead of at the elastic period, so that the first-order effect of period lengthening due to yielding can be accounted for. The damping ratio is not required in the computation of inelastic spectra.

In most current design practice, acceleration spectra are given as a function of elastic period. For
inelastic response at a given ductility ratio, the load reduction factor $R_\mu = S\mu - \mu S\chi$ is also given as a function of elastic period and ductility ratio. However, for a structure with a small post-yield stiffness ratio $\beta$, the response of the structure changes between the elastic phase and the post-yield phase, and the response may be dominated by the post-yield phase when the ductility ratio is large. In such cases, effective period, instead of elastic period, is a superior parameter for the actual response of the structure.

3 CAPACITY SPECTRA FOR NEAR-FIELD RECORDS

The capacity spectra of the Lucerne record with nearly full forward directivity effects are presented in Figure 2. Note that total acceleration instead of pseudo-acceleration is presented and the straight lines labelled with periods are calculated from Equation (2a) using the calculated peak displacement of bilinear models. The Lucerne station is on the northern end of a large surface rupture trace and this record is from an $M_w = 7.2$ strike-slip event at a distance of 1.5 km to the fault rupture plane. For both components, spectral acceleration decreases with increasing spectral displacement. At a short period of 0.2 s, the spectral accelerations are between 0.5-0.6 g for $\mu = 10$, reducing from about 1.5 g in elastic spectra. Up to 0.9 s, the spectral values in the two horizontal components are reasonably similar. At periods beyond about 1 s, the spectral values in the fault-normal direction are much larger than those in the fault-parallel direction, presumably due to forward directivity effect. At the effective period of 9 s, the spectral acceleration for $\mu = 10$ is about 0.1 g for the fault-normal direction and is only 0.05 g in the fault-parallel direction. Note that the amplitudes of the capacity spectra for a large ductility ratio are not always smaller than those of the elastic case or a smaller ductility ratio, especially at long periods, suggesting a complex interaction of period shift and energy dissipation due to the yielding of the structure.

Note that solid lines between spectra of different ductility ratios for a given spectral period are also plotted in the capacity spectra. For short periods, these solid lines are almost identical to the straight lines with periods labelled, suggesting good approximation of Equation (2a). The accuracy at long periods is still good. This feature is very useful when capacity spectra from different earthquake records are compared in the same plot.

The 1994 Northridge earthquake has a reverse faulting mechanism, and the record from the Rinaldi site shows strong forward directivity effects. The capacity spectra in Figures 3a show that the spectral accelerations of the fault-normal component for $\mu \geq 5$ are nearly constant at 0.8 g in a period range of 0.2-1.5 s. Spectral displacements of the fault-normal component vary from 0.3-0.7 m in an effective period range of 3-9 s. In the fault-parallel direction, spectral values gradually decrease with increasing effective spectral periods. At periods between 2.5 and 3 s, the elastic spectra of the fault-normal and fault-parallel components are very similar, but for $\mu = 10$ the spectral values of the fault-parallel component are considerably smaller than those of the fault-normal component. Note that in the long period end, spectral values for $\mu \geq 2$ are actually larger than those of the elastic case, suggesting a complicated interaction between period change and energy dissipation.

A number of near-source records from the 1999 Chichi earthquake show large permanent displacement which is often referred to as fault fling. These records were processed according to the method by Boore (2001) so that permanent displacement is preserved. The permanent displacements obtained are generally consistent with those derived from GPS data (Boore, 2001). We processed the records in the EW and NS components first and then rotated to a direction such that one component had zero permanent displacement (which is referred to as the minimum-fling direction) while the other perpendicular direction is referred to as the maximum-fling direction.

Figure 4 shows the capacity spectra of the TCU 068 record from the 1999 Chichi earthquake. This site is at the northern end of the surface rupture trace and the permanent displacement is over 9 m (Boore 2001). The peak acceleration is 0.67 g for the maximum-fling component and 0.6 g for the minimum-fling component. For large ductility ratios ($\mu \geq 5$), the spectral values of the two components are similar at short effective periods, and the spectral values of the maximum-fling component are slightly smaller than those of the minimum-fling component in an effective period range of 0.3-1.5 s. Beyond a 1.5 s period, spectral accelerations of the minimum-fling component decrease quickly with increasing period, while the maximum-fling component shows a less rapid
decrease. At the long period end, the displacement demand of the maximum-fling component is extremely large (nearly 5 m for $\mu = 10$) compared with about 2 m for the minimum-fling component.

