Direct displacement based design – a definition of damping

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ABSTRACT.
The response of a range of structures with different hysteretic rules was investigated for a variety of strong earthquake ground motions. It is shown that the use of substitute viscous damping, instead of equivalent viscous damping, with the direct displacement based design approach, improves the accuracy and generality of the method, and gives predictions which are marginally better than those obtained with force based design. Analyses of a wide variety of earthquake records show that the values of substitute viscous damping are relatively insensitive to the type of earthquake but they vary with the structural period, ductility level and hysteretic form.

1 INTRODUCTION

It is widely accepted that the maximum deformation imposed on a structure is an important indication of the level of damage that is sustained. As deformation is closely related to displacement, design methods seek to limit the maximum lateral displacement resulting from design level earthquakes.

The way in which the design displacement limit, generally expressed as a maximum inter-storey deflection, is considered in design is radically different with force based and displacement based design methods. With force based design (FBD) a ductility level is selected, design strengths are determined and the corresponding lateral deflections are found. With displacement based design the designer starts with a limiting displacement and determines the design forces and ductility level corresponding to that value. In both cases a trial and error approach is required. There are several methods of displacement based design that have been proposed in the literature. In this paper the "Direct Displacement Based Design" (DDBD) method as proposed by Kowalsky et al. (1994, 1995) is the main focus of the discussion.

2 BASIC APPROACH FOR FORCE BASED AND DISPLACEMENT BASED DESIGN

The fundamental generic steps involved in force-based and displacement-based design are briefly outlined below. More comprehensive descriptions of the different approaches may be found in Judi et al. (1998, 2000), Kowalsky et al. (1994, 1995) and Priestley & Kowalsky (2000). It should be noted that in practice some of the steps are simplified in codes of practice. In particular the variation of stiffness of members with reinforcement content is generally ignored in practice.

2.1 *Force Based Design*

A ductility level, which is appropriate to the form of structure, is selected together with a limiting lateral displacement. Trial member sizes of structural steel members, or for reinforced concrete section sizes and reinforcement contents, allow the initial stiffness values of members prior to yielding to be assessed. Using these an elastic analysis is made to find the design forces. In this analysis a level of damping appropriate to the elastically responding structure (typically 5%) is used, together with a design acceleration response spectrum for the chosen ductility level and hysteretic form. Generally codes of practice neglect the influence of hysteretic form on the design spectrum and only one, which is usually based on bilinear response, is given. A logical development of the approach would give response spectra for different hysteretic forms. In some codes the elastic response spectrum values are reduced to allow for ductility while in others the elastic spectrum is used. Where the second option is followed the design actions are found by dividing the analysis values to allow for the design ductility level. The assumed stiffness values are checked against member sizes required to sustain the actions found in the analysis and the process is repeated with revised stiffness values if necessary. The ultimate lateral deflections are assessed using a multiplier to increase the values found in the elastic analysis to allow for the expected inelastic deformation, which relates the elastic analysis deflections to the ultimate values. This multiplier, which is generally embedded in a table in the design code, depends on the initial elastic period of the structure and the design ductility level. Its value is derived using the “equal energy” and “equal displacement” concepts. The process is repeated until acceptable convergence is obtained between the resultant ultimate displacements and the predicted values.

2.2 *Displacement Based Design*

A limiting design displacement is selected. On the basis of assumed structural steel member size, or for reinforced concrete the section dimensions and reinforcement contents, the deflection that can be sustained at the effective elastic limit is assessed. This is the ductility 1 displacement. Dividing the design displacement limit by the ductility 1 displacement gives an estimate of the ductility, which can now be used to find an equivalent viscous damping for an associated elastically responding model. This model has its stiffness value based on the secant stiffness between zero and maximum displacement as illustrated in Figure 1. From a set of design displacement response spectra, constructed for different viscous damping values, the fundamental period of the model is found. This value leads to the model stiffness and hence to the design strength, as illustrated in Figure 1. The design strength is now checked against the initial assumed sizes and ductility 1 displacement. The process is repeated until satisfactory convergence is achieved. For multi-degree of freedom structures some assumption has to be made to distribute the strength into the structure.

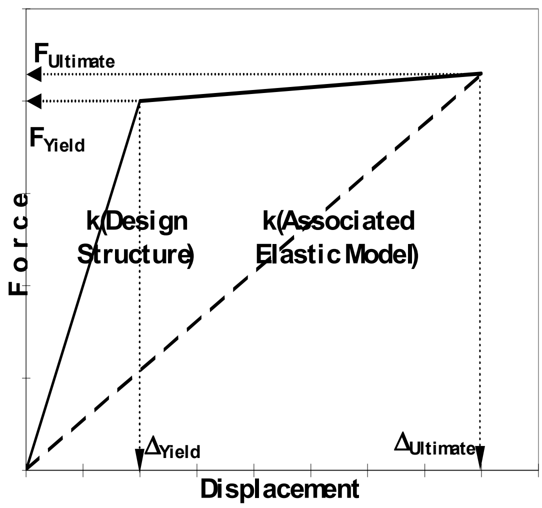


Figure 1 – The Design Structure & Associated Elastic Model in DDBD

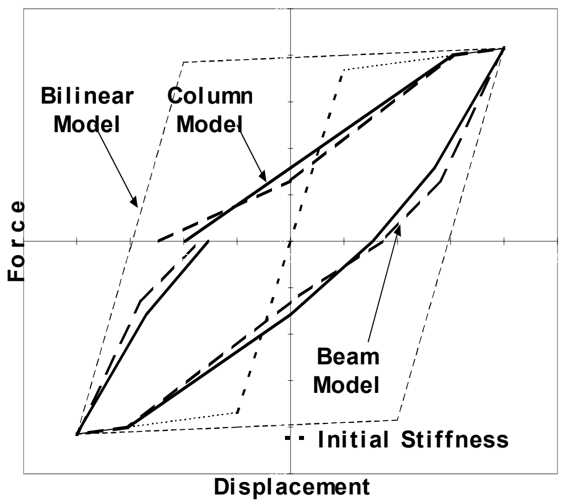


Figure 2 – The Hysteretic Models used in the Study

The force based design method is based on the equal displacement and equal energy concepts. With this approach the influence of different hysteretic models on response can be recognised by the use of different ductility acceleration response spectra for the different hysteretic forms, even though this is not done at present. Displacement based approaches are established on the assumption that a hysteretically responding design structure sustains the same displacement as an associated elastically responding model with an appropriate level of viscous damping. As noted above the associated elastic model has its stiffness value based on secant stiffness for the position of maximum displacement. A basic key to the approach is to find the appropriate damping value for this method. If this can be found differences in hysteretic form can be incorporated in the analysis by changing the level of viscous damping.

3 BACKGROUND TO DAMPING FOR ASSOCIATED ELASTIC MODEL IN DISPLACEMENT BASED DESIGN

Two different concepts have been proposed for finding the required level of viscous damping for the associated elastic model, namely “equivalent viscous damping” and “substitute viscous damping”. Equivalent viscous damping value is derived by equating the energy dissipated in one cycle by the oscillator, which is displaced under steady state conditions between + and – its maximum displacement, to the viscous energy dissipated by the associated elastic model undergoing the same displacement. Substitute viscous damping is derived by equating the total energy dissipated by the hysteretic oscillator, when it is subject to a strong earthquake motion, to the corresponding energy dissipated due to viscous damping in the associated elastic model.

Equivalent viscous damping is based on a concept first proposed by Jacobsen (1930). He proposed that the maximum displacement of an oscillator with complex damping mechanisms, when subjected to steady state vibratory motion, could be found from an analysis of a viscously damped associated elastic model. The complex damping mechanism in the oscillator could include hysteretic behaviour due to yielding. He proposed that the appropriate level of viscous damping for the associated elastic model could be found by equating the energy dissipated by the oscillator to that dissipated by the associated elastic model.

Jacobsen found that his theory, when implemented for mechanical systems under forced steady state vibration, was in close agreement with exact solutions for systems with relatively low levels of non-linearity. For the non-regular vibratory motion Jacobsen suggested that a time average damping would be more representative than the equivalent viscous damping, but that the equivalent viscous damping is more convenient to use

Gulkan & Sozen (1974) carried out dynamic tests on a series of reinforced concrete frames. They found that the substitute viscous damping value found in their tests, could be assessed with sufficient accuracy for seismic design purposes, by the equivalent viscous damping concept presented by Jacobsen. However, this deduction is only valid for structures that behave in a similar manner to their test specimens. These formed reversing plastic hinge zones, and as such they exhibited stiffness degrading hysteretic behaviour typical of reversing reinforced concrete plastic hinges.