Figure 5a shows the spectral ratio between fault-normal and fault-parallel components of the Lucerne record. The peak ratios appear to shift towards longer periods with increasing ductility, and the peak values for $\mu = 2$ and $\mu = 3$ are slightly larger than that of the elastic case. Note that the first-order effect of period shift has been accounted for by presenting the spectral ratio as a function of effective period. Figure 5b shows the spectral ratio between fault-normal and fault-parallel components of the Rinaldi record. The dominant peak ratio for the inelastic spectra of the Rinaldi record appears to shift to about 1.2 s from 0.8 s in the elastic case. Note that, for $\mu = 10$, the spectra ratios at the short period end and a period range of 2-6 s are considerably larger than for the elastic case. These differences in the inelastic responses to the fault-normal and parallel components, which are significantly large to affect the response of structures, would not be accounted for when the design process is based on the elastic spectra only.

4 A POSSIBLE PROCEDURE FOR CONSTRUCTION OF CAPACITY SPECTRUM FROM SMOOTHED CODE DESIGN SPECTRA

In order to construct an inelastic spectrum from a 5% damped elastic design spectrum, a strength reduction factor $R_\mu$ or response reduction factor $R_D$ (see Figure 6a) is required. $R_P$, the ratio of elastic spectral acceleration $SA_{\mu}$ at the elastic period to the elastic spectral acceleration $SA_{\text{eff}}$ at an effective period, can be considered to be a result of period shift, and is therefore governed to a large extent by the elastic spectral shape. The response reduction factor is given by $R_D = \frac{SA_{\text{eff}}}{SA_{\mu}}$, where $SA_{\mu}$ is the intercept of the post-yield load path and the inelastic spectra for a given ductility. It is primarily governed by energy dissipation due to inelastic deformation. Because the strength reduction factor $R_\mu$ and the total response reduction factor $R_T$ depend on the spectral shape as well as energy dissipation, their estimates derived from earthquake records may not be appropriate for smoothed design spectra.

The approach suggested here is to select and modify a suite of earthquake records that will produce response spectra which are closely matched to the targeted elastic design spectra. These records are used to compute $R_D$ for different ductility ratios.

As an illustration, we constructed a smoothed design spectrum which has a constant spectral acceleration between 0.2-0.5 s period, constant spectral velocity between periods of 1-4 s, and constant spectral displacement beyond a 5 s period. Peak ground acceleration is assumed to be the peak spectral acceleration divided by 2.5, and a straight line is used for linear interpolation between 0-0.2 s period range. Straight line interpolation in log-log scale was used between the spectral values that were specified. The spectra have very large displacement demand in intermediate and long periods. This would give large period ranges of constant acceleration, constant velocity and constant displacement portions and two transition portions. Two types of near-source records were chosen - records from strike-slip faults with nearly full forward directivity effect in the fault-normal component, and records with a large fault fling. The design spectra in the 0.03-15 s period range were matched by using the response spectral ratio of the design spectra to the spectra of a record to modify the Fourier spectra of the record in an iterative manner. Filters were applied to eliminate the ground motions beyond a 15 s period. The spectra of the modified ground motion are usually very close to the target design spectra, as shown in Figure 6b.

Figure 7(a) shows the reduction factor $R_D$ for $\mu = 10$ calculated from near-source records from strike-slip events. The trend of the data suggests that linear interpolation in a log-log scale between the periods at which the design spectra change period dependency would be a reasonable fit to the computed data. The mean value of the reduction factor for the fault-parallel components was generally slightly higher than that for the fault-normal component for periods up to 2 s for $\mu = 10$. It appears possible that by using one component, either fault-normal or fault-parallel, depending on the nature of the targeted spectra, the variability of the fitted model could be reduced so as to obtain a robust model.

Figure 7(b) shows the reduction factor derived from the records with fault fling effects. The dominant fault-fling effect appears in the fault-parallel component for the YPT record from the Turkey 1999 Izmit earthquake. The mean values from maximum-fling components are considerably larger than the
mean values from the minimum-fling components in the 0.5–1.5 s, and 3-5 s period bands, while they are similar for the other periods. The reduction factor of the YPT record in the maximum-fling direction is much larger than that in the minimum-fling direction for periods up to 1.5 s. For most period bands, the reduction factor from the maximum-fling direction of the Chichi records is also generally smaller than that of the minimum-fling direction. It is clear that maximum-fling components should be used to construct inelastic design spectra that are intended for ground motions with a large fault fling, so as to reduce the scatter of the derived reduction factor.