Priestley & Kowalsky (2000) show that the equivalent viscous damping level varies with the hysteretic model. This is a convenient feature as it enables the DDBD method to accommodate a wide range of hysteretic behaviour modes. Moreover, Kowalsky et al. (1995) indicated that the equivalent viscous damping concept, by virtue of its geometric background, is a function of the ductility level anticipated for the design structure and it is independent of the structural period. Hence equivalent viscous damping was adopted into the DDBD method.

In some papers upper empirical limits were proposed for the equivalent viscous damping with these values depending on the hysteretic form. In particular Loeding, Kowalsky & Priestley (1998) recommended that empirical maximum values, which are significantly lower than the maximum analytical equivalent viscous damping values, be used. However, in other papers these upper limits do not appear to be used (Priestley & Kowalsky 2000).

Judi et al. (1998) found that the equivalent viscous damping approach resulted in non-conservative designs for the case of bilinear behaviour. The large area enclosed in the bilinear hysteretic loop to the maximum inelastic excursion gives a high level of equivalent viscous damping (~60% at a ductility of 6). With this value the direct displacement based design gives much lower strength levels than time history analyses indicate are required.

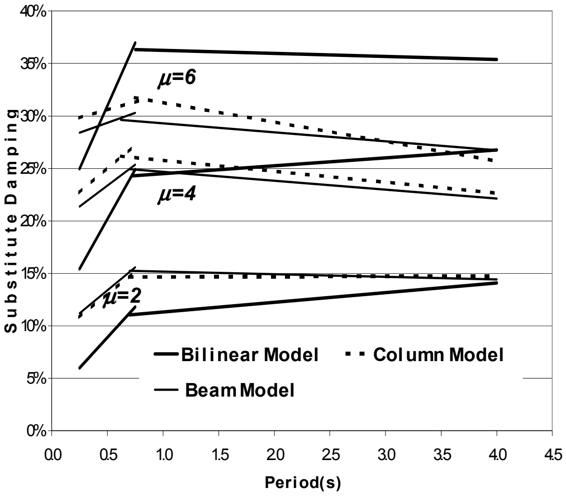
Judi et al. (2000) investigated the background of the substitute viscous damping, which Jacobsen referred to as 'time average damping'. They proposed that though it is more convenient to calculate the equivalent viscous damping, the use of substitute viscous damping is more logical. By carrying out a number of designs for single degree of freedom structures with different hysteretic relationships and evaluating these with time history analyses, they noted that this approach appeared to give more consistent results than designs based on the equivalent viscous damping concept. Based on a study of eight earthquake records they found that this concept is convenient for inclusion in DDBD design method. The justification for this is twofold. Firstly, it was found that the substitute viscous damping relationships are relatively insensitive to the earthquake record. Secondly, the substitute viscous damping concept presents a reasonable analytical representation of the design structure for the different hysteretic models, including the bilinear, and there is no need for artificial manipulations of the relationships as appears to be required with equivalent viscous damping. The substitute viscous damping was found to be dependent on the ductility level and to a lesser extent on the elastic period of the design structure. In the study reported in this paper use of equivalent and substitute viscous damping is examined in greater detail for a wider range of ground motions and hysteretic models than was the case in the previous paper.

4 DETERMINATION OF SUBSTITUTE VISCOUS DAMPING VALUES

In this investigation three different hysteretic models were used, namely a bi-linear model, a column model and a beam model. They are illustrated in Figure 2. Structural steel eccentrically braced frames and reinforced concrete frames that develop unidirectional plastic hinges have load deflection characteristics that approach those of the bilinear model. The column model is based on the response of a reinforced concrete column, which develops a reversing plastic hinge under cyclic loading. In this case both the loading and unloading stiffness values degrade, which gives it a greater tendency to self-centre that the bilinear model. The beam model is based on the response of a reversing plastic hinge in a reinforced concrete beam. It is similar to the column model except that, firstly the hysteretic loop becomes pinched in shape under repeated inelastic cyclic loading due to shear deformation in the plastic hinge zone, and secondly the unloading stiffness does not degrade to the same extent as occurs in the column model. The rules for the column and beam hysteretic models were developed from experimental results (Fenwick & Davidson 1994). It should be noted that both loading and unloading stiffness values degrade under steady state cyclic loading with these models in a similar way to that observed in many structural tests.

The substitute viscous damping values were assessed for 21 ground motions. The first group of four ground motions were the original records of Imperial Valley, El Centro NS Record 1940, Edgecombe Earthquake, Matahina Dam Base NS Record 1987, Hachinohe Earthquake, Tokachi Oki NS Record 1968, and Taft Earthquake, Kern County Record NE 1952 for soil sites. The second group of seismic events were artificial earthquakes that were produced by normalising the first family so that their elastic acceleration response spectra at 5% damping matched that of the Loadings Code NZS4203: 1992 for intermediate soils. The third group is that of ground motions that are characterised by long duration (>40sec) for soil sites. They are Hachinohe Earthquake, Tokachi Oki EW Record 1968, Olympia Earthquake, Seattle Army Base NW Record 1949, Michoacan Earthquake, Tacy EW Record 1985 and Chile Earthquake, Vina del Mar SW Record 1985. The last two groups of earthquakes investigated were those for near fault events both for soil and rock sites. The soil records are the Imperial Valley Earthquake, Meloland NS Record 1979, two records of Northridge Earthquake 1994, Sylmar NS and Rinaldi SW Records, Cape Mendocino Earthquake, Petrolia NS 1992 and Tabas Earthquake, Tabas NW Record 1978. The rock site events are Cape Mendocino Earthquake, Cape Mendocino NS Record 1992, Landers Earthquake, Lucerne EW Record 1992, Loma Prieta Earthquake, Los Gatos Presentation Centre NS Record 1989 and Kobe Earthquake, JMA EW Record 1995.

Figure 3 shows how the trend lines for substitute viscous damping values found from the first three groups of earthquakes change with hysteretic model, period and ductility level. The corresponding near fault values, for the last two groups, are shown in Figure 4. The reason for this separation is that due to the strong pulses in these records, design results with either FBD or DDBD with either damping approach, do not produce satisfactory convergence with time history analytical values. However, it can be seen that the substitute viscous damping values are similar to those of the other earthquake records.



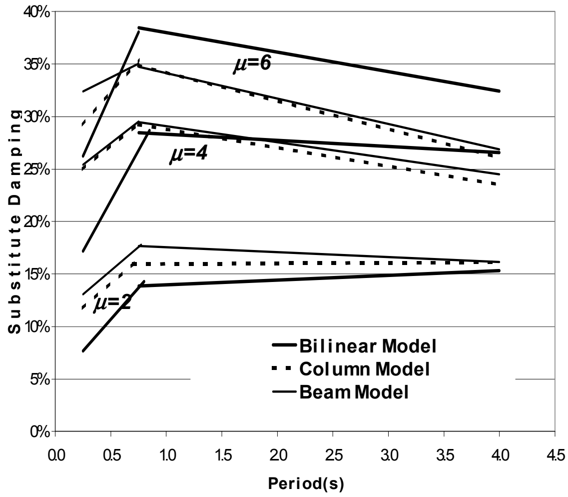


Figure 3 – Substitute Viscous Damping Variation (Excluding Near Fault)

Figure 4 – Substitute Viscous Damping Variation (Near Fault)

From Figure 3 it can be seen that there is little difference in the values of substitute viscous damping for the column and beam models. For low values of ductility ($\mu \leq 2$) the substitute viscous damping obtained with the bilinear model is less than the corresponding values for the column and beam models, while the reverse is true for higher ductility levels ($\mu = 6$). Another relevant observation that could be made from Figures 3 and 4 is that the substitute viscous damping values increase sharply with period in the period range of 0 to 0.7 seconds, while above this level the influence of period is relatively small. Table 1 contains a representative sample of the mean values and coefficients of variation for the substitute viscous damping values found from the analyses for the earthquake records in the first three groups. These values indicate the variation that was obtained with both the bilinear and column models.

Table 1: Substitute viscous damping values for bilinear and column models

Period (s)	$\mu = 2$				$\mu = 4$				$\mu = 6$			
	Bilinear		Column		Bilinear		Column		Bilinear		Column	
	Avg	CoV	Avg	CoV	Avg	CoV	Avg	CoV	Avg	CoV	Avg	CoV
0.25	5.7%	0.30	10.47%	0.15	14.4%	0.28	21.82%	0.25	23.3%	0.22	29.63%	0.16
0.50	9.5%	0.23	13.70%	0.16	22.2%	0.29	26.69%	0.08	34.3%	0.21	31.02%	0.08
0.75	11.5%	0.19	14.87%	0.14	23.9%	0.17	26.44%	0.10	35.3%	0.15	31.16%	0.11
1.00	11.7%	0.20	15.07%	0.11	24.7%	0.19	24.40%	0.12	36.3%	0.17	30.58%	0.14
1.00–4.00	12.7%	0.07	14.72%	0.06	25.5%	0.05	24.12%	0.05	35.9%	0.04	28.52%	0.09