Figure 8(a) shows the capacity spectra calculated for the fault-normal components of records with nearly full forward directivity effect from strike-slip events, including Arrays 6, 7 and 8 from the 1979 Imperial Valley event, Kobe University record in the 1995 Kobe event, and Lucerne record from the 1992 Landers earthquake. Figure 8(b) shows the capacity spectra constructed from the maximum-fling components of records from stations TCU 052, 065 and 068 from the 1999 Chichi earthquake and YPT and SKR sites from the 1999 Izmit Turkey earthquake. A restriction of \( RD = 1 \) if \( RD < 1 \) was imposed for establishing an inelastic capacity design spectra, even though \( RD < 1 \) is possible for unmodified records (see Figures 3(b) where elastic spectral values are smaller than those for \( \mu > 1 \) at long period). The standard error of the response reduction factor derived from the maximum-fling components is considerably reduced compared with that using both maximum and minimum-fling components. At long periods and large ductility ratios, the capacity spectra derived from records with forward directivity and fault fling pulses are considerably different.

5 CONCLUSIONS

The following conclusions can be reached from the present study:

1) When the damping force is assumed to be dependent on the tangent stiffness of a single degree of freedom structure with nonlinear hysteresis force-displacement behaviour, then the capacity spectra provide a simple design tool with acceptable accuracy for a 5 % damping ratio. The effective period is used to account for the first-order effect of period shift due to inelastic deformation,

2) Capacity spectra for near-source records from strike-slip and dip-slip events are presented for components with forward directivity effect or maximum-fling effect and the other components without these effects. The peak spectral ratio between the spectra in the fault-normal and fault-parallel directions appears to shift towards a longer effective period as the ductility ratio increases, even though the first-order effect of period lengthening due to inelastic structural deformation is accounted for by using the effective period,

3) In order to construct inelastic capacity spectra, we defined a response reduction factor, the ratio of elastic spectral acceleration and inelastic spectral acceleration at the effective period for a given ductility ratio. Different ground motion records modified to match a smoothed and 5 % damped elastic design spectrum produced different values of response reduction factor for a given ductility ratio, and a piecewise-linear interpolation on logarithm scales was used to fit the computed response reduction factor from different records. Consistent types of strong motion that a design spectrum is intended to represent (for example, ground motions with forward directivity or fault fling) should be used in deriving the response reduction factor, so as to reduce the variability of the derived model, and

4) 5 % damped inelastic capacity design spectra derived from different types of ground motions can be significantly different at long period and large ductility ratios.
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Figure 1  Errors arising from the assumption of pseudo-acceleration spectra (a) for elastic spectra, (b) for inelastic spectra with a tangent-stiffness-dependent damping ratio of 5 %. Note that at long periods and large damping ratios the error from the elastic pseudo-acceleration assumption could be up to 25 % for this particular record.
Figure 2 Capacity spectra of the Lucerne record from the 1992 Landers earthquake, (a) fault-normal component, and (b) fault-parallel component. Note the forward directivity effect in the fault-normal component at periods beyond 3 s. The thin sloping parallel lines labelled with effective periods are computed from Equation 2a and the total acceleration from nonlinear analyses is presented as the spectral acceleration.

Figure 3 Capacity spectra of the Rinaldi record from the 1994 Northridge earthquake, (a) fault-normal component; and (b) fault-parallel component. The spectral values of the fault-parallel component are generally much smaller than those of the fault-normal component. Note that, at 2.5-3.0 s effective periods, even though the elastic spectral values of the fault-parallel component are close to those of the fault-normal component, the spectral values for $\mu = 10$ of the fault-parallel component are considerably smaller than those of the fault-normal component.

Figure 4 Capacity spectra of the TCU 068 record from the 1999 Chichi Taiwan earthquake, (a) maximum-fling component; and (b) minimum-fling component. Note the extremely large displacement demand of the maximum-fling component at effective periods beyond 3 s.
Figure 5 Spectral ratio of the fault-normal and fault-parallel components for records with forward directivity pulse from strike-slip events: (a) Lucerne record from the 1992 Landers earthquake, and (b) Rinaldi record from the 1994 Northridge earthquake. Note the apparent shift of peak ratios to longer effective periods with increasing ductility ratio for the Lucerne record.

Figure 6 Illustration for strength reduction factor $R_{\mu}$, reduction factor $R_P$ due to period lengthening, total reduction factor $R_T$ and response reduction factor $R_D$ (which is primarily the effect of energy dissipation) in (a); and the target elastic design spectra of 5 % damped, and the close match to the target spectra by the spectra from the modified strong motion in (b).

Figure 7 Computed $R_D$ for $\mu = 10$ (a) from the modified strong motion records with forward directivity pulse in the fault-normal component, together with the fitted piecewise-linear (on logarithm scales for both the response reduction factor and effective periods) function of effective period; and (b) from the modified strong motion records with fault-fling pulse.
Figure 8  Inelastic capacity spectra constructed from the target design spectra and response reduction factor from (a) fault-normal component of near source records with forward directivity, and (b) maximum-fling component of records from the 1999 Chichi Taiwan and the 1999 Izmit Turkey earthquakes.