Avg = Average, CoV = Coefficient of Variation

5 ANALYSIS AND DISCUSSION

Using the bilinear, elastic-perfectly-plastic model, a set of 14 structures was designed by FBD to the NZ Standard spectrum for intermediate soils for μ of 1, 2, 4 & 6. The elastic periods were 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.5, 2.0, 2.5, 3.0 & 4.0s. This gave a total of 14 design displacements and 56 designs.

The 14 design displacements found in this process were used with the DDBD method to determine the required strengths. In this process the yield displacements were varied so that ductility levels of 1, 2, 4 & 6 were achieved. As with the FBD there was a total of 56 designs. This process was carried out twice, firstly with the damping for the associated elastic model being based on equivalent damping values and secondly with substitute damping values.

The process described above was repeated with the bilinear model being replaced by the column model. Time history analyses were then made for all the 336 designed structures using the appropriate hysteretic model and the four earthquake ground motions (group 2) that were nomalised to the NZ Loadings Standard response spectrum. This gave a total of 1344 analyses. For each analysis the ratio of the maximum time history deflection to the design deflection was calculated. The results for all the analyses are shown for individual sets in Figures 5, 6, 7 & 8. One point stands out from these figures and that is that the use equivalent viscous damping with the bilinear model gives a very poor prediction of displacement particularly in the low deflection (period) range. However, with the column model it appears to give reasonable predictions.

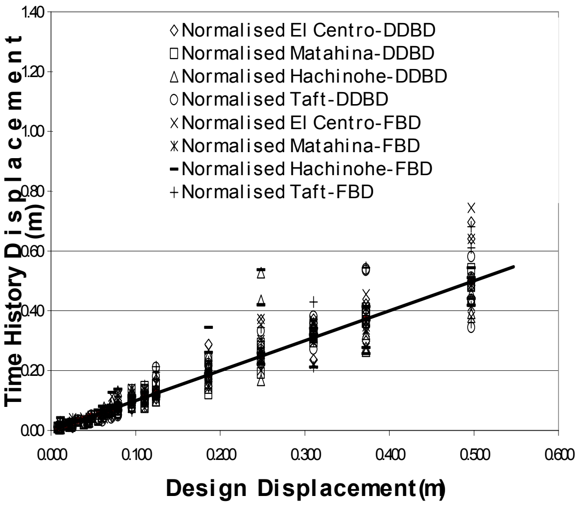


Figure 5 – T/H vs. Design Displacement Values (DDBD / Substitute – FBD) – Bilinear Model

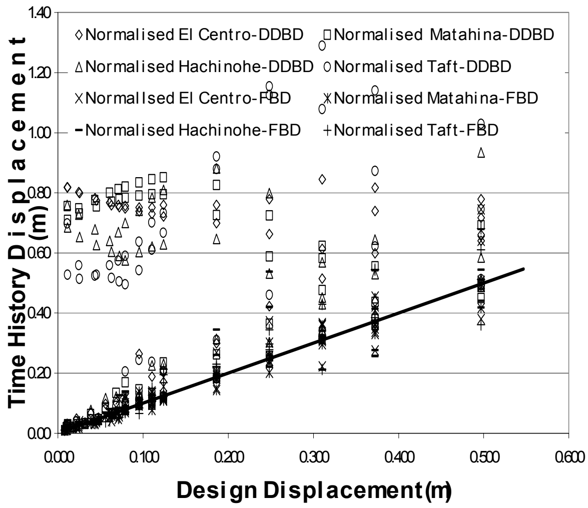


Figure 6 – T/H vs. Design Displacement Values (DDBD / Equivalent – FBD) Bilinear Model

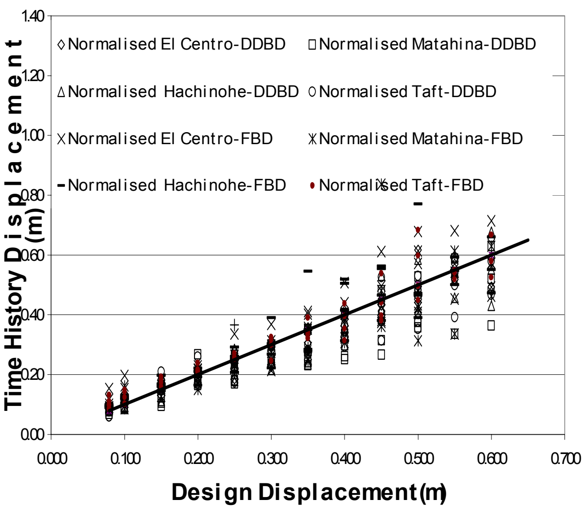


Figure 7 – T/H vs. Design Displacement Values (DDBD / Substitute – FBD) – Column Model

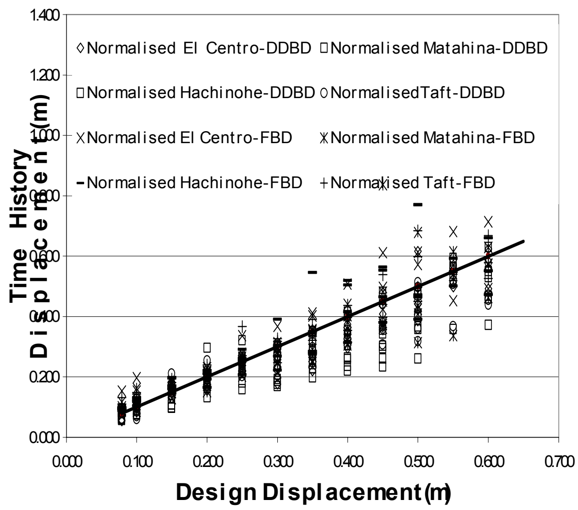


Figure 8 – T/H vs. Design Displacement Values (DDBD / Equivalent – FBD) Column Model

Table 2 summarises the results of all the analyses. From this it can be seen that direct displacement based design with substitute damping gives the best overall deflection predictions of the three methods. The use of equivalent viscous damping instead of substitute viscous damping reduces the accuracy of prediction. For the bilinear hysteretic response this substitution leads to inaccurate values and it clearly should be avoided, or, possibly as an alternatively empirical coefficients should be introduced to modify the damping values that are used. The margin in accuracy between force based design and direct displacement based design with substitute damping is not large, particularly for structures with periods greater than 0.7s.

Table 2: Average design results (T/H displacement/ design displacement ratio), for the normalised earthquake records

Hysteretic Model	Design Displacement (m)	DDBD - Substitute		DDBD - Equivalent		FBD	
		Avg	CoV	Avg	CoV	Avg	CoV
Bilinear	$\Delta \leq 0.079$	0.82	0.08	12.16	11.01	1.19	0.42
	$0.079 \leq \Delta \leq 0.500$	1.03	0.06	2.98	1.36	1.11	0.08
Column	$\Delta \leq 0.079$	0.99	0.04	0.91	0.02	1.31	0.14
	$0.079 \leq \Delta \leq 0.500$	0.91	0.08	0.84	0.09	1.06	0.12

6 CONCLUSIONS

1 Substitute viscous damping values have been derived from a range of different types of earthquake ground motions. The differences between these values are found to be relatively small. For design purposes it is possible to give values in tables or in graphical form.

2 Substitute damping values are found to depend strongly on ductility level. For period range between 0 and 0.7s there is sharp increase in the damping value with period. Above the 0.7s the variation depends on the ductility level. For a ductility of 2 the substitute damping value increases with period, for a ductility level of 4 there is little variation and for a ductility of 6 the substitute damping values decrease with period.

3 There was little difference between substitute damping values found with the column and beam hysteretic models. Both models represent stiffness degrading structures but the beam model has a pinched hysteretic typical of that found in structures which form reversing plastic hinge zones in reinforced concrete beams.

4 The comparison of the maximum displacements from time history analyses with design displacements obtained by the different methods indicates that

- displacement based design with substitute viscous damping gives the best deflection predictions
- use of equivalent viscous damping instead of substitute viscous damping with displacement based design reduces the accuracy and gives misleading values where the hysteretic response is bilinear or close to bilinear
- the difference in accuracy obtained by direct displacement based design with substitute viscous damping and force based design is not very significant.

5 The use of substitute viscous damping with displacement based design overall gives more consistent deflection predictions than the use of equivalent viscous damping. This substitution also avoids the need for the empirical manipulations of equivalent viscous damping values, as has been suggested by Loeding et al (1998).

7 ACKNOWLEDGEMENTS

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